Price Discrimination through Multi-Level Loyalty Programs

Serdar Sayman

Murat Usman

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Serdar Sayman is Associate Professor of Marketing
College of Adm. Sciences, Koç University
Rumeli Feneri Yolu, Sariyer
Istanbul, 34450, Turkey
Tel: +90 (212) 338 1616, Fax: +90 (212) 338 1653
E-mail: ssayman@ku.edu.tr

Murat Usman is Associate Professor of Economics
College of Adm. Sciences, Koç University
Rumeli Feneri Yolu, Sariyer
Istanbul, 34450, Turkey
Tel: +90 (212) 338 1553, Fax: +90 (212) 338 1653
E-mail: musman@ku.edu.tr

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Abstract

Loyalty programs often feature multiple rewards with different requirements; for instance a coffee shop offering one free muffin with 6 coffee purchases, and two muffins with 10 coffees. This research focuses on the role of multi-level rewards as a price discrimination mechanism. More specifically, we propose that a program with two rewards can be designed in such a way that it is more profitable than a one reward program, and two rewards are for segments differing in purchase rate or time discounting. Incentive compatibility requires that the larger reward is for frequent buyers or for customers with lower discount rate. Two-level program works like a menu-based, second-degree price discrimination mechanism.

Keywords: Loyalty Programs, Price Discrimination, Buyer Heterogeneity, Purchase Rate

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1. Introduction

Loyalty programs are widely used in many industries such as retailing and travel. These programs reward the customer based on a measure of accumulated spending (e.g. volume, miles, or points). Many loyalty programs offer multiple rewards on a tiered requirement structure, an example of which would be a coffee shop offering one free muffin with 6 coffee purchases or two free muffins with 10 coffees. In this paper, we use an analytical approach to provide insight regarding the role of such multi-reward programs as a price discrimination mechanism. We argue that multi tiered loyalty programs may be profitable for the firm when customers differ in their purchase frequency or time discounting -- such that the larger reward is intended for customers buy more frequently or who discount future less, and vice versa.

In our framework one of the two selling firms offers a loyalty program to attract customers from its competitor. As a baseline case we examine a market where all buyers purchase one unit of the product every period. We let one firm to choose a single reward for n purchases to maximize profits. We then consider a two-segment market where light buyers buy every other period. The firm is better off now with two rewards, each optimally chosen for one segment, than a single reward. It turns out that, offering one reward optimized for regular buyers and one optimized for light buyers is not incentive compatible -- both types of buyers prefer the reward that is supposedly for the light buyers. We therefore examine an alternative two-level reward structure that satisfies the incentive compatibility constraint, and increases profits over the single reward case. While trying to maintain incentive compatibility, the firm would have to forego some profits as compared to the case with two optimally chosen rewards. The main tenet of the paper, and our main contribution, is to show that such a program can be designed.

Research on loyalty programs examined various issues; however, the question of "why multi-level rewards?" has not received much attention. One answer is from Nunes and Drèze (2004); they provide experimental evidence that smaller rewards serve as intermediate and attainable goals, and hence can motivate customers to stay with the seller. A second argument is that larger rewards (vis-à-vis the smaller reward) motivate customers to spend more (O'Brien and Jones 1995). A third argument is that variety of rewards or levels is offered for customer convenience; individuals have different preferences or situational factors (Nunes and Drèze 2004) -- implying horizontal differentiation between rewards. Our interest is not in the validity of these arguments; rather, we argue that firms can use multi-reward programs as a (second degree) price discrimination mechanism to exploit consumer heterogeneity to increase profits. And unlike the prior arguments, we propose that the larger vs. smaller reward choice should be related to consumer's purchase rate or time discounting.

The paper proceeds as follows. We first introduce our Hotelling type model where two sellers compete in prices, and examine the case of no loyalty programs. Then we analyze the loyalty program introduced by one firm where all the consumers buy one unit every period. We introduce purchase rate heterogeneity by assuming that part of the market is light users who buy every other period, and compare profits from certain reward structures. A discussion of our analysis, including the case of discount rate heterogeneity follows. The conclusion puts our paper further into perspective.

2. The Model

2.1. Market Structure

We assume a market of size 1 served by two firms 1 and 2, selling differentiated products, 1 and 2 respectively. Firms simultaneously choose their prices P_1 and P_2 in each time period. Buyers purchase one unit of the product; they choose between firms 1 and 2 according to the following:

$$\begin{cases} \text{choose 1} & \text{if } P_2 > P_1 + \gamma \\ \text{choose 2} & \text{if } P_2 \leq P_1 + \gamma \end{cases}$$

where γ is uniformly distributed with $U[-d, d]$. This is essentially a Hotelling framework where γ represents individual specific product preference.[1] Demand for firm i ($i = 1, 2$) is:

$$q_i = \begin{cases} 1 & \text{if } P_i < P_j - d \\ \dfrac{P_j - P_i + d}{2d} & \text{if } P_j - d \leq P_i \leq P_j + d \\ 0 & \text{if } P_i > P_j + d \end{cases} \qquad \text{where } j = 2 - i$$

As price i decreases within the relevant range, demand for firm i increases linearly; when price i reaches price j minus d, firm i captures the whole market. Furthermore, total demand for two firms is constant.

We assume that the unit marginal cost c is constant and same for firms 1 and 2. We also assume that the buyers' reservation price satisfies $P_R > c + d$; so that all buyers purchase the product.

[1] The framework adopted here is similar to others that examine loyalty programs (e.g. Von Weizsacker 1984; Klemperer 1987.)

2.2. Equilibrium with No Loyalty Programs

In the absence of a loyalty program, buyers will consider only the current prices and their relative preferences for products 1 and 2. Proposition 1 describes the Nash equilibrium of the game in which the firms set their prices simultaneously:

PROPOSITION 1. For the case of no loyalty programs, equilibrium prices, demands, and profits are:

(a) $P_1 = P_2 = c + d$

(b) $q_1 = q_2 = 1 / 2$

(c) $\Pi_1 = \Pi_2 = d / 2$

Proof: All proofs are in the Technical Appendix.

Firms split the market equally; firm 1 and 2's customers are those with $\gamma < 0$ and $\gamma > 0$, respectively; the buyer with $\gamma = 0$ is the marginal one. The price P_1 will be used in the loyalty program discussed next.

3. Introducing the Loyalty Program

3.1. A Single-Reward Loyalty Program

We study the case in which firm 1 starts a single-level reward program: if a buyer makes n purchases from firm 1, she qualifies for a reward of value R_n to her. We assume that firm 1 does not change its price P_1 after introducing the program. The justification for this assumption is that firms engaging in loyalty programs typically do not raise the prices in the short term; primary expectation would be an increase in volume (see Lal and Bell 2003 for a marketplace example).

In our case, firm 1 expects to attract a portion of buyers with $\gamma > 0$. We also assume that firm 2 does not respond either by changing its price or starting its own loyalty program.[2] Our focus is on single versus multi-level reward programs rather than the strategic interaction between the two firms. We specify the cost of the reward to firm 1 as kR_n where $0 < k \leq 1$ is a measure of savings or cost efficiency. Products offered as rewards often belong to own product portfolios or they are procured with some discount from other manufacturers. Note that the reward value R_n to the buyer is independent of the preference parameter γ.

We find the optimal reward R_n and profit from the program for a given purchase requirement n. We start by specifying the buyer response to reward R_n. Given our framework, firm 1's existing customers will continue to choose firm 1. When firm 1 keeps its price at the same level as firm 2 but offers a reward R_n with n purchases, a customer of firm 2 would consider switching to firm 1. More specifically, the buyer (rational and forward looking) would compare the discounted value of R_n with the sum of the disutility of buying a less preferred product in each period. Considering period $t = 0$ as the current period, switching to firm 1 will bring the reward in period $t = n-1$. Hence, a consumer with a taste parameter γ_0 would switch to firm 1 if:

$$R_n \, \delta^{n-1} > \gamma_0 + \gamma_0 \, \delta + \gamma_0 \, \delta^2 + \ldots + \gamma_0 \, \delta^{n-1}$$

where δ is the discount factor of the buyer. We note that firm 2's customers were those with $0 < \gamma \leq d$. For a given R_n, the marginal customer has $\gamma_h = R_n \dfrac{\delta^{n-1} - \delta^n}{1 - \delta^n}$ such that firm 2 customers with

[2] The essence of this assumption is that firm 2 does not respond; with an appropriate distribution for the preference

$0 < \gamma_0 \leq \gamma_h$ will switch to firm 1. Total demand for firm 1 as a function of R_n is as follows:

$$q_1 = \frac{1}{2} + \frac{1}{2}\frac{R_n(\delta^{n-1} - \delta^n)}{d(1-\delta^n)}$$

For the profitability of the reward program to firm 1, we should consider the whole n periods. Demand will be higher throughout periods $t = 0$ and $t = n-1$, and rewards will be redeemed in period $n-1$. Sum of discounted profits from the reward program is:

$$\Pi_{1n} = \left(\frac{1}{2} + \frac{1}{2d}\frac{R_n(\delta^{n-1} - \delta^n)}{(1-\delta^n)}\right)\left(d\frac{1-\delta^n}{1-\delta} - kR_n\delta^{n-1}\right)$$

where δ is the discount factor of the firm that we assume to equal the buyers' discount factor. The rightmost term $k\,R_n\,\delta^{n-1}$ is the cost of the rewards redeemed in the last period, and the term before that is the sum of margins d in n periods. Proposition 2 presents the optimal reward, and firm 1's share and profits with the optimal reward.

PROPOSITION 2. The optimal reward and the corresponding demand and profit are:

(a) $R_n^* = \frac{d}{2}\frac{1-k}{k}\left(\frac{1}{\delta^{n-1}} \cdot \frac{1-\delta^n}{1-\delta}\right)$

(b) $q_{1R}^* = \frac{1}{2} + \frac{1-k}{4k}$

(c) $\Pi_{1n}^* = d\frac{(1+k)^2}{8k} \cdot \frac{1-\delta^n}{1-\delta}$

parameter γ, our framework can accommodate the case where firm 2 already has a program.

If $k = 1$, optimal reward R_n* is 0, and firm 1 cannot profit from the loyalty program. When there is no cost efficiency, giving the reward is similar to offering a price discount, thus not profitable.[3] When we have $k < 1$, offering a positive reward increases profits, because the cost of the reward to the firm is less than an equivalent price reduction.

We now present comparative static effects of discounting and program length. Assuming $k < 1$, optimal reward R_n* is decreasing in δ: when there is less time discounting (higher discount factor), firm 1 offers a smaller reward. Correspondingly, profit Π_{1n}* increases as δ increases. As would be expected, the reward R_n* increases with increasing *n*.

To find the optimal purchase requirement *n**, we will compare the present value of profits when the loyalty program is repeated. We let Π_{1n}* denote the present value of profits from a repeated program of length n and reward R_n*:

$\Pi_{1ntot}^* = \Pi_{1n}^* + \Pi_{1n}^*\ \delta^n + \Pi_{1n}^*\ \delta^{2n} + \ldots$

We find that Π_{1ntot}* is independent of *n*. This implies that, if both *n* and R_n are decided by the firm, optimal loyalty program is not unique.[4] This finding is driven by linear utility and constant discounting. This characteristic of the framework is actually favorable because it implies that any additional profit from introducing a second reward must be due to structural changes rather than the program length (see the following section).

3.2. Heterogeneity in Purchase Rate and Two-Rewards

We now consider the case where a fraction of the buyers is light users. Light users buy

[3] This contrasts with Kim, Shi, and Srinivasan (2001) who argue that "inefficient" rewards can be optimal as well. In their framework, prices increase and a segment of buyers do not redeem rewards.

every other period; hence, the relevant discount factor while evaluating the program is $\delta_L = \delta^2 < \delta$. In order to keep the per period market size constant, we assume that half of the light users buy in even numbered periods and the other half in odd periods. The size of the light user segment is 2α, and the size of the regular user segment is $(1-\alpha)$. The preference parameter γ of the light users is also uniformly distributed on the interval $[-d, d]$.

We examine three alternative reward program designs below. Design A offers a second reward optimally designed for the light users. If the firm can offer each reward exclusively to the designated buyer segment, then this is the most profitable design. If, however, buyers can choose, then both segments will choose the second reward. Design B offers a single reward. Design C offers a reward for light users with purchase requirement $n/2$ but the reward is smaller than the optimal level. Design C satisfies the "incentive compatibility" requirement: buyers can choose either reward, but each segment will "self-select" and choose the reward that is intended for their segment.

3.2.1. Design A - Two Optimal Rewards: Assume that, in addition to R_n*, firm 1 offers a second reward R_{nl}* optimally devised for the light user segment. R_{nl}* is obtained by substituting δ^2 for the discount factor in the expression for R_n* provided in Proposition 2. The profit from this design is the highest among all designs A, B, C; however, this scheme is not incentive compatible.

We noted earlier that optimal reward is decreasing in δ; this implies $R_{nl}^* > R_n^*$ because the relevant discount factor for light users is $\delta_L = \delta^2 < \delta$. But, if there are two different rewards

[4] A framework that would lead to an optimal *n* should include contrasting effects; for instance, buyers' increasing dislike versus decreasing reward redemption for higher *n* (or prospect for price increase).

with the same requirement n, and $R_{nl}^* > R_n^*$, then it is obvious that both segments will choose R_{nl}^*. The next proposition states that this problem persists for any reward R_{ml}^* that is optimally devised for light users and has a purchase requirement m -- where $R_m^* = \frac{d(1-k)(1-\delta^{2m})}{2k(\delta^{2m-2}-\delta^{2m})}$.

PROPOSITION 3. If the firm offers "R_{ml}^* for m purchases" and "R_n^* for n purchases", optimized for light and regular users, respectively; then both segments choose R_{ml}^* for all n and m.

Proposition 3 shows that no such scheme with a second optimal reward R_{ml}^* will be incentive compatible.

3.2.2. Design B - Single Optimal Reward: An alternative scheme is offering one common reward R_{nc}^* for n purchases such that R_{nc}^* maximizes the total profits from two segments. Considering the time it takes for light users to qualify for the reward, we find R_{nc}^* by maximizing profits in 2n periods. More specifically:

$$\Pi_{nc} = (1-\alpha)\left(\frac{1}{2}+\frac{1}{2}\frac{R_n(\delta^{n-1}-\delta^n)}{d(1-\delta^n)}\right)\left(\frac{d(1-\delta^{2n})}{1-\delta} - kR_n\delta^{n-1} - kR_n\delta^{2n-1}\right)$$

$$+\ \alpha\left(\frac{1}{2}+\frac{1}{2}\frac{R_n(\delta^{2n-2}-\delta^{2n})}{d(1-\delta^{2n})}\right)\left(\frac{d(1-\delta^{2n})}{1-\delta} - kR_n\delta^{2n-2} - kR_n\delta^{2n-1}\right)$$

Regular users receive the reward twice in periods $n-1$ and $2n-1$. Half of the light users receive the reward in period $2n-2$, and the other half in period $2n-1$. In addition, discount factor for the duration between purchases is δ^2 for light users -- reflected in the first term in the second part. One aspect of this design is that $R_n^* < R_{nc}^* < R_{nl}^*$; that is, the reward addressing both segments is in between the rewards optimal for these segments.

3.2.3. Design C - Incentive Compatible Menu: In section 3.2.1 above we explained that regular users will prefer optimal R_{nl}* (or any R_{ml}* that addresses light users) over R_n*. We now analyze the case where firm 1 offers a second reward "$\overline{R}_{n/2}$ for n / 2 purchases" such that regular users are indifferent between "R_n* for n purchases" and "$\overline{R}_{n/2}$ for n / 2 purchases". For simplicity purposes, we assume that n is even, and regular users pursue R_n* when indifferent. It is straightforward to show that $\overline{R}_{n/2} < \frac{1}{2} R_n$*. The discussion regarding design A implies that regular users would prefer the optimal "$R_{n/2}$* for n / 2 purchases" if it were offered; this requires $\overline{R}_{n/2} < R_{n/2}$*. More importantly, we show in the Appendix that, given R_n* and $\overline{R}_{n/2}$, if regular users are indifferent, then it necessarily follows that light users prefer $\overline{R}_{n/2}$ over R_n*; that is, if the incentive compatibility constraint is satisfied for regular users, it is also satisfied for light users. So, the menu of rewards R_n* and $\overline{R}_{n/2}$ constitutes an incentive compatible scheme separating the two segments.

The intuition for buyers' choices is as follows. The light buyer prefers to receive the smaller reward twice over receiving the larger reward once. This is because the first of the two smaller rewards will be reached earlier than the larger reward. On the other hand, facing the same menu of rewards, a regular buyer might prefer to receive the larger reward. The reasoning is that it would take shorter to reach the larger reward for the regular buyer compared to the light buyer; hence, the larger reward has less discounting advantage against the two smaller rewards when faced by the regular buyer.

In terms of the profits from design C vis-à-vis other designs, it is evident that $\Pi_C < \Pi_A$. Proposition 4 presents the more insightful and important comparison of designs C and B:

PROPOSITION 4.

(a) Given the optimal reward "R_n* for *n* purchases" targeting regular users, the profit maximizing reward for *n*/2 purchases is the incentive compatible $\overline{R}_{n/2} = \frac{d(1-k)}{2k} \frac{(1-\delta^{n/2})}{(1-\delta)\delta^{(n/2)-1}}$.

(b) Two-level reward program C is more profitable than the optimal single-reward program B, as long as $\alpha < \alpha_0$ where $\alpha_0 \in (0, 1)$.

Part (b) implies that unless the light user segment dominates the market, offering R_n* for regular users and $\overline{R}_{n/2}$ for light users is better. When the light user segment is large (that is, $\alpha > \alpha_0$), suboptimal profit from light users due to suboptimal $\overline{R}_{n/2}$ cannot be compensated by the profit from regular users. In this case, the firm is better off by offering one common reward R_{nc}* (which would be closer to the optimal reward for light users than the optimal reward for regular users). To give an idea, the threshold α_0 is in the range of 0.7 - 0.9 for applicable parameter values of *n*, *k*, δ, and *d*.

3.3. Discussion

Our analysis indicates that the firm can increase profits by offering two rewards with different purchase requirements when there are both regular users and light users in the market. We examined a particular menu of rewards where the larger reward R_n* is chosen to maximize profits from the regular users segment and the smaller reward $\overline{R}_{n/2}$ is chosen such that the incentive compatibility constraint is satisfied for both the regular and the light users. We have shown that when the smaller reward $\overline{R}_{n/2}$ is chosen such that regular users are indifferent between R_n* and $\overline{R}_{n/2}$, light users will prefer $\overline{R}_{n/2}$. As R_n* is offered for *n* purchases and $\overline{R}_{n/2}$ is offered for *n* / 2 purchases, regular users receive a larger reward per purchase than light users do, because $2\overline{R}_{n/2} < R_n$*. In other words, a multi-level loyalty program can be used as a non-linear (second-degree) price discrimination mechanism.

We are cautious not to claim that Design C, and Proposition 4, yields maximum profits from any incentive compatible two-reward structure. There is room for improving over Design C. Consider the finding "offering two rewards is more profitable unless the light user segment is large"; this is based on the menu of rewards R_n* and $\overline{R}_{n/2}$. If the smaller reward was chosen as optimal $R_{n/2}$* and the larger reward R_n was tailored with consideration to the incentive compatibility constraint, we presume that we would find the opposite (i.e. "two rewards is more profitable unless the regular user segment is large"). Presumably one may choose the two rewards in different ways depending on segment sizes, and devise a two-reward program that is better than a one-reward one for any α. More generally, the firm could choose the rewards R_n and R_m, and the requirements *n* and *m* to maximize its lifetime profits subject to the incentive compatibility constraints. Such an approach should yield higher profits than Design C, and hence extend the range of α over which two-rewards is more profitable than one.

Consumer preference for small versus large rewards may well be explained by their discount factors. Assume that customers *x* and *y* both purchase the item every period but *x* discounts future payoffs more than *y* does. In other words, customer *x* has a lower discount factor. For illustrative purposes, assume that $\delta_x = \delta_y^{1/2}$. If, for customer *x*, we define a new time period length which is half of the original period, she can be considered as having the same discount rate as *y*, but purchasing in every other period. Hence, customer *x* may prefer receiving the small reward if a menu of rewards is offered by the firm (for consistency with the earlier analysis, we can assume that the discount factor of the firm coincides with customer *y*). Therefore, the firm may introduce multiple rewards in response to discount rate heterogeneity as well. And so, individuals preferring the larger reward may be regular users or may have lower discounting (higher discount factors).

In our analysis, we abstracted from the cost of introducing and administering a loyalty program; such costs would decrease the program profitability. There are additional costs that come with multiple rewards, such as time and effort spent on managing and communicating multiple rewards, a higher cost parameter *k* due to a decrease in economies of scale vis-à-vis the one-reward case, etc. Including an additional (fixed) cost parameter for multi-rewards would narrow the range of α where offering two rewards is more profitable than offering only one. This may explain why many loyalty programs in the marketplace offer a single reward -- after all, there should be some heterogeneity among buyers.

4. Conclusion

The current paper argues that each reward in a multi-level loyalty program is intended for a different type of buyer. As the larger reward is also larger per purchase requirement, compared to the smaller reward (to ensure incentive compatibility), buyers who redeem the larger reward end up paying a lower net price per unit product. And so, a multi-reward program is essentially a menu based (second degree) price discrimination mechanism. We note that in our framework, rewards differ on a vertical dimension, such as quality or amount -- and if the rewards were immediate payoffs without any requirements, all consumers would prefer the larger reward to the same degree. The premise of this paper is that there would be a systematic difference in the purchase rates and / or time discounting of buyers who prefer the larger versus smaller rewards in a loyalty program.

Other studies that consider regular (heavy) and light users typically focus on profitability from these segments. In Lal and Bell (2003), there are loyal customers and casual shoppers (cherry pickers), and the trade-off for the firm is about the increased sales to casual shoppers versus rewards given to already loyal customers. Wansink (2003) reports that brand managers may overestimate the importance of targeting heavy users, and may underestimate the effectiveness of programs targeting light users which may generate higher incremental sales and profits. In our model regular and light users are not different in terms of loyalty to the firm or likelihood of switching; they differ in their demand per period of time. Loyalty program benefits the firm by attracting both types of customers from the competitor.

Our analysis mainly relies on a comparison of the profitability of a multi-level reward structure versus a single-level one. In our framework, the no-loyalty programs case provides us with the price $P = c + d$ which is a starting point for the analysis of loyalty programs. This enables us to examine our problem without having to assume an arbitrary price term. The qualitative nature of the findings would not change if we used a general price term. We recognize that our framework is not necessarily suited to examining the optimality / profitability of loyalty programs in general (see Kopalle and Neslin 2003). We exclude features such as firm’s ability to raise prices, category expansion, and consumers’ overestimation of reward redemption likelihood (see Accenture 2006; Drèze and Hoch 1998).

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Technical Appendix

Proof of Proposition 1

Given firm j's price P_j, firm i's profit as a function of P_i is follows:

$$\Pi_i = \left(\frac{P_j - P_i + d}{2d}\right)(P_i - c)$$

We solve $\frac{\partial \Pi_i}{\partial P_i} = 0$ for a given P_j to obtain the best-response function for firm i. Because the firms are symmetric, in Nash equilibrium $P_1 = P_2$, so we obtain $P_1 = P_2 = c + d$. Firms have equal market shares: $q_1 = q_2 = 1/2$; and equal profits: $\Pi_1 = \Pi_2 = d/2$ ■

Proof of Proposition 2

Firm 1's profit with the loyalty program (R_n, n) is $\Pi_{1n} = \left(\frac{1}{2} + \frac{1}{2}\frac{R_n(\delta^{n-1} - \delta^n)}{d(1-\delta^n)}\right)\left(\frac{d(1-\delta^n)}{1-\delta} - kR_n\delta^{n-1}\right)$.

Given n, the profit maximizing reward R_n is defined by $\frac{\partial \Pi_{1n}}{\partial R_n} = 0$, and equals $R_n* =$ $\frac{d(1-k)(1-\delta^n)}{2k(\delta^{n-1} - \delta^n)}$. We substitute R_n* into the demand function given in the text to obtain $q_{1R}* = \frac{1}{2} + \frac{1-k}{4k}$. We substitute R_n* into Π_{1n} to find the profit $\Pi_{1n}* = \frac{d(1+k)^2(1-\delta^n)}{8k(1-\delta)}$ ■

Characteristics of the Optimal Loyalty Program (Homogeneous Buyers)

Claim: (i) R_n* is decreasing in δ, (ii) $\Pi_{1n}*$ is increasing in δ, (iii) $R_n*$is increasing in n.

Proof: (i) $\frac{\partial R_n*}{\partial \delta} = \frac{d(1-k)}{2k\delta^n(1-\delta)^2} \cdot \text{A}$ where A = $(1 - \delta^n - n + \delta n)$.

$\frac{\partial A}{\partial \delta} = n(1-\delta^{1-n}) < 0$, and for an arbitrarily small δ > 0, A < 0; so A < 0 for all δ < 1.

Since $\frac{d(1-k)}{2k\delta^n(1-\delta)^2} > 0$ for $k < 1$, this proves that $\frac{\partial R_n^*}{\partial \delta} < 0$.

(ii) We observe that $\frac{1-\delta^n}{1-\delta}$ is increasing in δ.

(iii) $\frac{\partial R_n^*}{\partial n} = \frac{d(k-1)Log(\delta)}{2k\delta^{n-1}(1-\delta)} > 0$; because $Log(\delta) < 0$; $k - 1 < 0$. ■

The Choice of Program Requirement *n*

<u>Claim:</u> When $R_n = R_n^*$, the maximum profits are independent of n.

<u>Proof:</u> To compare profits across programs with different n values, we use profits from infinitely repeated programs: $\Pi_{1ntot}^* = \Pi_{1n}^* + \Pi_{1n}^* \delta^n + \Pi_{1n}^* \delta^{2n} + \ldots = \sum_{t=0}^{\infty} \Pi_{1n}^* \delta^{nt} = \frac{d(1+k)^2}{8k(1-\delta)}$.

Hence, total discounted profit does not depend on n. ■

Heterogeneous Buyers

A. Two Optimal Rewards

Proof of Proposition 3

Suppose the firm offers two reward programs, (R_n^*, n) that is optimal for the regular users, and (R_{ml}^*, m), that is optimal for the light users. $R_{ml}^* = \frac{d(1-k)(1-\delta^{2m})}{2k(\delta^{2m-2}-\delta^{2m})}$ is obtained by replacing δ by δ^2 in Proposition 2. A regular user will prefer R_{ml}^*, if $\sum_{t=1}^{\infty} R_{ml}^* \delta^{mt-1} > \sum_{t=1}^{\infty} R_n^* \delta^{nt-1}$. We substitute for R_n* and R_{ml}*, and rewrite the inequality as $\frac{d(1-k)}{2k}\left(\frac{1-\delta^{m-1}+\delta^{m}+\delta^{m+1}}{(\delta^{m-1}-\delta^{m+1})(1-\delta)}\right) > 0$.

As this inequality holds, for any m, a regular user will choose (R_{ml}*, m) over (R_n*, n). ■

B. Single Optimal Reward

Because the light users reach the reward in 2n periods, we consider profits over 2n periods:

$$\Pi_{nc} = (1-\alpha)\left(\frac{1}{2}+\frac{1}{2}\frac{R_n(\delta^{n-1}-\delta^{n})}{d(1-\delta^{n})}\right)\left(\frac{d(1-\delta^{2n})}{1-\delta}-kR_n\delta^{n-1}-kR_n\delta^{2n-1}\right)$$

$$+\ \alpha\left(\frac{1}{2}+\frac{1}{2}\frac{R_n(\delta^{2n-2}-\delta^{2n})}{d(1-\delta^{2n})}\right)\left(\frac{d(1-\delta^{2n})}{1-\delta}-kR_n\delta^{2n-2}-kR_n\delta^{2n-1}\right)$$

The profit maximizing R_n is defined by $\partial\Pi_{nc}/\partial R_n = 0$. Solving this for R_n yields:

$$R_{nc}^* = \frac{d(1-k)(1-\delta^{2n})(\delta+\delta^{n+1}-\delta\alpha+\delta^{n}\alpha)}{2k(1-\delta)\delta^{n}(1+2\delta^{n}+\delta^{2n}-\alpha-2\delta^{n}\alpha+\delta^{2n-2}\alpha+2\delta^{2n-1}\alpha)}$$

Corresponding maximum profit Π_{nc}* is a lengthy expression and is omitted here.

<u>Claim:</u> R_n* < R_{nc}* < R_{nl}*.

<u>Proof</u>: Profit in 2n periods can be considered as $\Pi_{nc} = (1-\alpha) f_1 + \alpha f_2$, where R_n = R_n* maximizes f_1 and R_n = R_{nl}* maximizes f_2, and these maximums are equal. Therefore, R_n = R_{nc}* that maximizes Π_{nc} cannot be smaller than R_n* or larger than R_{nl}*.

C. Incentive Compatible Menu

We consider two reward programs, (R_n*, n), and (R_{ml}, m), such that (i) R_n* is optimally chosen for regular users for a given n, and (ii) a regular user chooses (R_n*, n) over (R_{ml}, m). Since a regular user prefers (R_{ml}*, m) over (R_n*, n), we must have R_{ml} < R_m*. A regular user will choose

(R_n*, n) over (R_{ml}, m) if $\sum_{t=1}^{\infty} R_n * \delta^{nt-1} \geq \sum_{t=1}^{\infty} R_{ml} \delta^{mt-1}$. The largest R_m that satisfies the condition is

$\overline{R}_{ml} = R_n * \frac{\delta^{n-1}}{1-\delta^n} \frac{1-\delta^m}{\delta^{m-1}}$.

<u>Claim:</u> The light users will choose ($\overline{R}_{ml}$, m) over (R_n*, n) for $m < n$.

<u>Proof:</u> The light users will choose (R_{ml}, m) if $R_{ml} \frac{\delta^{2m-2}}{1-\delta^{2m}} \geq R_n * \frac{\delta^{2n-2}}{1-\delta^{2n}}$. We substitute for $\overline{R}_{ml}$ and rewrite as $R_n * \left(\frac{\delta^{n-1}}{1-\delta^n} \frac{1-\delta^m}{\delta^{m-1}} \right) \frac{\delta^{2m-2}}{1-\delta^{2m}} \geq R_n * \frac{\delta^{2n-2}}{1-\delta^{2n}}$, and simplify to $\frac{\delta^{m-1}}{1+\delta^m} > \frac{\delta^{n-1}}{1+\delta^n}$, which holds if $m < n$. ■

Let $\overline{R}_{n/2}$ designate $\overline{R}_{ml}$ when $m = n/2$; $\overline{R}_{n/2} = R_n * \frac{\delta^{n-1}}{\delta^{n-1} + \delta^{(n/2)-1}} = \frac{d(1-k)}{2k} \frac{(1-\delta^{n/2})}{(1-\delta)\delta^{(n/2)-1}}$.

We have $\overline{R}_{n/2} < \frac{1}{2} R_n *$ because $\delta^{(n/2)-1} > \delta^{n-1}$.

Proof of Proposition 4

Profits from the light users in design C in $2n$ periods is:

$$\Pi_C = \alpha \left(\frac{1}{2} + \frac{1}{2} \frac{R_{n/2}(\delta^{n-2} - \delta^n)}{d(1-\delta^n)} \right) \left(\frac{d(1-\delta^{2n})}{1-\delta} - kR_{n/2}\delta^{n-2} - kR_{n/2}\delta^{n-1} - kR_{n/2}\delta^{2n-2} - kR_{n/2}\delta^{2n-1} \right)$$

(a) Π_C is a quadratic function of $R_{n/2}$; so it is increasing in $R_{n/2}$ for $R_{n/2} \leq \overline{R}_{n/2}$.

(b) We can write the profit difference $\Pi_C - \Pi_B$ as (E×F)/G, where

$E = d(1-k)^2 \alpha \delta^{(n/2)-2}(1+\delta)(-1+\delta^{n/2} - \delta^n + \delta^{3n/2})$

$F = (\delta^{(3n/2)+2} + \delta^{(3n/2)+3} - 2\delta^{2n+2} + (1-\alpha)(-2\delta^3 + \delta^{(n/2)+2} + \delta^{(n/2)+3}) + \alpha(\delta^{5n/2} - 2\delta^{2n+1} + \delta^{(5n/2)+1})$,

G = $8k(1-\delta)(1+\delta^{n/2})(\delta^{2n+2}+(1-\alpha)(\delta^2+2\delta^{n+2})+\alpha(\delta^{2n}+2\delta^{2n+1}))$

Let $\alpha \in (0, 1)$. Then E < 0 and G > 0; therefore, the overall sign depends F.

Claim: F is strictly increasing in α. F < 0 if $\alpha = 0$, F > 0 if $\alpha = 1$.

Proof: $\dfrac{\partial F}{\partial \alpha} = 2\delta^3 - \delta^{(n/2)+2} - \delta^{(n/2)+3} + \delta^{5n/2} + \delta^{(5n/2)+1} - 2\delta^{2n+1}$

$> 2\delta^3 - \delta^{(n/2)+2} - \delta^{(n/2)+2} + \delta^{5n/2} + \delta^{5n/2} - 2\delta^{2n+1}$

$= (2\delta^3 - 2\delta^{2n+1}) - (2\delta^{(n/2)+2} + 2\delta^{5n/2}) = 2\delta^3(1-\delta^{2n-2}) - \delta^{(n/2)+2}(1-\delta^{2n-2}) \geq 0$

Let $\alpha = 0$. Then F = $\delta^2(-2\delta + \delta^{n/2} + \delta^{(n/2)+1} + \delta^{3n/2} + \delta^{(3n/2)+1} - 2\delta^{2n})$

$< \delta^2(-2\delta + \delta^{n/2} + \delta^{n/2} + \delta^{(3n/2)+1} + \delta^{(3n/2)+1} - 2\delta^{2n})$

(note that substituting $\delta^{n/2}$ instead of $\delta^{(n/2)+1}$ increases the expression more than the decrease due to substituting $\delta^{(3n/2)+1}$ instead of $\delta^{3n/2}$)

$< \delta^2\left[-2\delta + 2\delta^{n/2} + \delta^{3n/2}(2\delta - 2\delta^{n/2})\right] < 0$

Now let $\alpha = 1$. Then F = $\delta^{3n/2}(1+\delta)(\delta - \delta^{n/2})^2 > 0$.

The claim together with the intermediate value theorem imply that there is a unique $\alpha_0 \in (0, 1)$ such that F < 0 if $\alpha < \alpha_0$, and F > 0 if $\alpha > \alpha_0$. This implies that $\Pi_C - \Pi_B > 0$ when $\alpha < \alpha_0$. ■