Public Education Funding: Foundation System vs. Power-Equalizing System with Property Taxes*

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Abstract

The existing literature on public education finance suggests that a Power-Equalizing system would be chosen over a Foundation system under majority voting. Recently several states, however, have been implementing school finance reforms towards a Foundation system. This paper analyzes the welfare effects of such reforms by using a public education finance model including a housing market. Housing market decisions play an important role in public education finance as property tax revenues constitute a big part of public education expenditures. With the introduction of a housing market into the model, property tax rates are higher under a Power-Equalizing system compared to a Foundation system. This results in lower housing wealth of households in the Power-Equalizing system. Due to this housing market effect, it might be that a Foundation system would be chosen over a Power-Equalizing system even though the majority of school districts benefit from higher redistribution of funds under a Power-Equalizing system. The model suggests that a high preference for education in the utility function, lower mean income in a state, and lower income inequality in a state results in a Foundation system being chosen by a majority. I provide suggestive evidence supporting these theoretical results. By comparing the growth rate of per capita income, gini coeffient and per pupil spending in public schools of the states that changed their public education finance system with various reference groups, I find that these switches can be rationalized through the lens of my model.

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1 Introduction

Public education is considered to be one of the most important public policy areas in the U.S.. According to recent estimates from the National Association of State Budget Officers State Expenditure Report, educational expenditures constitute the largest budgetary category at the state level. For Fiscal Year 2013, around 20 percent of all state spending was devoted to elementary and secondary (K-12) education.

U.S. Public education was founded on the principles of local financing and control. There were over $110,000$ school districts in the country in the early 1900s, and funding was mostly supported by local property tax revenues. During the 1930s, a wave of school finance reform centralized the funding process. Since then, many states modified aid formulas to account for differences in property tax bases in the state in order to equalize per pupil spending across districts. These reforms are often mandated by court decisions. In this process of reforms, however, different states adopted different systems in equalizing spending per pupil across districts. Most states mandate a minimum level of spending per pupil by district which is called the Foundation amount. If districts cannot create enough revenue to meet this Foundation amount, the state remedies the difference using state aid. This guarantees that every child in the public education system receives a minimum level of education regardless of family income or the wealth level of the district. I refer to these as *Foundation* systems. Other states offer a guaranteed tax base to districts. This guaranteed tax-base ensures that two districts with the same property tax rates raise the same tax revenue regardless of differences in property values. State aid matches any difference between the actual property values in the district and the guaranteed tax-base. I refer to these as *Power-Equalizing* systems.

Fernandez and Rogerson (2003) argue that a Power-Equalizing system would be chosen over a Foundation system if they were subject to majority voting. This is due to higher redistribution of funds in the former. The Foundation system is currently the most-used however. Indeed, Pennsylvania, New York, Colorado, and Massachusetts have switched from a Power-Equalizing system to a Foundation system since 1990.

In this paper, I extend Fernandez and Rogerson (2003) by introducing a housing market to the model. Housing market decisions are crucial to public education finance with an average of 35 percent of local expenditures being funded through property tax revenues. Nearly 80 percent of district funding for public education is supported by these revenues. Accounting for changes in housing market decisions is important when we compare finance systems. From one system to another, property tax rates are different which distorts housing demand and property tax revenues differently. In fact, the model predicts that most districts, including the median voter, have higher property tax rates when subject to a Power-Equalizing compared to a Foundation system. As a result, a majority of districts face a higher gross price and a lower net of tax price for housing which results in lower housing wealth in a Power-Equalizing system. Even though the majority of districts benefit from more redistribution under a Power-Equalizing system, it is possible that this lower housing wealth makes a Foundation system more appealing for the majority. As such, a Foundation system might be preferred to a Power-Equalizing system when we introduce the housing market to the analysis.

Then, this model offers an explanation for the recent public education finance system switches towards Foundation. The explanation provided is as follows. The states that switched from a Power-Equalizing system to a Foundation system are such that the housing market effect is stronger than the redistribution effect. This is possible if a state has a stronger preference for education, or lower per capita income, or lower income inequality. In order to test this hypothesis, I analyze the recent switches into a Foundation system. By comparing the growth rates of per pupil spending, per capita income, and gini coefficient for these switcher states with different reference groups, I provide evidence that the states that switched are characteristically different than the states that did not switch as the model predicts.

Although there are many other systems that states use to decrease inequality in per pupil spending, the Foundation and Power-Equalizing systems are the most commonly used. According to the categorization of state aid formulas in Jackson et al. (2014), twelve states use a pure Foundation system and seven states use a pure Power-Equalizing system. In addition, there are 30 states that employ both of these systems to decrease intrastate spending inequality. These two systems constitute the core of public education funding.

Many researchers have examined state level public education funding policies and their economic effects. Among the theoretical papers, Fernandez and Rogerson (1998) compare the effects of a locally financed system and state-financed system on income distribution, intergenerational income mobility, and welfare. They compare two extreme public education finance systems; local finance system which has no mechanism to control for intrastate spending inequality and state finance system which fully equalizes spending across districts. While they compare those two systems, they include the effects coming from a housing market. In conclusion, systems decreasing spending inequality the most lead to higher average income, and higher education spending as a fraction of income. Steady-state welfare is higher under a more equal public education finance system. This paper, following Fernandez and Rogerson (2003), analyzes more commonly used and less extreme public education finance systems. Fernandez and Rogerson (2003), however, do not take the effects of different finance systems on housing markets into account. As described above, the latter is crucial to rationalize recent switches from Power-Equalizing to Foundation systems. In a similar spirit, Epple and Ferreyra (2008) examine general equilibrium effects of school funding reform of 1994 in Michigan. This reform has two components: property tax reduction and centralization of school funding at the state level, with increases for low-revenue districts and revenue caps for high-revenue districts. In their model, the main effect of the reform is the capitalization of lower property taxes and revenue changes with an increase in school quality in low-wealth districts. With a change in income distribution that favored low-income households between 1990 and 2000, their model predicts that observed housing appreciation can be decomposed into the capitalization of lower taxes and revenue changes, and an appreciation pattern related to changes in the income variance. They also present empirical evidence to support these predictions for the Detroit metropolitan area. Also, Ferreyra (2009) applies the model in Epple and Ferreyra (2008) to study the effects of school finance reform on the Detroit metropolitan area. She estimates a general equilibrium model of multiple jurisdictions with 1990 data from Detroit. She then validates the model by comparing model's predictions with 2000 data. According to counterfactual simulations using the estimates, she concludes that feasible revenue-based reforms that ensure spending equity or adequacy have little impact on school quality or household demographics in Detroit. In addition to the evidence presented in this paper, these empirical findings are consistent with the predictions of the current model in the sense that a Foundation system may socially be beneficial over a Power-Equalizing system after accounting for the effects coming from the housing market and property tax revenues. While these results are consistent with this paper, Epple and Ferreyra (2008) do not model the political economy behind the Michigan reform but rather describe its effects. This paper analyzes welfare gains and provides conditions under which a system would be more likely to be chosen by majority voting.

There are a few empirical papers that compare the effects of different public education systems. Evans et al. (1996) argue that court-ordered finance reforms over the last 40 years decreased spending inequality within states significantly. This decrease was a result of the increases in public education spending in poor districts being higher than the decreases in the rich districts. Thus, the recent changes in public education finance systems lead to a "leveling up" while decreasing the inequality of spending in public schools. Conversely, Hoxby (2001) argues that Power-Equalizing systems cause more "leveling down" compared to Foundation systems. Including the effects of changes in the housing market and property tax revenues, she concludes that Power-Equalizing decreases total resources devoted to public education. Card and Payne (2002) analyze the effects of school finance reform on student achievement. They show that reforms leading to lesser intrastate public school spending inequality narrow differences in SAT score outcomes across family backgrounds. In addition to the evidence presented in this paper, these empirical findings are consistent with the predictions of the current model. In the sense that a Foundation system may socially be beneficial over a Power-Equalizing system after accounting for the effects coming from the housing market and property tax revenues. In contrast, a Power-Equalizing system yields higher benefits for the majority without the housing market effects.

The rest of this paper is organized as follows. Section 2 presents the theoretical model and results. Section 3 provides empirical evidence to support the predictions of the model. Section 4 concludes.

2 Model Setup and Theoretical Results

In this section, I first introduce the basics of the model. Then, I solve the model for the Foundation and Power-Equalizing systems separately. Following that, I compare the solutions to both systems and present the main results of the model by first analyzing first the redistribution effect and then the housing market effect. In the last subsection, I provide comparative statics to guide the empirical analysis in Section 3.

2.1 The Basics

This economy consists of a finite number, N, of households. Each household is endowed with one child, and has preferences over private consumption goods c, housing services h, and the the education of the child q.

$$u(c, h) + v(q).$$

The function u and v are assumed to be strictly concave, increasing, and twice continuously differentiable. The function u is separable, increasing in both arguments and defines homothetic preferences over c and h. Specifically, I employ the following utility function:

$$u(c, h) + v(q) = \frac{a_c c^\alpha + (1 - a_c)h^\alpha}{\alpha} + A\frac{q^\gamma}{\gamma},$$

with $A > 0$, $\alpha < 0$, $\gamma < 0$, $0 < a_c < 1$.

Districts in a state are assumed to differ only in initial income endowments, y^j, having a cumulative distribution described by $F(y)$ with mean, y_μ, to be greater than median, y_M. This is a plausible yet an important assumption for the theoretical results presented in this paper. There are multiple i indexed districts, and the distribution of households into districts is exogenous and constant with the same number of households in each district. In this paper, I focus on the perfect income sorting of individuals into districts as in Fernandez and Rogerson (2003). Thus, every individual in a given district i has the same income, y_i, and each district has a representative household. So districts can be sorted by income as $y_1 < y_2 < y_3 < ... < y_N$. In addition, these districts are characterized by a proportional tax on housing expenditures, t_i, a net of tax housing price, p_i, and a quality of education, q_i, which the representative household takes as given. Tax revenues are used exclusively to fund local public education. All residents of a given district receive the same quality of education and education cannot be privately supplemented. How education is funded will depend on the state financing system. The next two subsections contain a detailed explanation for each system.

Each district has its own housing market, with the supply of housing in district i given by $H_i(p_i)$. $H_i(p_i)$ is assumed to be increasing, continuous, and equal to zero when the net of tax price, p_i, is zero. I use the following functional form of $H_i(p_i) = ap_i^b$ with $a > 0$, $0 < b \leq 1$ for all districts. The gross-of-tax housing price in i is given by $\pi_i = (1 + t_i)p_i$. Houses in each district are owned by the households in that district.

The interaction among districts in a state can be described as a three-stage game. In the first stage, households learn their district of residence, income distribution across districts, and the state education finance system. Given these, districts choose statewide policy variables through majority voting. The set of these state policy variables depends on the education finance system described below. In the second stage, districts choose property tax rates given the variables from the previous stage. In the last stage, households make housing and consumption choices and children receive education. Households know prices, tax rates, education spending and state's public education finance system at this stage. For any given state income distribution, and state finance system, we can solve the three-stage game by backward induction. Next, we solve the model separately for each finance system.

Later I ask which finance system would be chosen by majority voting which can be viewed as Stage 0 of this game.

2.2 Foundation System

In this system, districts are required to tax income at some minimum level, τ_f, in order to match state-mandated minimum per pupil spending. They are free to choose local property tax rates in order to increase per pupil education spending. So, we have per pupil spending in district i:

$$q_i = \tau_f y_\mu + t_i p_i h_i, \text{with } t_i \in [0, 1]$$

In Step 3, district i is characterized by a foundation income tax rate, τ_f, gross-of-tax housing price, and the quality of education, (π, q, τ_f). Given π, q and τ_f, a household with income y and housing wealth H solves the following problem:

$$\max_{h,c} u(c, h) + v(q)$$

$$\text{s.t. } c + \pi h = (1 - \tau_f)y + pH.$$

With separable and homothetic preferences, the solution to this problem is of the form:

$$c^* = \psi h^*, \psi = \frac{\left(\frac{1-a_c}{a_c}\right)^{\frac{1}{\alpha-1}}}{\pi^{\frac{1}{\alpha-1}}}, h^* = \frac{(1 - \tau_f)y + ap^{b+1}}{\psi + \pi}. \tag{1}$$

With the housing market clearing condition, we have $H(p) = h_i$.

Housing supply is increasing in p and zero for $p = 0$ while housing demand is decreasing in p and positive for $p = 0$. Thus, there is a unique price that clears the housing market for each district. Using the Envelope Theorem, we conclude that housing prices are decreasing in property taxes. To see this, the market clearing condition for the housing market is given by:

$$ap^b - \frac{(1-\tau_f)y + ap^{b+1}}{\psi(p,t) + \pi(p,t)} = 0.$$

By totally differentiating this condition, it is straightforward to show that

$$\frac{\partial p}{\partial t} < 0 \tag{2}$$

Also note here that

$$\frac{\partial c}{\partial t}, \frac{\partial h}{\partial t} < 0 \tag{3}$$

In Step 2, given τ_f and the solution to the above problem, agents maximize indirect utility by choosing a property tax rate, t_i:

$$\max_{0 \le t \le 1} u(c^*, h^*) + v(q)$$

$$\text{s.t. } q = \tau_f y_\mu + tph^*,$$

$$c^*, h^* \text{given by } (1).$$

The first-order condition for this problem is given by

$$\underbrace{\frac{\partial u}{\partial c}\frac{\partial c}{\partial t} + \frac{\partial u}{\partial h}\frac{\partial h}{\partial t}}_{-MC_F} + \underbrace{\frac{\partial v}{\partial q}\frac{\partial q}{\partial t}}_{MB_F} = 0 \tag{4}$$

and allows us to solve for the optimal property tax rate, t^*.

$\frac{\partial u}{\partial c}\frac{\partial c}{\partial t} + \frac{\partial u}{\partial h}\frac{\partial h}{\partial t}$ is negative because u is increasing in c and h and c and h are decreasing in t (see (3)). Also $\frac{\partial u}{\partial h}\frac{\partial h}{\partial t}$ is positive because v is increasing in q and q is increasing in t from the budget constraint. $-\left(\frac{\partial u}{\partial c}\frac{\partial c}{\partial t} + \frac{\partial u}{\partial h}\frac{\partial h}{\partial t}\right)$ is the marginal utility cost (MC_F) of increasing the property tax rate which decreases in t. On the other hand, $\frac{\partial v}{\partial q}\frac{\partial q}{\partial t}$ is the marginal utility benefit (MB_F) of increasing the property tax rate which also decreases in t. Because $u(c,h)$ and $v(q)$ are both concave we can show that for low values of t,

$MC_F < MB_F$; for high values of t, $MC_F > MB_F$. Hence, there exists a unique solution to (4). The second-order condition for this problem is given by:

$$\frac{\partial MB_F}{\partial t} - \frac{\partial MC_F}{\partial t} < 0 \tag{5}$$

and I refer to this condition in the proof of *Proposition 2*. Finally, the richer districts have a higher property tax rate, $\frac{\partial t}{\partial y} > 0$. This is used in the proof of *Proposition 1*.

In Step 1, districts decide on the Foundation amount which will be funded through a state income tax. Given the solutions to the previous steps:

$$\max_{0 < \leq \tau_f \leq 1} u(c^*, h^*) + v(q^*)$$

$$\text{s.t. } c^*, h^* \text{given by } (1) \text{ and } t^* \text{given by } (4).$$

The solution to this problem is as as follows. For $y > y_\mu$, districts would prefer no redistribution, with $\tau_f = 0$. This is because they are the ones supporting the system. Any positive level of redistribution would be a net loss. On the other hand, districts having $y < y_\mu$, would prefer positive redistribution. Because they are poor, they benefit from redistribution. As this tendency increases with income, $\frac{\partial \tau_f}{\partial y} > 0$, richer districts would prefer to increase spending, so their preferred foundation tax rate would increase up to the mean income. Overall, preferences for the foundation tax rate, τ_f, are single-peaked. Thus, the Median Voter Theorem applies for the solution of τ_f. Since those districts with income exceeding y_μ would vote with the lower part of the income distribution, the median voter for τ_f, M_f, has a lower income, y_{M_f}, than the median income, y_M, which is also lower than the mean income,y_μ, by assumption.

2.3 Power-Equalizing System

In a Power-Equalizing system, there is no minimum level of per pupil spending. Instead, a guaranteed tax base, z_R, enables poor districts to raise the same revenue as the rich districts when applying the same property tax rate, $\tilde{t}_i$. The difference between actual and guaranteed tax base is met by state aid to the districts. Revenues generated under this system are independent of the district tax base and given by $\tilde{q}_i = \tilde{t}_i z_R$.

The difference between aggregate expenditures on education and the amount raised by each district is assumed to be funded by a state-wide tax τ_R on income.

$$\tau_R \sum_i y_i = \sum_i \tilde{t}_i (z_R - \tilde{p}_i \tilde{h}_i),$$

with $\tau_R \geq 0$.

We can characterize the equilibrium by applying the same solution method used for the Foundation system. In Step 3, given $\tilde{\pi}$, $\tilde{q}$, z_R and τ_R a representative household with income y and a housing wealth $\tilde{H}$ solves the following problem:

$$\max_{\tilde{h},\tilde{c}} u(\tilde{c}, \tilde{h}) + v(q)$$

$$\text{s.t. } \tilde{c} + \tilde{\pi}\tilde{h} = (1 - \tau_R)y + \tilde{p}\tilde{H}.$$

Again, as a result of homothetic preferences, we have

$$\tilde{c}^* = \tilde{\psi}\tilde{h}^*, \tilde{\psi} = \frac{\left(\frac{1-a_c}{a_c}\right)^{\frac{1}{\alpha-1}}}{\tilde{\pi}^{\frac{1}{\alpha-1}}}, \tilde{h}^* = \frac{(1-\tau_R)y + a\tilde{p}^{b+1}}{\tilde{\psi} + \tilde{\pi}} \tag{6}$$

As in the Foundation system, we solve for $\tilde{p}_i$ using the housing market clearing condition. It is decreasing in the property tax rate, $\frac{\tilde{\partial} p}{\tilde{\partial} t} < 0$. Also, if $\tau_f = \tau_R$ and $(\pi, q) = (\tilde{\pi}, \tilde{q})$ then $(c^*, h^*) = (\tilde{c}^*, \tilde{h}^*)$. Hence the results we have for Step 3 from the Foundation system apply here. In particular,

$$\frac{\partial \tilde{p}}{\partial \tilde{t}} < 0, \tag{7}$$

$$\frac{\partial \tilde{c}}{\partial \tilde{t}}, \frac{\partial \tilde{h}}{\partial \tilde{t}} < 0. \tag{8}$$

In Step 2, districts maximize their indirect utility by choosing a property tax rate given a guaranteed tax base, z_R, and the solution to the above problem.

$$\max_{0 \leq \tilde{t} \leq 1} u(\tilde{c}^*, \tilde{h}^*) + v(\tilde{q})$$

$$\text{s.t. } \tilde{q} = \tilde{t}_i z_R,$$

$$\tilde{c}^*, \tilde{h}^* \text{given by } (6).$$

The first-order condition for this problem is given by:

$$\underbrace{\frac{\partial u}{\partial \tilde{c}} \frac{\partial \tilde{c}}{\partial \tilde{t}} + \frac{\partial u}{\partial \tilde{h}} \frac{\partial \tilde{h}}{\partial \tilde{t}}}_{-MC_{PE}} + \underbrace{\frac{\partial v}{\partial \tilde{q}} z_R}_{MB_{PE}} = 0, \tag{9}$$

and allows us to solve for the optimal property tax rate, $\tilde{t}^*$.

Similar to under Foundation system, $-\left(\frac{\partial u}{\partial \tilde{c}} \frac{\partial \tilde{c}}{\partial \tilde{t}} + \frac{\partial u}{\partial \tilde{h}} \frac{\partial \tilde{h}}{\partial \tilde{t}}\right)$ is the marginal utility cost (MC_{PE}) and $\frac{\partial v}{\partial \tilde{q}} z_R$ is the marginal utility benefit (MB_{PE}) of increasing the property tax rate. The marginal utility cost is decreasing in the property tax rate and the marginal utility benefit is decreasing in the tax rate as well. A unique solution for (9) exists as it does under a Foundation system. For district i, the optimal property tax rate under a Foundation system, t_i^*, is potentially different than the optimal property tax rate under a Power-Equalizing system, $\tilde{t}_i^*$. This is mainly a result of the chosen guaranteed tax base, z_R, being different than the actual property tax base of the district. If the district is poor, the tax base of the district is lower than z_R, then MB is higher under a Power-Equalizing system than a Foundation system for every property tax rate. I refer to this result in the proof of *Proposition 2* in the next subsection when I compare the two systems. In addition, the second-order condition for this problem is given by:

$$\frac{\partial MB_{PE}}{\partial t} - \frac{\partial MC_{PE}}{\partial t} < 0, \tag{10}$$

which I also use in the proof of *Proposition 2*.

In Step 1, given the solutions to the previous steps, districts choose a guaranteed tax base and the corresponding income tax rate. In order to determine the median voter district, we solve the following maximization problem for each district with income y

and housing wealth $\tilde{H}$:

$$\max_{z_R,\tau_R} u(\tilde{c}^*, \tilde{h}^*) + v(\tilde{t}^* z_R)$$

$$\text{s.t. } z_R = \frac{\sum_i \tilde{t}_i \tilde{p}_i \tilde{h}_i + \tau_R \sum_i y_i}{\sum_i \tilde{t}_i},$$

$$\tilde{c}^*, \tilde{h}^* \text{given by } (6).$$

For a solution to this problem to exist, preferences must have the single crossing property in (τ_R, z_R). As our policy space is one dimensional under a Foundation system, we don't need such a property. Single-peaked preferences in the policy variable is the only condition we need for the Median Voter Theorem to apply. Under a Power-Equalizing system, we have a two-dimensional policy space. Therefore, we must meet two conditions to guarantee that the single crossing property holds, as is in Fernandez and Rogerson (2003) and Epple and Ferreyra (2008).[1] First, any two indifference curves of any two individuals may cross only once. Second, an indifference curve through any (τ_R, z_R) point must be increasing in income. In what follows I assume that parameters are such that the single-crossing property holds.[2] Given this, the Median Voter Theorem applies and the district with the median income is the decisive district. That is, hence, the preferred guaranteed tax base is monotonically increasing in income, we have $y_{M_{PE}} = y_M$. Furthermore, the median voter for this problem has an indirect utility that decreases in the income tax which he therefore sets to the lowest possible level, $\tau_R^*(y_M) = 0$. Finally, we solve for z_R in the state by solving this maximization problem for $y = y_M$.

2.4 Foundation vs. Power-Equalizing

In this section, I argue that it is possible for a Foundation system to be chosen over Power Equalizing system by majority voting in Step 0 of the game. The reason behind

[1] A necessary condition for single-crossing property: $\frac{d\tau_R}{dz_R}|_{u_i} > 0$, $\frac{\partial(d\tau_R \backslash dz_R|_{u_i})}{\partial y_i} > 0$.

[2] For a reasonable number of set of parameters, I have checked that the model has those properties.

this is that there are two competing effects. First, there are more districts benefitting from state redistribution under a Power-Equalizing system than under a Foundation system. This redistribution effect favoring the Power-Equalizing system over the Foundation system under majority voting is the main mechanism in Fernandez and Rogerson (2003). However, their model abstracts from housing markets and applies taxes on income rather than property as I do here. Adding the effects of such a switch between markets on housing markets, introduces a second effect. At an interior solution, for some districts with housing wealth below the guaranteed tax base, property tax rates under a Power-Equalizing system are higher compared to the Foundation system. This is because the cost of increasing education expenditures under a Power-Equalizing system is lower compared to that under a Foundation system for a majority of districts. These districts would find it optimal to increase education expenditures, which necessitates increasing property tax rates. Higher property tax rates results in a lower net of tax price of housing and hence lower housing wealth in the district. This favors the Foundation system over the Power-Equalizing system under majority voting. Next, I analyze the redistribution effect and housing market effect in detail.

2.4.1 Redistribution Effect

The next proposition says that the guaranteed tax base, z_R, is greater than the property tax base of the district with the mean income, $\tilde{p}_{y_\mu}\tilde{H}_{y_\mu}$. This implies that the mean income district, y_μ, will benefit from redistribution, because his property tax base is lower than the guaranteed tax base in the state. Conversely, y_μ does not benefit from redistribution under a Foundation system. Because I assume that the median of the income distribution is lesser than the mean of the income distribution, $y_M < y_\mu$, the majority tends to prefer a Power-Equalizing system over a Foundation system as a result of purely the redistribution effect.

Proposition 1: For any non-negative income tax rate, the guaranteed tax base chosen by the median voter district in the state, z_R, is greater than the property tax base of the mean income district, $\tilde{p}_{y_\mu}\tilde{H}_{y_\mu}$.

Proof: By definition, $\tau_R \sum_i y_i = \sum_i \tilde{t}_i(z_R - \tilde{p}_i\tilde{H}_i)$. As richer districts set a higher property tax rate, $\tilde{t}_i < \tilde{t}_{i+1}$, and they have higher property values, $\tilde{p}_{\tilde{t}_i}\tilde{H}_{\tilde{t}_i} < \tilde{p}_{\tilde{t}_{i+1}}\tilde{H}_{\tilde{t}_{i+1}}$, we get $\tau_R \sum_i y_i < \sum_i \tilde{t}_{z_R}(z_R - \tilde{p}_i\tilde{H}_i)$. This implies that $Nz_R \geq \sum_i^N \tilde{p}_i\tilde{H}_i$. Suppose that $z_R \leq \tilde{p}_{y_\mu}\tilde{H}_{y_\mu}$. So $\tilde{p}_{y_\mu}\tilde{H}_{y_\mu} \geq \frac{\sum_i^N \tilde{p}_i\tilde{H}_i}{N}$. This contradicts $\tilde{p}_i\tilde{H}_i$ being a strictly convex function of income. Thus, $\tilde{p}_{y_\mu}\tilde{H}_{y_\mu} < z_R$.■

In order to see the redistribution effect more clearly, we can compare the cost of increasing education spending by a dollar for both systems.

$Cost_F(y_i) = \frac{\tau_f y_i + t_i p_i H_i}{\tau_f y_\mu + t_i p_i H_i}$ and $Cost_{PE}(y_i) = \frac{\tilde{t}_i\tilde{p}_i\tilde{H}_i}{\tilde{t}_i z_R}$

Both are increasing in income; the closer income gets to the point of redistribution, the less the districts benefits from it. As $\tilde{p}_{y_\mu}\tilde{H}_{y_\mu} < z_R$ by Proposition 1, the cost for y_μ is less than one in the Power-Equalizing system and equals one in the Foundation system. For the median income district, y_M, additionally:

$Cost_{PE}(y_M) < \frac{\tilde{p}_{y_M}\tilde{H}_{y_M}}{\tilde{p}_{y_\mu}\tilde{H}_{y_\mu}}$ and $\frac{y_M}{y_\mu} < Cost_F(y_M)$

Furthermore, we have $\frac{\tilde{p}_{y_M}\tilde{H}_{y_M}}{y_M} < \frac{\tilde{p}_\mu\tilde{H}_\mu}{y_\mu}$ as $\tilde{p}\tilde{H}$ is a convex function. Thus, we have $Cost_{PE}(y_M) < Cost_F(y_M)$. This implies that y_M is better off under Power-Equalizing than under Foundation as a result of purely the redistribution effect. This is the only effect in Fernandez and Rogerson (2003). Hence, they conclude that a Power-Equalizing system will always be chosen over a Foundation system under majority voting.

2.4.2 Housing Market Effect

When a housing market is in place, there will be potential differences in property tax rates across systems. The next proposition tells us that, in the Power-Equalizing system, districts which benefit from the redistribution of funds, $\tilde{p}_i\tilde{H}_i < z_R$, choose higher property tax rates than they do in the Foundation system. This is a result of these districts having a lower cost of increasing education spending in the Power-Equalizing system than in the Foundation system as shown above. With a lower cost of increasing education spending, they choose to have higher education spending, and this is only possible with higher property tax rates under a Power-Equalizing system. Higher property tax rates result in higher gross price and lower net-of-tax price for housing.

This shifts housing demand down and decreases supply. Hence, housing wealth and housing consumption of the districts is lower under a Power-Equalizing system than a Foundation system. So, the housing market effect favors a Foundation system over a Power-Equalizing system. While more districts benefit from the redistribution under a Power-Equalizing system than under a Foundation system, the majority tend to prefer a Foundation system over a Power-Equalizing system as a result of purely the housing market effect.

Proposition 2: At an interior solution, for every district with a property tax base lower than the guaranteed tax base in the state, $\tilde{p}_i\tilde{H}_i < z_R$, if $\tau_R = \tau_f$, property tax rates under the Power-Equalizing system are greater than property tax rates under the Foundation system, i.e. $t_i^* < \tilde{t}_i^*$.

Proof: Since $\tau_R = \tau_f$, Stage 3 is identical in both systems if $(\pi, q) = (\tilde{\pi}, \tilde{q})$. Now, we recall the first-order conditions in each system for the optimal property tax rate:

F: $\overbrace{-(\frac{\partial u}{\partial c}\frac{\partial c}{\partial t} + \frac{\partial u}{\partial h}\frac{\partial h}{\partial t})}^{MC_F} = \overbrace{\frac{\partial v}{\partial q}\frac{\partial q}{\partial t}}^{MB_F}$

PE: $\underbrace{-(\frac{\partial u}{\partial \tilde{c}}\frac{\partial \tilde{c}}{\partial \tilde{t}} + \frac{\partial u}{\partial \tilde{h}}\frac{\partial \tilde{h}}{\partial \tilde{t}})}_{MC_{PE}} = \underbrace{\frac{\partial v}{\partial \tilde{q}}z_R}_{MB_{PE}}$

There are four properties used in this proof:

1. For every property tax rate, t, marginal cost is the same under both systems, $MC_F(t) = MC_{PE}(t)$.

2. For every property tax rate, t, marginal benefit under the Power-Equalizing system is higher than it is under the Foundation system as $\frac{\partial q}{\partial t} < z_R$ for the districts with $\tilde{p}_i\tilde{H}_i < z_R$, $MB_F(t) < MB_{PE}(t)$.

3. Both sides of both of the first-order conditions are decreasing in property tax rates, $\frac{\partial MC_F(t)}{\partial t}, \frac{\partial MB_F(t)}{\partial t}, \frac{\partial MC_{PE}(t)}{\partial t}, \frac{\partial MB_{PE}(t)}{\partial t} < 0$

4. The second-order condition for each system implies that the MB is steeper than MC, i.e. $\frac{\partial MB}{\partial t} < \frac{\partial MC}{\partial t}$.

The first and the second property imply that for any district with $\tilde{p}_i\tilde{H}_i < z_R$, property tax rates under two systems are different, $t^* \neq \tilde{t}^*$. Suppose $t^* > \tilde{t}^*$, then the second and the third property imply that MC has to be steeper than MB which would contradict with the fourth property. Thus, it has to be the case that $t_i^* < \tilde{t}_i^*$.■

Proposition 2 is for $\tau_R = \tau_f$. The previous subsection argues that the income tax rate in the Power-Equalizing system is zero, $\tau_R = 0$. Because the optimal property tax is higher with a lower income tax, $\frac{\partial \tilde{t}}{\partial \tau_R} < 0$, we still have $t_i^* < \tilde{t}_i^*$.

2.5 Comparative Statics

The model argues that when we compare a Power-Equalizing system with a Foundation system, there are two effects we must consider: the redistribution effect and the housing market effect. If the redistribution effect dominates the housing market effect for a majority of districts, the state would opt for the Power-Equalizing system if they were put to a vote. On the other hand, if the housing market effect dominates the redistribution effect for a majority of districts, the state would opt for the Foundation system instead.

Using two examples, I derive comparative statics to see under what conditions the redistribution effect is weakened and the housing market effect is strengthened in order to rationalize observed switchers from a Power-Equalizing system to a Foundation system. I find that, first, the redistribution effect is smaller if the districts have similar income levels. In other words, if the variance of the income distribution is smaller, then the redistribution effect is smaller. Second, the housing market effect is bigger if the preference parameter for education, A, is greater or per capita income in the state, y_μ, is lower.

First, the size of the redistribution effect is the difference between the cost of increasing education spending by a dollar under two systems, $Cost_F(y_i) - Cost_{PE}(y_i)$, as defined in Section 2.4.1. Second, the size of the housing market effect is the difference between property tax rates under the two systems, $\tilde{t}^* - t^*$. Consider the case in which we have a log-utility function and a linear housing supply function: $\alpha = 0$, $\gamma = 0$,

$a = 1$, $b = 1$, $a_c = 0.5$. Then, for a mean preserving spread of the income distribution, the redistribution effect is bigger because $Cost_{PE}(y_i)$ decreases faster than $Cost_F(y_i)$. The reason for this is that the wealth difference between the median voter and rich districts is higher so the median voter sets the level of redistribution under the Power-Equalizing system, z_R, higher. Also, with the above parameter values we get

$$\tilde{t}^* - t^* = 2A^2 - A\frac{\tau_f y_\mu}{(1-\tau_f)y} + \frac{\tau_f y_\mu}{(1-\tau_f)y}.$$

The housing market effect is therefore increasing in A and decreasing in y_μ. In other words, a higher A or a lower y_μ leads to a bigger housing market effect. I also analyzed for parameter values in Fernandez and Rogerson (2003) and a concave house supply function as in Fernandez and Rogerson (1998): $\alpha = -1$, $\gamma = -1$, $a = 1$, $b = 0.5$, $a_c = 0.5$. Same intuition follows.

In the next section, I examine the switches between public education finance systems in the recent years and see if those switches can be rationalized with the results produced by this model.

3 Suggestive Evidence

Comparative statics above suggest at least three possible reasons for a state to be more likely to prefer a Foundation system over a Power-Equalizing system in the model: a stronger preference for education, A, lower per capita income, y_μ, and lower income inequality, $var(y)$. Even though this model has no implications on the transition from one system to another, these theoretical results could be tested empirically by analyzing the switches between systems in the recent years. In the light of these comparative statics states with these characteristics would be good candidates to have a Foundation system or even switch into a Foundation system if they have a Power-Equalizing system currently. For those switcher states, y_μ and $var(y)$ are identified by per capita income and gini coefficient respectively.[3] For the preference parameter, the choice is not obvious. In this paper, I employ expenditure per pupil in public schools in the

[3]Different inequality measures does not seem to have large impacts on the results.

state to represent the preference parameter for education, A, in the model.[4] Though this measure is far from perfect, it carries significant information on the preference parameter for education. That is, if per pupil expenditure in a state is growing at a faster rate, this could potentially result from many factors not modeled here. For example, a demographic structure with a higher percentage of young parents and children might lead to a higher preference for K-12 education and therefore higher expenditures per pupil. Similarly, many other factors increasing per pupil expenditures unrelated to income levels can be interpreted as a high preference for education, A.

There are at least two possible ways to compare the variables of interest across states. First way would be to report the levels of these variables and sort the states into systems with respect to their relative levels using the model. Then, we could verify whether the results actually match. For example, the model predicts that the states with a Foundation system should have at least one of the following three: a high level of per pupil spending, a low level of per capita income, or a low gini coefficient. Moreover, those states with a Power-Equalizing system should have at least one of: a low level of per pupil spending, a high level of per capita income, or a high gini coefficient. Second way, which I use in this paper, is to report the growth rates of those three variables for each state and determine if the reported growth rates are significantly different for those states that have switched into a different finance system recently. One advantage of using growth rates is that the states differ in many other characteristics, such as geographical or industrial factors, that are not captured by the current model. By comparing the growth rate of each variable, we can difference out these characteristics across states.

The experiment in this section operates as follows. I compare the growth rate of the variables of interest for switchers with respect to the reference group. We focus our attention on two types of switchers: states that switched from a Power-Equalizing

[4] I could work with a measure of demographic differences across states. One could argue that states with a lower percentage of older households would have a stronger preference for education. Other possible measures for demographic differences across states include: fertility rate, population density, educational attainment, enrollment levels in public schools, et cetera.

system to a Foundation system, and states that switched in the opposite direction. Reference groups differ depending on the finance system in place prior to the switch. Table 1 and Table 2 are defined by the following:
S: Switcher. Either from Power-Equalizing to Foundation or in the opposite direction.
RG: Reference group. Either pure Power-Equalizing or pure Foundation, depending on the original type.[5]
I report the yearly percentage change in each variable and compare it with the average change in the reference group.

- $EXP_t^S = \%\Delta\text{Per Pupil Expenditure}_t^S - \%\Delta\text{Per Pupil Expenditure}_t^{RG}$
- $INC_t^S = \%\Delta\text{Per Capita Income}_t^S - \%\Delta\text{Per Capita Income}_t^{RG}$
- $INE_t^S = \%\Delta\text{Gini Coefficient}_t^S - \%\Delta\text{Gini Coefficient}_t^{RG}$

Data on per capita income and gini coefficient by states come from the U.S. Dept. of Commerce Bureau of Economic Analysis. Data on expenditure per pupil in K-12 public schools by state come from The National Center for Education Statistics. Lastly, data on public education finance systems by state are from Jackson et al. (2014). All data referenced in this research is for the period from 1990 to 2011.[6] Table 1 reports the above three variables for states switching: from Power-Equalizing to Foundation. These states are Pennsylvania (2008), New York (2006), Colorado (1994), and Massachusetts (1993). The reference group includes the pure Power-Equalizing states between the years 1990 and 2011: Delaware, Kansas, New Jersey, North Carolina, Rhode Island, Washington, Wisconsin, and Oregon after 1992.

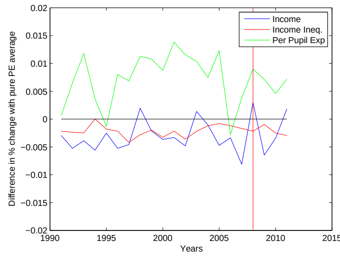
For the four switcher states of Table 1, the model suggests that their growth rates of expenditure per pupil should be higher, and that per capita income and gini coefficient should be lower than the average of the reference group. The third column reports

[5] I use two other types of reference groups and report these results in the Appendix. The choice of reference group doesn't affect the results qualitatively.
[6] The start date is arbitrary, and I am in the process of extending the time horizon.

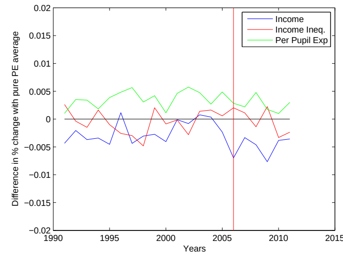
Table 1: From Power-Equalizing to Foundation with respect to pure Power-Equalizing average

STATE	VARIABLE	MODEL	BEFORE	AFTER	AVG
PA (2008)	EXP	(+)	+0.74	+0.63	+0.72
	INC	(-)	-0.30	-0.27	-0.30
	INE	(-)	-0.21	-0.21	-0.21
NY (2006)	EXP	(+)	+0.36	+0.25	+0.34
	INC	(-)	-0.25	-0.46	-0.30
	INE	(-)	-0.03	-0.07	-0.04
CO (1994)	EXP	(+)	+0.18	+0.33	+0.30
	INC	(-)	-0.17	-0.08	-0.09
	INE	(-)	+0.07	-0.14	-0.10
MA (1993)	EXP	(+)	+0.22	+0.33	+0.32
	INC	(-)	-0.11	-0.20	-0.19
	INE	(-)	-0.03	-0.13	-0.12

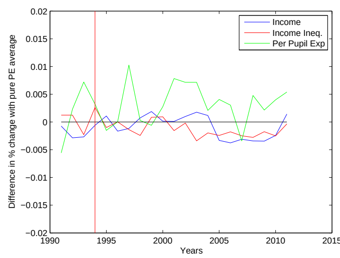
these signs. The fourth column is the average before the switch, the fifth is the average after the switch and the last column is the entire period. As the model would suggest, it appears that these four states are different than the reference group. In Pennsylvania prior to the switch in 2008 for instance, the growth rate of expenditure per pupil in public schools was 0.74 percent higher than was the average for pure Power-Equalizing states between 1990 and 2008. The model suggests that at least one of the variables should be different from the reference group for a switch to occur. Since the model does not offer an interpretation for the magnitudes of growth rates for a variable or across variables, I focus on the sign of the growth rate. I check that switcher states will match at least one of the three signs. As it happens, all four switcher states have variables with the expected signs. Plotting the values illustrates the difference between these states and the reference group in more detail.



(a) PA (2008)



(b) NY (2006)



(c) CO (1994)

(d) MA (1993)

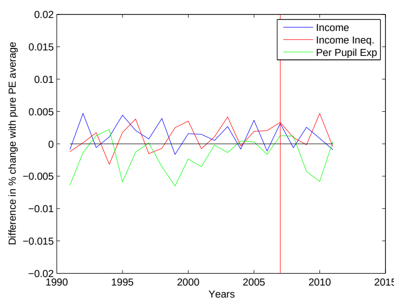
I plot the three variables of interest for each switcher state. The growth rate of per pupil expenditure is higher for the switcher states compared to the reference group. This is as the model predicts. For per capita income and gini coefficient, switchers have lower growth rates than the reference group for each year for these two variables. These observations are in line with the predictions of the model.

Now, I examine the switchers in the opposite direction. The model suggests that these are the states with a higher redistribution effect (higher growth rate for gini coefficient) or a lower housing market effect (lower growth rate of per pupil spending or a higher growth rate of per capita income). Table 2 reports these three variables for the switchers in the opposite direction to Table 1. Our switchers are now the states who switched from a Foundation system to a pure Power-Equalizing system between 1990 and 2011: North Dakota (2007), Ohio (2002), Maryland (2002), Alaska (1999), New Mexico (1998), Nebraska (1997), Utah (1997), Wyoming (1995), Alabama (1994), Arkansas (1994), and Indiana (1994). The group of states that stayed in a pure Foundation system between 1990-2011 serve as our reference group: California, Idaho, Iowa, Kentucky, Minnesota, Mississippi, Nevada, South Carolina, South Dakota, Tennessee, West Virginia, and Massachusetts after 1993.

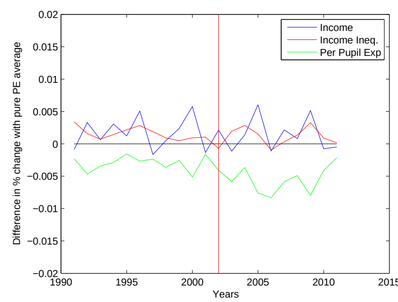
Table 2: From Foundation to Power-Equalizing with respect to pure Foundation average

STATE	VARIABLE	MODEL	BEFORE	AFTER	AVG
ND (2007)	EXP	(-)	-0.17	-0.21	-0.18
	INC	(+)	+0.15	+0.05	+0.13
	INE	(+)	+0.11	+0.13	+0.11
OH (2002)	EXP	(-)	-0.04	-0.36	-0.18
	INC	(+)	+0.17	+0.13	+0.15
	INE	(+)	+0.14	+0.13	+0.13
MD (2002)	EXP	(-)	-0.34	-0.31	-0.33
	INC	(+)	+0.09	+0.16	+0.12
	INE	(+)	+0.10	+0.05	+0.08
AK (1999)	EXP	(-)	-0.25	-0.35	-0.31
	INC	(+)	+0.03	+0.08	+0.06
	INE	(+)	+0.12	+0.07	+0.09
NM (1998)	EXP	(-)	-0.42	-0.48	-0.46
	INC	(+)	+0.08	+0.11	+0.10
	INE	(+)	+0.13	+0.08	+0.10
NE (1997)	EXP	(-)	-0.42	-0.39	-0.40
	INC	(+)	+0.12	+0.07	+0.09
	INE	(+)	+0.19	+0.14	+0.15
UT (1997)	EXP	(-)	-0.04	-0.24	-0.17
	INC	(+)	+0.04	+0.07	+0.06
	INE	(+)	+0.13	+0.13	+0.13
WY (1995)	EXP	(-)	-0.49	-0.44	-0.45
	INC	(+)	+0.26	+0.17	+0.19
	INE	(+)	+0.15	+0.16	+0.16
AL (1994)	EXP	(-)	+0.15	-0.08	-0.04
	INC	(+)	+0.01	+0.19	+0.16
	INE	(+)	+0.08	+0.09	+0.09
AR (1994)	EXP	(-)	-0.25	-0.14	-0.16
	INC	(+)	-0.14	+0.19	+0.13
	INE	(+)	+0.18	+0.13	+0.14
IN (1994)	EXP	(-)	-0.11	-0.32	-0.28
	INC	(+)	+0.04	+0.08	+0.07
	INE	(+)	+0.09	-0.02	0.00

In Table 2 North Dakota is a suitable example: prior to the switch in 2007, the growth rate of per pupil expenditure in public schools was 0.17 percent lower than was the average for pure Foundation states between 1990-2007. If we look at each state in Table 2, we notice that there is at least one variable that grows in the direction suggested by the model. For most of the switchers, almost all three variables grow in the direction suggested by the model. From the plots below, we see that for this type of switchers, the growth rate of expenditure per pupil is consistently lower than that of the reference group as predicted by the model. This trend in growth rates is the polar opposite of the respective growth rate of per capita income and gini coefficient. They are consistently greater than the respective averages of the growth rates of the reference group. Overall, the implications of the model appear to be verifiable by the documented switches in either direction.

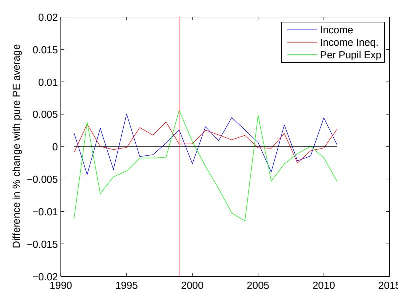


(a) ND (2007)

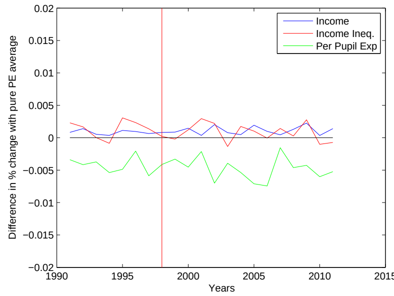


(b) OH (2002)

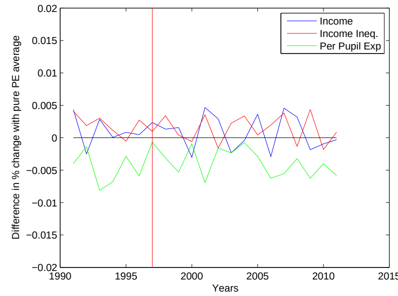
(c) MD (2002)



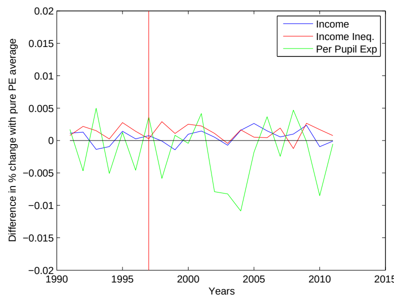
(d) AK (1999)



(a) NM (1998)

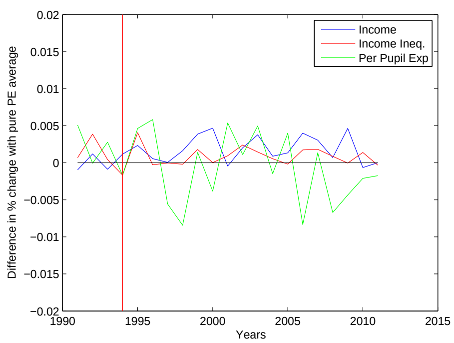


(b) NE (1997)

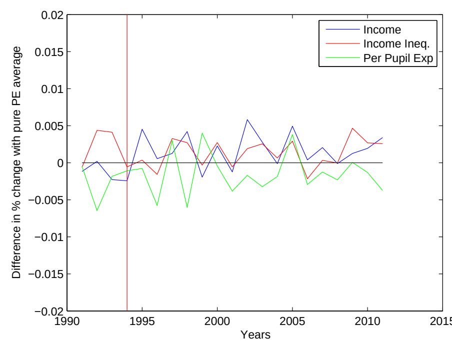


(c) UT (1997)

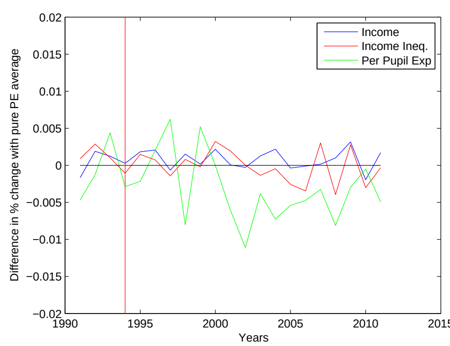
(d) WY (1995)



(a) AL (1994)



(b) AR (1994)



(c) IN (1994)

4 Conclusion

An important result in the public education finance literature is that the Power-Equalizing system is socially preferred over the Foundation system, as established by Fernandez and Rogerson (2003). Yet more states have been using a Foundation system in recent years. Indeed, some states such as Pennsylvania (2008), New York (2006), Colorado (1994) and Massachusetts (1993) switched from using a Power-Equalizing system to using a Foundation system. In this paper, I build a model that helps us understand these switches. This model builds on Fernandez and Rogerson (2003) by introducing a housing market. In a model with no housing market, a majority of individuals prefers Power-Equalizing over Foundation, because redistribution is always higher in the former than in the latter. With the introduction of the housing market however, another effect appears. For most districts, property tax rates are higher in a Power-Equalizing system as a result of lower costs of increasing educational spending for the majority of districts compared to those in a Foundation system. Higher property tax rates result in lower property values and lower housing wealth in the district. This decrease in housing wealth and increase in gross price of housing makes the Power-Equalizing system less attractive for the median voter.

As a result, we have two different effects in a model with a housing market. The redistribution effect works in favor of a Power-Equalizing system while the housing market effect works against it. For the housing market effect dominate the redistribution effect, model requires the following conditions: a higher value for the coefficient of education in the utility function, A, a lower mean income in the state, y_μ, or a lower variance of income distribution, $var(y)$. I test these implications of the model by examining data on income, per pupil expenditure, and the type of public education finance system by state from 1990 to 2011. By using per pupil expenditure in public schools in the state for A, per capita income in the state for y_μ and gini coefficient in the state for $var(y)$, I provide suggestive evidence on the implications of the model.

I report that the yearly growth rates of these three variables for those states that have changed their public education finance system in the recent years are significantly different than those of their reference group. For the states that switched from Power-Equalizing to Foundation, I use pure Power-Equalizing states as the reference group; for those that switched in the opposite direction, I use pure Foundation states as the reference group. The reported tables indicate that the states switching from Power-Equalizing to Foundation experienced at least one of the following: a higher growth rate in per pupil expenditure, a lower growth rate in per capita income, or a lower growth rate in gini coefficient each year. This evidence demonstrates that the model can rationalize the switches in this direction. Additionally, the states that switched in the opposite direction experienced growth rate differences with the reference group in an opposite manner. This confirms the ability of the model to explain switches in the opposite direction as well.

For future work, the model has testable implications concerning house prices in the switcher states versus the reference group. The model suggests that as a result of higher property taxes, property values in Power-Equalizing states should be lower than in the Foundation states. Alternatively, the states that switching from Power-Equalizing to Foundation systems should experience an increase in the value of houses in the state. In the spirit of the experiments presented above, the growth rate of the property values in the states that switched from Power-Equalizing to Foundation should be higher than that for pure Power-Equalizing states.

Finally, there are a number of states using properties from both the Foundation and the Power-Equalizing system to finance public education. After understanding the reasoning behind the choice of the Foundation system over the Power-Equalizing system, I plan to extend the model in a direction in which it is possible to have a mixed system. This could lead to a better understanding of the distribution of states into public finance systems as pure Foundation states, pure Power-Equalizing states, and mixed states.

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5 Appendix

Second experiment: I group the states with respect to their percentage of school aged children in total population in 2010, dividing them into three groups as High, Medium and Low. So, the group they are in according to this categorization is their reference group and numbers are reported in Table 3 and Table 4:

High: AK, AR, AZ, CA, CO, GA, ID, IL, IN, KS, MS, NE, NM, NV, OK, TX, UT.

Medium: AL, CT, IA, KY, LA, MD, MI, MN, MO, NC, NJ, OH, SD, TN, WA, WI, WY.

Low: DC, DE, FL, HI, MA, ME, MT, ND, NH, NY, OR, PA, RI, SC, VA, VT, WV.

Third experiment: I group the states with respect to population density in 2010 and divide them into three groups as High, Medium and Low. So, the group they are in according to this categorization is their reference group and numbers are reported in Table 5 and Table 6:

High: DC, NJ, RI, MA, CT, MD, DE, NY, FL, PA, OH, CA, IL, HI, VA, NC, IN.

Medium: MI, GA, SC, TN, NH, KY, LA, WI, WA, TX, AL, MO, WV, VT, MN, MS, AZ.

Low: AR, IA, OK, CO, ME, OR, KS, UT, NV, NE, ID, NM, SD, ND, MT, WY, AK.

Table 3: From Power-Equalizing to Foundation with respect to Reference 2

STATE	VARIABLE	MODEL	BEFORE	AFTER	AVG
PA (2008)	EXP	(+)	+0.63	+0.64	+0.63
	INC	(-)	-0.20	-0.19	-0.20
	INE	(-)	-0.16	-0.20	-0.17
NY (2006)	EXP	(+)	+0.25	+0.24	+0.25
	INC	(-)	-0.15	-0.34	-0.20
	INE	(-)	+0.02	-0.05	0.00
CO (1994)	EXP	(+)	+0.18	+0.30	+0.28
	INC	(-)	-0.11	-0.03	-0.05
	INE	(-)	+0.08	-0.07	-0.04
MA (1993)	EXP	(+)	+0.18	+0.23	+0.22
	INC	(-)	-0.02	-0.09	-0.08
	INE	(-)	+0.02	-0.08	-0.07

Table 4: From Foundation to Power-Equalizing with respect to Reference 2

STATE	VARIABLE	MODEL	BEFORE	AFTER	AVG
ND (2007)	EXP	(-)	-0.15	-0.27	-0.17
	INC	(+)	+0.19	+0.03	+0.16
	INE	(+)	+0.07	+0.08	+0.07
OH (2002)	EXP	(-)	-0.20	-0.48	-0.32
	INC	(+)	+0.07	+0.04	+0.06
	INE	(+)	+0.05	+0.04	+0.05
MD (2002)	EXP	(-)	-0.23	-0.23	-0.23
	INC	(+)	+0.09	+0.17	+0.12
	INE	(+)	+0.11	+0.07	+0.09
AK (1999)	EXP	(-)	-0.23	-0.25	-0.24
	INC	(+)	+0.04	+0.03	+0.04
	INE	(+)	+0.07	+0.07	+0.07
NM (1998)	EXP	(-)	-0.38	-0.39	-0.39
	INC	(+)	+0.10	+0.07	+0.08
	INE	(+)	+0.07	+0.08	+0.08
NE (1997)	EXP	(-)	-0.40	-0.30	-0.33
	INC	(+)	+0.14	+0.03	+0.07
	INE	(+)	+0.15	+0.12	+0.13
UT (1997)	EXP	(-)	-0.01	-0.15	-0.10
	INC	(+)	+0.06	+0.02	+0.03
	INE	(+)	+0.09	+0.12	+0.11
WY (1995)	EXP	(-)	-0.42	-0.33	-0.35
	INC	(+)	+0.18	+0.07	+0.09
	INE	(+)	+0.07	+0.08	+0.07
AL (1994)	EXP	(-)	+0.24	+0.02	+0.06
	INC	(+)	-0.08	+0.09	+0.06
	INE	(+)	0.00	0.00	0.00
AR (1994)	EXP	(-)	-0.18	-0.07	-0.09
	INC	(+)	-0.10	+0.15	+0.10
	INE	(+)	+0.14	+0.12	+0.12
IN (1994)	EXP	(-)	-0.04	-0.25	-0.21
	INC	(+)	+0.09	+0.04	+0.05
	INE	(+)	+0.05	-0.03	-0.02

Table 5: From Power-Equalizing to Foundation with respect to Reference 3

STATE	VARIABLE	MODEL	BEFORE	AFTER	AVG
PA (2008)	EXP	(+)	+0.78	+0.89	+0.80
	INC	(-)	-0.28	-0.18	-0.26
	INE	(-)	-0.22	-0.18	-0.21
NY (2006)	EXP	(+)	+0.40	+0.47	+0.41
	INC	(-)	-0.24	-0.35	-0.27
	INE	(-)	-0.04	-0.04	-0.04
CO (1994)	EXP	(+)	+0.15	+0.24	+0.23
	INC	(-)	-0.07	+0.01	0.00
	INE	(-)	+0.06	-0.08	-0.06
MA (1993)	EXP	(+)	+0.28	+0.41	+0.39
	INC	(-)	-0.07	-0.17	-0.15
	INE	(-)	-0.02	-0.03	-0.11

Table 6: From Foundation to Power-Equalizing with respect to Reference 3

STATE	VARIABLE	MODEL	BEFORE	AFTER	AVG
ND (2007)	EXP	(-)	0.00	-0.03	-0.01
	INC	(+)	+0.11	+0.04	+0.09
	INE	(+)	+0.02	+0.07	+0.03
OH (2002)	EXP	(-)	-0.32	-0.59	-0.43
	INC	(+)	+0.11	+0.06	+0.09
	INE	(+)	+0.11	+0.10	+0.11
MD (2002)	EXP	(-)	-0.35	-0.34	-0.34
	INC	(+)	+0.13	+0.19	+0.16
	INE	(+)	+0.17	+0.13	+0.15
AK (1999)	EXP	(-)	-0.23	-0.32	-0.29
	INC	(+)	+0.10	+0.07	+0.08
	INE	(+)	+0.08	+0.03	+0.05
NM (1998)	EXP	(-)	-0.40	-0.46	-0.44
	INC	(+)	+0.16	+0.10	+0.13
	INE	(+)	+0.07	+0.05	+0.06
NE (1997)	EXP	(-)	-0.39	-0.38	-0.38
	INC	(+)	+0.20	+0.07	+0.11
	INE	(+)	+0.15	+0.10	+0.12
UT (1997)	EXP	(-)	-0.01	-0.22	-0.15
	INC	(+)	+0.12	+0.06	+0.08
	INE	(+)	+0.09	+0.09	+0.09
WY (1995)	EXP	(-)	-0.49	-0.46	-0.47
	INC	(+)	+0.22	+0.10	+0.13
	INE	(+)	+0.13	+0.13	+0.13
AL (1994)	EXP	(-)	+0.17	-0.11	-0.06
	INC	(+)	-0.04	+0.12	+0.09
	INE	(+)	+0.08	+0.06	+0.06
AR (1994)	EXP	(-)	-0.20	-0.13	-0.14
	INC	(+)	-0.05	+0.20	+0.15
	INE	(+)	+0.12	+0.10	+0.10
IN (1994)	EXP	(-)	-0.07	-0.31	-0.26
	INC	(+)	+0.13	+0.09	+0.10
	INE	(+)	+0.03	-0.05	-0.03