Income Inequality and Sovereign Default*

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Abstract

In this paper, we study the role of income inequality in government's borrowing and default decisions. We consider a standard endogenous sovereign debt default model and extend it to allow for heterogeneous agents. In addition to shocks to the average income level, we consider the effect of shocks to income distribution. Consistent with the data, income dispersion across individuals increases during a recession and decreases in an expansion. The model is calibrated to match a number of stylized facts for Argentina. We show that (i) rising income inequality within a country increases the probability of default significantly; (ii) the effect of inequality shocks is larger than the effect of output shocks; (iii) the joint effect of these two shocks can generate a high default probability consistent with the Argentine data; (iv) progressive income taxes can reduce the default risk.

Keywords: Sovereign debt; Default; Income Inequality; Redistribution.

JEL codes: F3, F4, E5, D5

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1 Introduction

Emerging markets tend to experience high rates of sovereign default and they are also characterized by a high income inequality that has been changing over time. In sovereign default literature, the line of research following Eaton and Gersovitz (1981) has focused mainly on the effects of output shocks on the government's borrowing and default decisions. In this paper, we study the role of income inequality in government's borrowing and default decisions. Does higher income inequality increase the probability of default? Furthermore, how do changes in inequality compare to changes in output in explaining the variation in default probability?

Figure 1: Partial Regression Plots $E(ratings|X)$ **vs** $E(Gini|X)$

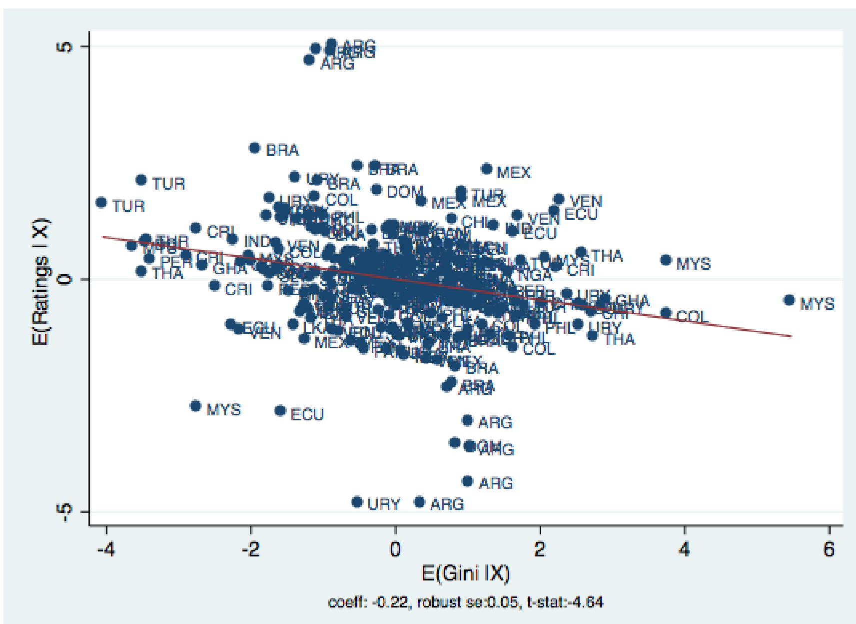


Figure 1 presents empirical evidence that shows the relationship between income inequality and the sovereign bond ratings. The figure shows the scatter plots and the negative relationship between income inequality measured by the Gini index and the credit worthiness of countries' bond ratings, which proxy default risk.[1] We estimate the partial regressions, controlling for external debt-to-gdp ratio, income per capita, country and time fixed effects, using a panel dataset that covers 23 emerging economies from 1994-2012.[2] The

[1]Reinhart et al. (2003) show that sovereign bond ratings are good predictors of default.

[2]Following the empirical papers that study the determinants of a country's default risk, we also use

results suggest that there is a positive correlation between income inequality and default risk.

In order to explore the relationship between endogenous default risk and income inequality, we consider a stochastic general equilibrium model following an approach similar to that of Eaton and Gersovitz (1981). We model a small open economy with two types of households. In addition to output shocks that affect the average level of endowment, we introduce shocks that affect its distribution, which we call inequality shocks.[3] The economy is subject to aggregate uncertainty about future endowments, and households cannot completely insure against the shocks. The output and inequality shocks have different effects on the endowments; an adverse output shock lowers the endowments of both types, but an adverse inequality shock raises the endowment of the rich households and reduces the endowment of the poor households, increasing the dispersion between the endowments. There is also a benevolent government that represents the preferences of the households and can issue non-state-contingent, one-period bond contracts to borrow from risk-neutral foreign lenders, retaining the option to default at any time. We assume that default entails exogenous drops in output and that the economy goes into autarky temporarily. The government internalizes how its borrowing decisions affect the default risk, as well as the price of bonds, which determines the interest rates.

In our model, the government would like to borrow on behalf of households for two reasons. First, the government uses bond contracts and rebates the proceeds of debt operations equally across households to help them smooth consumption. Second, the equilibrium interest rate is lower than the discount rate of the government, so the government would like to shift future consumption to today by borrowing. The level of existing debt and the size of the shocks are crucial for government's borrowing decision. As the debt accumulates, it becomes harder to roll over because the benefits of borrowing diminish. Defaults are particularly more attractive in recessions, in high inequality states and when there is high debt accumulation because foreign lenders offer bond contracts that have higher interest rates in those states, which creates a borrowing constraint for the government. The government's goal is to maximize household's expected lifetime utilities, so it achieves this goal by trying to equalize the marginal utilities of consumption between households and across time. Default can reduce the gap between in the marginal utilities of consumption between the two

ratings grade data and total external debt scaled by GDP in this analysis. See Cantor and Packer (1996), Reinhart (2002) and Reinhart et al. (2003).

[3] Even though our model treats the changes in income inequality as exogenous, these shocks can be motivated by the fact that idiosyncratic labor earnings risk exhibits countercyclical volatility, as shown by Storesletten et al. (2004).

types of households because the burden of debt payment can be eliminated. Consequently, in our model, default can serve as a redistribution mechanism that improves households' welfare. The main finding of this paper is that inequality shocks can increase the default risk significantly. The key intuition for this result is that when the economy is subject to both adverse inequality and output shocks, the marginal utility of consumption of the poor increases significantly relative to the marginal utility of consumption of the rich. This generates a large tax burden, particularly on poor households, and the government chooses to default more often to wipe out the debt burden.

When we consider the role of each shock, we show that there is a stark difference between the effects of the two. We find that inequality shocks can generate more than ten times the probability of default than output shocks. When there is only an output shock, its effect on the dispersion between marginal utilities of consumption of households is relatively small, so the government's borrowing is more about achieving smooth consumption intertemporally. The small dispersion is coupled with low heterogeneity in the endowments. The benefits from default are less because the tax burdens on households are similar. On the other hand, when there is only an inequality shock, even though the total endowment in the economy remains constant, it becomes harder to achieve equal marginal utilities between households and across time. As mentioned above, the high burden of debt finance, especially on the poor, and the redistributive effect of default make default an attractive policy for the government. However, the inequality shocks alone can generate only half of the probability of default observed when there are both shocks in the model. Thus, we show that the joint effect of these shocks helps the model generate a default probability consistent with the data. The reason behind this result is the VAR(1) process estimated from the Argentine data. Based on the estimates of the structural parameters, we find that high inequality at time $t-1$ leads to lower output at time t. Also, the estimates of the covariances of the shocks are negative, which implies that it is more likely to have an adverse output shock together with an adverse inequality shock. These characteristics play an important role in lenders' and the government's expectations about the future state of the economy. An adverse inequality shock not only amplifies the effect of a low output shock today, but also creates a deep-seated pessimism that the recession will be more severe in the future. As a result, foreign lenders ask for a higher premium even for smaller levels of debt. This increases the borrowing constraints for the government, and default becomes the optimal decision.

The model is calibrated using Argentine data between 1990-2002, and we simulate the model to generate the business-cycle statistics. Our model's results regarding the default probabilities can be compared to the results in Aguiar and Gopinath (2006).[4] Similar to Aguiar and Gopinath (2006), the default probability when the economy is hit by an output shock is 0.1 percent in our model. Aguiar and Gopinath (2006) also use shocks to the trend of output and generate a default probability of around two percent, while in our model, the inequality shocks generate a default probability of 1.8 percent. Using shocks to both output and inequality, our model can match countercyclical interest rates, high volatility of consumption and output, and a positive correlation between the trade balance and interest rate spreads. In addition to that, inequality is countercyclical with output and positively correlated with interest rate spreads.

We also calibrate the model to match the business cycle statistics of Greece and Turkey. Since their independence, Greece and Turkey experienced external debt defaults or restructuring for five and seven times, respectively. We find that the model also generates countercyclical interest rates, trade balance and procyclical consumption. Moreover, we also find that income inequality is negatively correlated with output for these countries. Overall, our model can account well for the business-cycle characteristics observed in these economies.

As a policy implication, we extend the model by introducing progressive income taxes and analyze the effect of taxes on the debt levels and the default probability. When it is costly to borrow for the government, i.e. the proceeds of the debt operations are negative, the government finances the existing debt by issuing progressive income taxes. We adopt the progressive tax regime that Heathcote et al. (2014) presents. However, when it is cheap to borrow, the government does not tax households, it simply distributes the transfers across households. We show that as the progressivity of the tax increases, the probability of default decreases. The tax system helps eliminate the effect of inequality shocks in the model and reduces the dispersion in the marginal utilities of consumption between households. Therefore, we obtain larger debt in the simulated economies.

This paper relates to the recent quantitative models that explore emerging markets' business cycles and sovereign debt. We contribute to the sovereign default literature by incorporating the role of income inequality as an additional source of default risk. The endogenous sovereign default literature starts with the seminal paper of Eaton and Gersovitz (1981) and continues with Aguiar and Gopinath (2006), Arellano (2008), Martin and Ventura (2010), Yue (2010), Pitchford and Wright (2011), Chatterjee and Eyigungor (2012a),

[4] Aguiar and Gopinath (2006) assume a representative agent model; yet their default penalty structure and calibration strategy are similar to ours.

Amador and Aguiar (2014) and Gennaioli et al. (2014), some of which were mentioned above.[5] Hatchondo and Martinez (2009), Arellano and Ramanarayanan (2012), and Chatterjee and Eyigungor (2012b) consider long maturity bonds in a representative agent framework. Cuadra and Sapriza (2008) and Hatchondo et al. (2009) study the role of political uncertainties in sovereign default risk. Martin and Ventura (2010) and Broner et al. (2008) show that well functioning secondary markets can eliminate the default risk. All these papers use representative agent models and focus on the role of output shocks. Our paper is also closely related to D'Erasmo and Mendoza (2012) and D'Erasmo and Mendoza (2013), the main focus of which is the relationship between wealth inequality and default using a heterogeneous agent framework. D'Erasmo and Mendoza (2012) have endogenous wealth heterogeneity that comes from idiosyncratic income shocks; however, the amount of bonds is determined by a fiscal reaction function and does not come from the maximization of household utility. As mentioned above, in our model, the government optimally chooses the level of next-period bonds taking into account the welfare of the households. Furthermore, we show that income inequality shocks tend to have a systematic relationship with output shocks, so we incorporate this dimension into our model to generate inequality. D'Erasmo and Mendoza (2013) study the distributional effects of sovereign debt default in a two-period, closed economy model, assuming an exogenous initial wealth distribution. In their closed economy setup, they study optimal debt and default decisions on domestic debt. However, in our model, we focus on borrowing and default on external debt in a small open economy. In this sense, our paper is complementary to these two papers. Cuadra et al. (2010) study fiscal policy and default risk using a representative agent model, in which tax on consumption is endogenously determined and the revenues are used to finance public goods. In our paper, we assume progressive taxes on income. Our paper is also related to the immense empirical literature that studies the determinants of sovereign default. Cantor and Packer (1996) show that income, external debt and economic development are significant determinants of credit risk. Reinhart et al. (2003) show that a country's past behavior about meeting its debt obligations can be a good predictor of its ability to pay future debt, pointing out the importance of financial institutions. Hatchondo et al. (2007) argue that countries are more likely to default during periods with low resources, high borrowing costs and changes in political circumstances, and González-Rozada and Yeyati (2008) examine the role of global factors, such as liquidity, risk appetite and contagion, in explaining the emerging market spreads.

[5]Also see Panizza et al. (2009), Wright (2011) and Aguiar and Amador (2013) for good reviews of this literature.

The rest of the paper is organized as follows: We provide a more formal analysis of the empirical results, showing the relationship between income inequality and credit scores in Section 2. We then present the model and define the recursive equilibrium in Section 3. We discusses the calibration, the quantitative analysis of the model and the simulation results with the counterfactual experiments in Section 4. Section 5 presents the business cycle statistics obtained for other countries, Greece and Turkey. Section 6 shows the effects of income taxes. Section 7 concludes.

2 Empirical Motivation

In this section, we provide empirical results that support the relationship between income inequality and default risk. We use credit ratings dataset as a measure of default risk. Reinhart (2002) shows that credit ratings can predict defaults well.[6] First, we show that income inequality is positively correlated with the creditworthiness of sovereign bonds. Next, we provide evidence on the fact that income inequality is countercyclical over the business cycle.

2.1 Income Inequality and Credit Ratings

Reinhart et al. (2003) show that there is a strong relationship between external debt and credit ratings. In order to present some empirical evidence for the effect of inequality on the credit worthiness of sovereign bonds, we follow an approach similar to that in Reinhart et al. (2003). We use the following specification to estimate the effect of inequality on credit scores:

$$\begin{aligned} \textit{Credit Score}_{i,t} &= \alpha_0 + \alpha_1 \text{Gini}_{i,t-1} + \alpha_2 \textit{Debt-to-GDP}_{i,t-1} \\ &+ \alpha_3 \textit{GDP per capita}_{i,t-1} + u_i + z_t + \textit{error}_{i,t} \end{aligned} \quad (1)$$

To measure the creditworthiness of sovereign bonds, we use the Fitch credit ratings data for long-term bonds that are issued under foreign currency. This dataset covers a period between 1994 and 2012. For income inequality, we use the Gini indices provided by the Standardized World Income Inequality Database (SWIID) (Solt, 2009). This is an unbalanced panel dataset that has information on inequality for 153 countries covering 1960 to 2012. Debt-to-GDP ratio is the external debt-to-GDP ratio from the Reinhart-Rogoff series that extends until 2010. Most of this dataset comes from IMF's Standard Data Dissemination Service, and it is defined as the outstanding amount of those actual current liabilities that require payments of principal and/or interest that residents of an economy owe to non-residents (Statistics, 2003). The GDP per capita series is from the World Bank, and we take its log for this estimation. The Net foreign assets-to-GDP (NFA/GDP) data used in Table 1 are from the External Wealth of Nations Mark II database by Lane and Milesi-Ferretti (2007). This series includes net foreign assets (NFA) using FDI or equity

[6] They show that this relationship is robust using various credit-score datasets such as Institutional Investor ratings, Standard and Poor's and Moody's.

assets and liabilities estimated using different methodologies. NFA is defined as the sum of the net debt position, the net equity position and the net FDI position in Lane and Milesi-Ferretti (2001). In order to perform a regression using the credit ratings, we assign a numerical value similar to that in Cantor and Packer (1996) and Reinhart (2002). Table 14 shows the conversion of the ratings to scores in Appendix.

We expect to obtain a negative coefficient on Gini and debt-to-GDP ratio and a positive coefficient on GDP per capita. This implies that higher inequality in country i at time $t-1$ reduces the credit score in the next period. The credit score of a country shows how risky that country's bond is, and higher inequality increases the riskiness, which is reflected by a lower credit score.

Table 1 shows the summary statistics for the variables used for the regression sample, which covers the period 1994-2010 and contains 45 countries. A couple of differences stand out when we compare observations of emerging markets and advanced economies. First, emerging markets have low ratings even though their debt-to-GDP ratios are not very high. When the ratio of net foreign assets to GDP is considered, emerging markets are, on average, more indebted than advanced countries. Second, they also have higher income inequality and lower GDP per capita than advanced economies have.

We estimate equation 1 using year (z_t) and country (u_i) fixed effects. We are interested in analyzing the effect of inequality that varies over time; therefore, country fixed effects will control for time-invariant characteristics unique to a country. In the first specification, we find that an increase in a country's external debt-to-GDP ratio is associated with lower credit worthiness in the next period. This is a standard result in the literature, as well. In the second specification, we introduce GDP per capita in log terms, and we find that an increase in income is associated with an increase in country's credit worthiness. Finally, the last specification shows the relationship between income inequality and credit ratings. We find that an increase in Gini index is negatively associated with the credit worthiness in the next period. The estimate is significant at ten percent and robust to country and time fixed effects. In order to get an economic interpretation of the estimates, we do a simple back-of-the-envelope calculation based on the third specification. The median score in the sample is 13, which corresponds to BB+. We estimate the third specification separately for each country. Then, we increase each variable by its one standard deviation and compare their effects on the score for each country. We find that, on average, a one standard deviation increase in external debt reduces the credit score by 0.97 and a one standard deviation increase in log GDP increases the credit score by only 0.01 point. On the other hand, a

Table 1: Country Ratings, Debt, Income Inequality, GDP per capita and Net Foreign Assets

Country	Average Fitch Rating	Average external debt/GDP	Average Inequality	Average GDP per capita	Average NFA/GDP
		Emerging market economies			
Argentina	CCC-/CCC	72.49	45.19	4,483	-21.82
Bolivia	CCC+/B-	55.64	50.16	1,086	-64.71
Brazil	B+/BB-	28.82	50.22	4,739	-32.47
Bulgaria	BB-/BB	85.21	28.38	3,613	-53.71
Chile	BBB+/A-	47.71	49.34	7,131	-30.29
China	BBB+/A-	12.99	48.27	1,573	8.65
Colombia	BB/BB+	30.64	50.38	3,386	-25.00
Costa Rica	BB-	29.18	44.08	4,680	-21.82
Dominican Rep.	CCC+	26.71	45.80	3,928	-35.65
Ecuador	CCC-/CCC	49.55	50.02	3,005	-49.82
Egypt, Arab Rep.	BB/BB+	33.13	35.43	1,220	-11.76
El Salvador	BB-/BB	40.51	45.47	2,698	-38.53
Ghana	B-/B	75.86	40.01	504	-53.53
India	BB/BB+	19.45	49.57	762	-19.71
Indonesia	B/B+	65.47	55.45	1,283	-60.76
Korea, Rep.	BBB+/A-	31.30	31.60	16,643	-12.94
Malaysia	BBB-/BBB	46.50	47.61	5,296	-21.18
Mexico	BB/BB+	30.22	47.03	7,586	-34.94
Nigeria	B+	18.33	42.46	920	-57.76
Panama	BB/BB+	56.91	49.64	4,747	-77.94
Peru	BB-/BB	40.70	50.92	3,038	-41.59
Philippines	BB-/BB	62.05	50.79	1,195	-42.29
Romania	BB-	38.69	30.06	4,447	-28.47
Russia	BB	43.34	30.31	4,928	1.24
Sri Lanka	CCC+/B-	44.52	41.16	1388	-45.24
Thailand	BB+/BBB-	46.69	52.70	2,623	-39.59
Turkey	B/B+	43.44	45.29	6,584	-35.29
Uruguay	B+/BB-	43.91	43.18	5,447	-11.53
Venezuela	B/B+	35.65	41.59	5,500	5.71
		Advanced economies			
Australia	AA-/AA	59.96	31.32	30,7901	-54.47
Canada	AA/AA+	69.65	30.18	30,870	-18.47
Denmark	AA/AA+	96.08	22.40	43,164	-12.53
Finland	AA-/AA	72.71	22.35	29,175	-46.24
Greece	BBB/BBB+	93.64	33.64	19,689	-53.94
Hungary	BBB-/BBB	85.08	28.36	9,881	-81.76
Italy	A+/AA-	84.03	33.60	29,355	-17.53
Japan	AA-/AA	33.66	28.38	34,743	33.94
New Zealand	AA-/AA	84.00	32.38	27,242	-84.35
Norway	AA+	44.65	23.41	57,064	38.59
Poland	BB/BB+	41.43	30.01	6,960	-39.41
Portugal	A-/A	138.62	35.05	17,497	-59.29
Singapore	AA/AA+	154.52	42.92	25,595	1.78
Spain	AA/AA+	90.52	32.88	23,920	-47.76
Sweden	AA-	92.11	23.39	34,421	-25.41
United States	AA+	60.68	36.80	41,165	-14.76

List of countries used in the panel regression. Time period covers 1994-2010. Data sources from left to right: Fitch, Rainhart-Rogoff series, SWIID, the World Bank and the External Wealth of Nations Mark II database.

Table 2: Panel Regressions Explaining Creditworthiness with Debt Ratios, GDP per capita and Inequality

Independent Variable	Dependent Variable: Score of country i in year t. (1)	(2)	(3)
External debt-to-GDP at $t-1$	-0.0221^{***}	-0.0146^{***}	-0.0122^{**}
	(0.0057)	(0.0050)	(0.0048)
GDP per capita at $t-1$	–	9.5976^{***}	10.0130^{***}
		(2.5606)	(2.5013)
Gini at $t-1$			-0.0698^{*}
			(0.0360)
Year fixed effects	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes
No of countries	45	45	45
N	568	568	568

Sample period is between 1994 and 2010. The dependent variable is the credit score of country i in year t. Estimation is by robust standard errors. Standard errors are reported in parentheses. Per capita GDP is in logs. (***,**,* represent significance level at 1%, 5% and 10%, respectively.)

one standard deviation increase in Gini reduces the credit score by 0.21 point. The largest effect comes from the external debt-to-GDP, but the change in the Gini index also has a substantial effect.

The regression results are also robust to the use of the Gini series obtained from The World Bank. The regression sample constructed using this series has 40 countries and covers the period 1994-2009. This sample has similar statistics for the median and standard deviation of debt and and GDP per capita data. The median inequality in this sample is 40.8, and it has a larger standard deviation of 5.57. The estimation is again done with the inclusion of country and time fixed effects. The results are reported in Table 15 in the Appendix. Using the same specification, we find that the estimate for inequality is larger and significant at five percent. However, external debt-to-GDP loses its significance when GDP per capita and the Gini index are included in the specification.

2.2 Income Inequality over the Business Cycle

In order to support our theory that income inequality plays a role in default decisions, we also need to determine whether there is countercyclical inequality over the business cycle. Using household-level data from several countries, Krueger et al. (2010) show that during recessions, earnings inequality increases.[7] We perform a similar exercise using our country-level data. We use the countries that have continuous series for Gini and GDP, leaving us with 77 countries. We compute the correlation between detrended GDP and inequality and find that, on average, inequality is countercyclical over the business cycle, with a mean correlation equal to -0.02. This result is robust to using the Gini series from The World Bank, as well. In this sample, there are only 46 countries and the mean correlation is equal to -0.03. Both results support the idea inequality is, on average, countercyclical over the business cycle in our sample.

3 Model

In this section, we present a model economy in order to structurally analyze the role of inequality in sovereign debt default. Our model is similar to the model presented by Arellano (2008) and belongs to the class of models in the standard framework of Eaton and Gersovitz (1981). We consider a discrete time, small open economy inhabited by heterogeneous agents that are hand-to-mouth and differ in the stochastic endowments they receive. The endowment is subject to aggregate output and inequality shocks that cannot be completely insured against. There is a benevolent government that represents the preferences of households and has access to international markets. The government can issue one-period bonds to foreign lenders and rebate the proceeds of the debt operations to the households. The government can choose to default fully on its debt at any time, because contracts are not enforceable. The penalty for default is that the economy is forced into financial autarky for a period of time, and there is an exogenous drop in output. Now, we move on to the details of the model.

3.1 Households

There are two types of infinitely-lived households indexed by $i = 1, 2$, and their preferences over consumption of the good, c_t, is assumed to be

$$u(c_t^i) = \frac{c_t^{i,1-\sigma}}{1-\sigma} \tag{2}$$

[7] They have several inequality measures, such as Gini coefficient, variance of logs, 50/10 and 90/50 percentile ratios, and the countries they study are Canada, Germany, Italy, Mexico, Russia, Spain, Sweden and the USA.

where σ is the constant relative risk-aversion parameter, and $\sigma > 0$ and $\sigma \neq 1$. The type 1 household receives a stochastic stream of a tradable good, $\frac{(1+\gamma)y}{2}$, and type 2 receives $\frac{(1-\gamma)y}{2}$, where y and γ denote output and inequality, respectively. The output y and the inequality γ follow a Markov process with a transition function $f\left(y', \gamma' | y, \gamma\right)$. Households also receive an equal amount of transfer from (or pay taxes on goods to) the benevolent government in a lump sum fashion. Households live hand-to-mouth, which means they do not make any individual saving or borrowing decisions.

3.2 Government

The government of the economy can trade one-period, non-state contingent bonds with foreign lenders that are risk neutral and competitive. As in a standard default model, when the government defaults, the economy faces two types of exogenous default penalties: direct output costs and a temporary exclusion from borrowing in the debt markets. The government's goal is to maximize social utility, which is the expected discounted sum of lifetime utilities of both types with equal weights given as

$$E_0 \sum_{t=0}^{\infty} \beta^t \Big[u(c_t^1) + u(c_t^2) \Big], \tag{3}$$

where β denotes the discount parameter and $\beta \in (0, 1)$. The government makes two decisions in this model. First, it decides whether to repay or default on its existing debt. Second, conditional on not defaulting, it chooses the amount of one-period bonds, B', to issue or buy. If the government chooses to buy bonds, the price it needs to pay is given as $q(B', y, \gamma)$. The discount bonds, B', can take a positive or negative value. If it is negative, this means that the government borrows $-q(B', y, \gamma)B'$ amounts of period t goods and promises to pay B' units of goods in the next period, if it does not default. Similarly, if B' is positive, then this implies that the government saves $q(B', y, \gamma)B'$ amounts of period t goods and will receive B' units of goods in the next period. The bond price function $q(B', y, \gamma)$ depends on the size of the bonds, B', income shock, y, and inequality shock, q. Government internalizes how its borrowing decisions affect the default risk and the price of the bond.

When the government chooses to repay its debt, the resource constraint for household

1 is

$$c^1 = \frac{(1+\gamma)y}{2} + \frac{B - q(B', y, \gamma)B'}{2}, \tag{4}$$

and the resource constraint for household 2 is

$$c^2 = \frac{(1-\gamma)y}{2} + \frac{B - q(B', y, \gamma)B'}{2}. \tag{5}$$

The economy faces three types of uncertainty that cannot be insured away with non-state-contingent bonds. The first one is the dispersion in incomes induced by shocks to γ. The second one is the output shock y that affects the aggregate output in the economy. Finally, the third one is the endogenous default risk. The goal of the government is to maximize the expected utilities of households, and it achieves this goal by trying to equalize the marginal utilities of consumption between households and across time. One government policy is to choose optimal B' that satisfies its goal, and the level of existing debt and the size of the shocks are crucial for this decision. As debt accumulates, it becomes harder to roll it over because of increasing default risk.

When the government chooses to default, consumption of the types are:

$$c^1 = \frac{1+\gamma}{2} y^d \tag{6}$$

and

$$c^2 = \frac{(1-\gamma)}{2} y^d, \tag{7}$$

where y^d is the level of output in default and $y^d = (1-\nu)y$. When the economy is in autarky, ν fraction of output is lost.

3.3 Foreign Creditors

Foreign creditors can perfectly monitor the state of the economy and have perfect information about the shock processes. They can borrow loans from international credit markets at a constant interest rate $r > 0$, which is the risk-free interest rate for this model. Taking the bond price function $q(B', y', \gamma)$ as given, they choose loans B' that maximize their expected profits ϕ, given as

$$\phi = q(B', y, \gamma)B' - \frac{1 - \delta(B', y, \gamma)}{1+r} B', \tag{8}$$

where $\delta(B', y, \gamma)$ is the probability of default and it is determined endogenously.

$$q(B', y, \gamma) = \begin{cases} \frac{1}{1+r} & B' \geq 0 \\ \frac{1-\delta(B', y, \gamma)}{1+r} & B' < 0. \end{cases}$$

The price function depends on the sign of B'. It is never optimal to default when the government saves ($B' \geq 0$), so in that case, the price is a constant function of the risk-free interest rate. On the other hand, if the government borrows ($B' < 0$), then the price reflects the default probability. This implies that as the default probability increases, the price of the bond falls.

3.4 Timing

The timing in the model is as follows.

1. The government starts with initial assets B.

2. The output shock y and the inequality shock γ are realized.

3. The government decides whether to repay its debt obligations or default.

   (a) If the government decides to repay, then taking as given the bond price schedule $q(B', y, \gamma)$, the government chooses B' subject to the resource constraint. Then creditors, taking $q(B', y, \gamma)$ as given, choose B'. Finally, households consume c^1 and c^2 with respect to their types.

   (b) If the government chooses to default, then the economy is in financial autarky and remains in autarky in the next period with probability θ. Households simply consume their endowments.

3.5 Recursive Equilibrium

We focus on a recursive equilibrium, in which there is no enforcement. Based on the foreign creditors' problem, government's debt demand is met as long as the gross return on the bond equals $(1 + r)$. Given loan size B', inequality state γ and income state y, the bond price is

$$q(B', y, \gamma) = \frac{1 - \delta(B', y, \gamma)}{1 + r}. \tag{9}$$

The value function for the government that has the option to default or pay its debt is given as $v^o(B, y, \gamma)$. Government chooses the option that maximizes the welfare of agents. The default option will be optimal only if the government has debt. The value of default is denoted by the function $v^d(y, \gamma)$, and the value of repayment is denoted by $v^c(B, y, \gamma)$.

$$v^o(B, y, \gamma) = \max_{c,d}\{v^c(B, y, \gamma), v^d(y, \gamma)\}. \tag{10}$$

The value of default is expressed by

$$\begin{aligned} v^d(y, \gamma) &= u\left(\frac{(1+\gamma)y^{def}}{2}\right) + u\left(\frac{(1-\gamma)y^{def}}{2}\right) \\ &+ \beta \int_{\gamma'} [\theta v^o(0, y', \gamma') + (1-\theta)v^d(y', \gamma')] f(y', \gamma'|y, \gamma) d(\gamma', y'). \end{aligned} \tag{11}$$

Under default, individuals only consume their income. The government can gain access to debt markets with probability θ, and the economy stays in autarky with probability $1 - \theta$. The transition probabilities are given by the joint density function, f. Similarly, the value of staying in contract is

$$\begin{aligned} v^c(B, y, \gamma) &= \max_{B'} u\left(\frac{(1+\gamma)y - q(B', y, \gamma)B' + B}{2}\right) + u\left(\frac{(1-\gamma)y - q(B', y, \gamma)B' + B}{2}\right) \\ &+ \beta \int_{y', \gamma'} v^o(B', y', \gamma') f(y', \gamma'|y, \gamma) d(\gamma', y'). \end{aligned} \tag{12}$$

If the government chooses to repay its debt, the value function for this choice reflects the future options for default and staying in contract. The government chooses the optimal bond contract that maximizes the sum of utilities of the households and expected discounted future value of option.

We can characterize the government's default policy by default and repayment sets. Let $A(B)$ be the set of y and γ for which repayment is optimal when assets are B, such that

$$A(B) = \left\{(y, \gamma) \in (\mathbb{Y}, \Gamma) : v^c(B, y, \gamma) \geq v^d(y, \gamma)\right\}, \tag{13}$$

and let $D(B) = \tilde{A}(B)$ be the set of y, γ for which default is optimal for a level of assets B:

$$D(B) = \left\{(y, \gamma) \in (\mathbb{Y}, \Gamma) : v^c(B, y, \gamma) < v^d(y, \gamma)\right\}. \tag{14}$$

Proposition 1. *Given an output shock y, inequality shock γ and bond positions $B^1 < B^2 \leq 0$, if default is optimal for B^2 then default will be optimal for B^1, and the probability of default at equilibrium satisfies $\delta(B^1, y, \gamma) > \delta(B^2, y, \gamma)$.*

Proof. See Appendix. □

This proposition formally states a feature of the model that Eaton and Gersovitz (1981) also have. It shows that in equilibrium default sets expand and probability of default increases, as the level of debt in a country increases. The following proposition states that equilibrium bond price decreases as the level of debt increases.

Proposition 2. *Given an output shock y, inequality shock γ and bond positions $B^1 < B^2 \leq 0$, equilibrium bond price satisfies $q(B^1, y, \gamma) \leq q(B^2, y, \gamma)$.*

Proof. See Appendix. □

Now we define the recursive equilibrium for this economy. Let $s = \{B, y, \gamma\}$ be the set of aggregate states for the economy.

Definition 1. *The recursive equilibrium for this economy is defined as a set of policy functions for (i) consumptions $c^1(s)$, $c^2(s)$; (ii) government's asset holdings $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$; and (iii) the price function for bonds $q(B', y, \gamma)$ such that:*

1. *Agents' consumption $c^1(s)$ and $c^2(s)$ satisfy the resource constraints, taking the government policies as given.*

2. *The government's policy functions $B'(s)$, repayment sets $A(B)$, and default sets $D(B)$ satisfy the government optimization problem, taking the bond price function $q(B', y, \gamma)$ as given.*

3. *Bonds prices $q(B', y, \gamma)$ reflect the government's default probabilities and default probabilities satisfy creditors' expected zero profits.*

In equilibrium, the bond price function should satisfy both the government's optimization problem and the expected zero profits in the lenders' problem. As mentioned, the probability of default endogenously affects the bond price. Using the default sets, we can express the probability of default such that:

$$\delta(B', y, \gamma) = \int_{D(B')} f(y', \gamma' | y, \gamma) d(y', \gamma'). \tag{15}$$

When default sets are empty, default is never optimal at the asset level B', so the probability of default equals zero, independent of the realized shock. When $D(B') = (\mathbb{Y}, \Gamma)$, government always chooses to default for all shock levels. Default sets are shrinking in assets.

4 Quantitative Analysis and Simulation

4.1 Quantitative Analysis

In this section, we describe the estimation procedure for the shock processes and then explain the calibration strategy. We use the model to analyze the debt dynamics in Argentina between 1990-2002, quantitatively. Focusing on an Argentine default episode enables us to compare our results with the ones in the existing literature.

4.1.1 Calibration and Functional Forms

We solve the model assuming that both output and inequality shocks are in play. We call this the benchmark model. In the benchmark model, output and inequality shocks are modeled as a VAR process. Next, in order to quantify the role of each shock and to assess the importance of the shocks in matching the high volatilities and particularly high default rates observed in emerging economies, we solve the model subject to only one shock at a time. Model II has only output shocks, and we assume that output follows an AR(1) process. Model III has only inequality shocks and, again, the inequality shock is modeled as an AR(1) process.

In the benchmark model, we assume that the VAR process for log output and inequality is as follows:

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix},$$

where

$$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \left[\boldsymbol{I} - \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \right] \begin{bmatrix} \mu_y \\ \mu_\gamma \end{bmatrix}$$

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

$$E[\boldsymbol{\varepsilon}] = \mathbf{0} \quad \text{and} \quad Var[\boldsymbol{\varepsilon}] = \begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix}.$$

The estimated values are derived from Argentina's GDP and income inequality data between 1991Q1 and 2005Q2. We use real output in quarterly, seasonally adjusted, real series and covering the period 1993Q1 to 2001Q4 from the dataset in Arellano (2008).[8] We take logs of GDP and detrend these series using an HP filter. The data pertaining to inequality is constructed using the distribution of income series in World Development Indicators provided by the World Bank. We choose the same period period as for GDP. In order to construct the inequality measure, we compute the total income share of the upper 50th percentile and lower 50th percentile. Then, we take the difference of the income shares and divide it by two, which gives us the dispersion from the mean income. Since only annual data are available, we adopt the Boots-Feibes-Lisman method to disaggregate the annual data into quarterly data. Both output and inequality shocks are then discretized into a 21-state Markov chain, using Tauchen (1986).

The discount factor β, probability of reentry θ and the output cost are jointly calibrated to target a debt-to-GDP ratio of 5.53 percent and trade balance volatility of 1.75. We choose to estimate these moments, so that we can compare them with the results in the literature.[9]

Table 3 shows the parameters that we use for the benchmark model's calibration. We set the risk-free interest rate to 1.7 percent to match the US five-year treasury bond

[8] Arellano (2008) uses the data provided by the Ministery of Finance of Argentina.

[9] The calibrated values for β, θ and ν are close to the values used in the default literature. For instance, Yue (2010) assumes that $\beta = 0.72$, and Aguiar and Gopinath (2006) assume that $\beta = 0.80$. Both of these papers assume that the drop in output is 2 percent. Reinhart and Rogoff (2009) find that output falls more than nine percent after default. The value of parameter θ implies that, on average, autkary takes four quarters, assuming that the distribution of default lengths is exponential (Tomz and Wright (2007) and Pitchford and Wright (2011)). Dias et al. (2007) empirically show that it takes 5.7 years, on average, for countries to regain partial access to international capital markets and Gelos et al. (2011) document that average exclusion from the international markets declined to two years in the 1990s; however, endogenous sovereign default models with exogenous entry to the debt markets calibrate the parameter θ around this value. (Arellano (2008) chooses 0.282 and Aguiar and Gopinath (2006) choose 0.10).

Table 3: A Priori Parameters for Model I

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Stochastic structure	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.95 & -0.38 \\ 0.00 & 0.95 \end{bmatrix}$	Argentina's GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0003 & -0.0001 \\ -0.0001 & 0.0001 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.12 \\ 0.01 \end{bmatrix}$	

Table 4: Calibrated Parameters for Model I

Name	Parameters	Calibrated Parameter	Target	Value Target
Discount rate	β	0.80	Default probability	3 percent
Loss of output in autarky	ν	0.05	Debt service-to-gdp	5.45 percent
Probability of reentry	θ	0.25	Trade balance volatility	1.75

quarterly yield. The risk-aversion parameter σ is set to 2, as it is standard in the macro literature. We also report the estimates of the parameters in the stochastic shock process. Note that the correlation of the output at t and the inequality at $t-1$, $\rho_{y\gamma}$, is negative. This means that high inequality generates low output in the next period. Similarly, since $\rho_{\gamma y}$ is equal to zero, the output in the previous quarter does not affect the inequality in the current period. This relationship between inequality and output is not unique to Argentina. We find that other frequently defaulting economies, such as Brazil, Costa Rica, Dominican Republic, Ecuador and Uruguay, also have similar results in terms of the signs of the estimates. These results are reported in Table 16 in the Appendix.

For Model II, we remove the stochastic inequality shocks by setting the level of inequality to the mean inequality up to the default episode. This corresponds to setting γ equal to 0.66. The stochastic process for output is assumed to be a log-normal AR(1) process such that

$$log(y_t) = \rho_y log(y_{t-1}) + \epsilon_{yt}, \tag{16}$$

where $E[\epsilon_{yt}] = 0$ and $E[\epsilon_{yt}^2] = \sigma_y^2$, which are estimated from Argentina's GDP. We again discretize the output process into a 21-state Markov chain using the Tauchen method. We keep all else the same as in the benchmark model. Table 5 presents the parameters for the

Table 5: A Priori Parameters for Model II

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount rate	$\beta = 0.80$	
Loss of output in autarky	$\nu = 0.05$	
Probability of reentry	$\theta = 0.25$	
Inequality	$\gamma = 0.66$	Mean income inequality in Argentina
Stochastic structure	$\rho_y = 0.9351$	Argentina's GDP
	$\sigma_y = 0.0190$	

Table 6: A Priori Parameters for Model III

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Discount rate	$\beta = 0.80$	
Loss of output in autarky	$\nu = 0.05$	
Probability of reentry	$\theta = 0.25$	
Stochastic structure	$\rho_\gamma = 0.9851$	Argentina's Inequality
	$\sigma_\gamma = 0.0037$	
	$\mu_\gamma = 0.38$	

second model.

Similarly, we need to estimate the stochastic inequality process for Model III. We estimate the following AR(1) process:

$$\gamma_t = (1 - \rho_\gamma)\mu_\gamma + \rho_\gamma \gamma_{t-1} + \epsilon_{\gamma t}, \tag{17}$$

where $E[\epsilon_{\gamma t}] = \mu_\gamma$ and $Var(\epsilon_{\gamma t}) = \sigma_\gamma^2$, which are estimated from Argentina's inequality data. As with Model III, we discretize the inequality process into a 21-state Markov chain using the Tauchen method. We keep all else the same as in benchmark model. The parameters for the third model are presented in Table 6.

4.2 Model Solution

In this section, we begin with the analysis of the benchmark model's results and then elaborate on the intuition behind the workings of the model. Our solution algorithm is given in the Appendix.

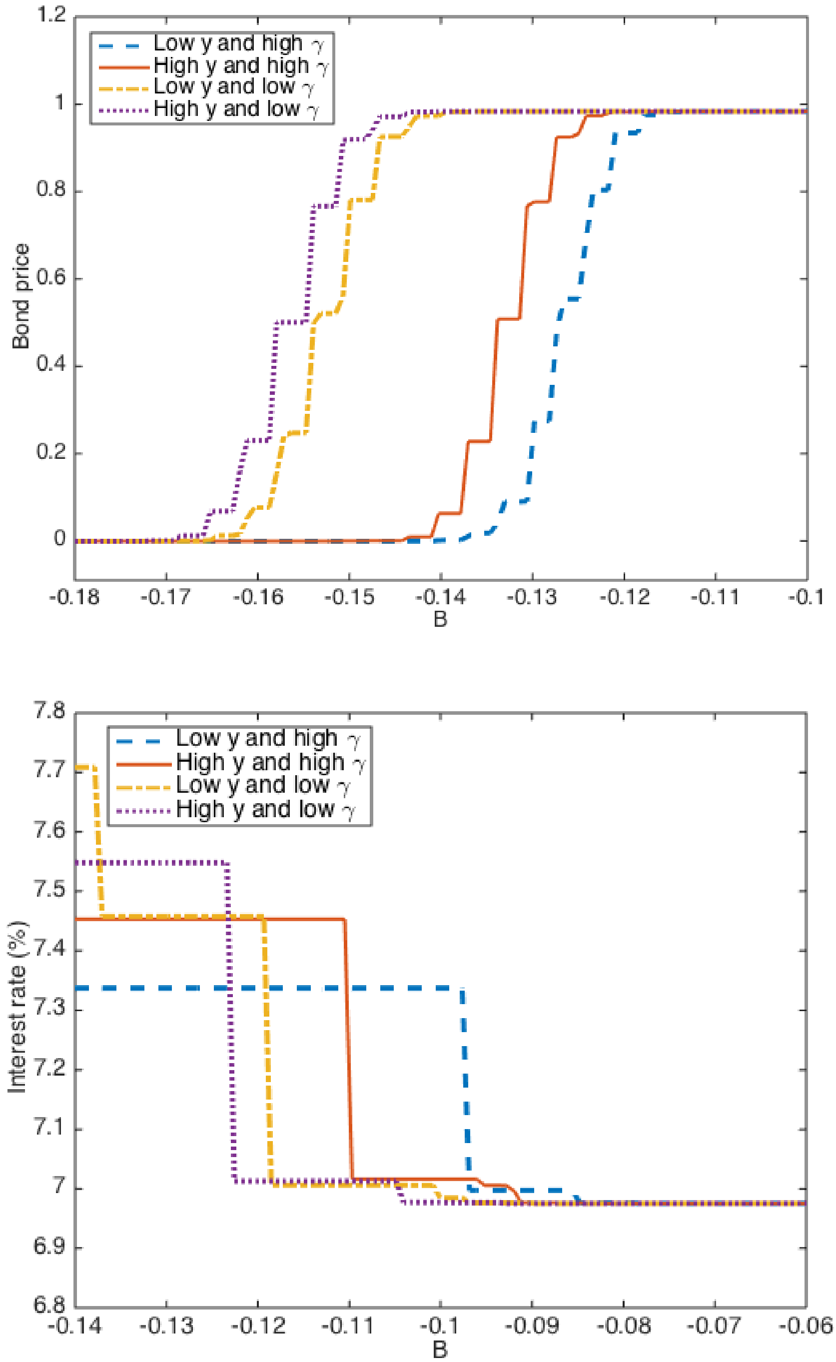
In our model, the benevolent government has two policy decisions to make: whether to repay the existing debt or default; and how much to borrow or save using one-period bonds. The government borrows to help households have smooth consumption and to shift future consumption to today because the equilibrium interest rate is lower than government's discount rate. The level of optimal debt depends on the current assets and the state of the world. Since lenders have full information about the state of the world and contracts are not state-dependent, borrowing constraints can bind for the government, particularly in bad states of the world, such as high inequality and low output. Therefore, we observe that bond prices depend on the level of assets and the types of shocks that the economy is subject to.

In the model, since the endowment is shared unequally among households, even in the absence of the shocks, poor agents' marginal utility of consumption is higher than that of rich agents. An adverse output shock increases both agents' marginal utility of consumption, but an adverse inequality shock raises the marginal utility of the poor and reduces the marginal utility of the rich, increasing the dispersion between the marginal utilities of consumption. Defaults are more likely when there are adverse shocks and high levels of debt because the lenders offer bond contracts that have higher interest rates in these states. This makes the government borrowing-constrained and imposes large taxes on households in order to finance the debt. An adverse inequality shock exacerbates the burden of the tax, particularly on the poor, because it increases the poor's marginal utility of consumption disproportionately. In this case, the government can choose to default and use default as a redistribution mechanism. This policy improves welfare because, by eliminating the tax burden, the government can alleviate the dispersion.

First, we analyze our results related to policy functions and value functions in the benchmark model. We report the results based on four different combinations of output and inequality shocks. A low (high) shock is one standard deviation below (above) its mean for each type of shock. The level of assets is denoted as a fraction of GDP. Then, we look at the business-cycle statistics that the model generates.

Figure 2 shows the bond price schedule and the interest rate generated by the model.

Figure 2: Bond prices and interest rate (Model I)



On the x-axis we have assets as a fraction of output. Similar to the results presented in the standard default literature, such as Arellano (2008) and Aguiar and Gopinath (2006), we observe that bond prices are an increasing function of assets, such that high levels of debt entail a low bond price and a high interest rate. Fixing the level of inequality shocks, we observe that it is easier to borrow during expansions than during recessions. However, the results also show that the effect of a high output shock can be dominated by the effect of a high inequality shock. In other words, an economy that is subject to both high output and high inequality shocks can have a bond price that is lower than that when there are low output and low inequality shocks.

The lower panel in Figure 2 shows the annual interest rates generated by the model. The interest rate is calculated as $1/q(B', y, \gamma) - 1$. Inequality shocks generate another source of risk that is reflected in interest rates. The highest level of borrowing is possible when there is high output and low inequality in the economy. Government borrowing is subject to higher interest rates, even for small amounts of debt that are above the level of default in high-inequality and or low-output states.

The top panel in Figure 3 shows the saving policy function conditional on not defaulting. Our results show that the government borrows more in expansions and when there is low inequality. This result is consistent with the countercyclical interest rates, since it becomes more costly to borrow in bad states of the world. The bottom panel of Figure 3 is the value function for the option to default or repay as a function of assets. Again, the inequality plays a significant role in the default decision. The flat regions of the value function show the range of debt for which default is optimal. The value functions show that the highest debt can be supported, when there is high output and low inequality in the economy.

4.3 Business Cycle Results

4.3.1 Data

First, we document the business-cycle characteristics of the Argentine economy. For the business-cycle statistics, we use real output, consumption and trade balance data in quarterly, seasonally adjusted, real series for the period 1993Q1 and 2001Q4 from the dataset in Arellano (2008).[10] We take logs of GDP and consumption series and apply a linear trend on

[10] Arellano (2008) uses the data provided by the Ministery of Finance of Argentina.

Figure 3: Savings and value functions (Model I)

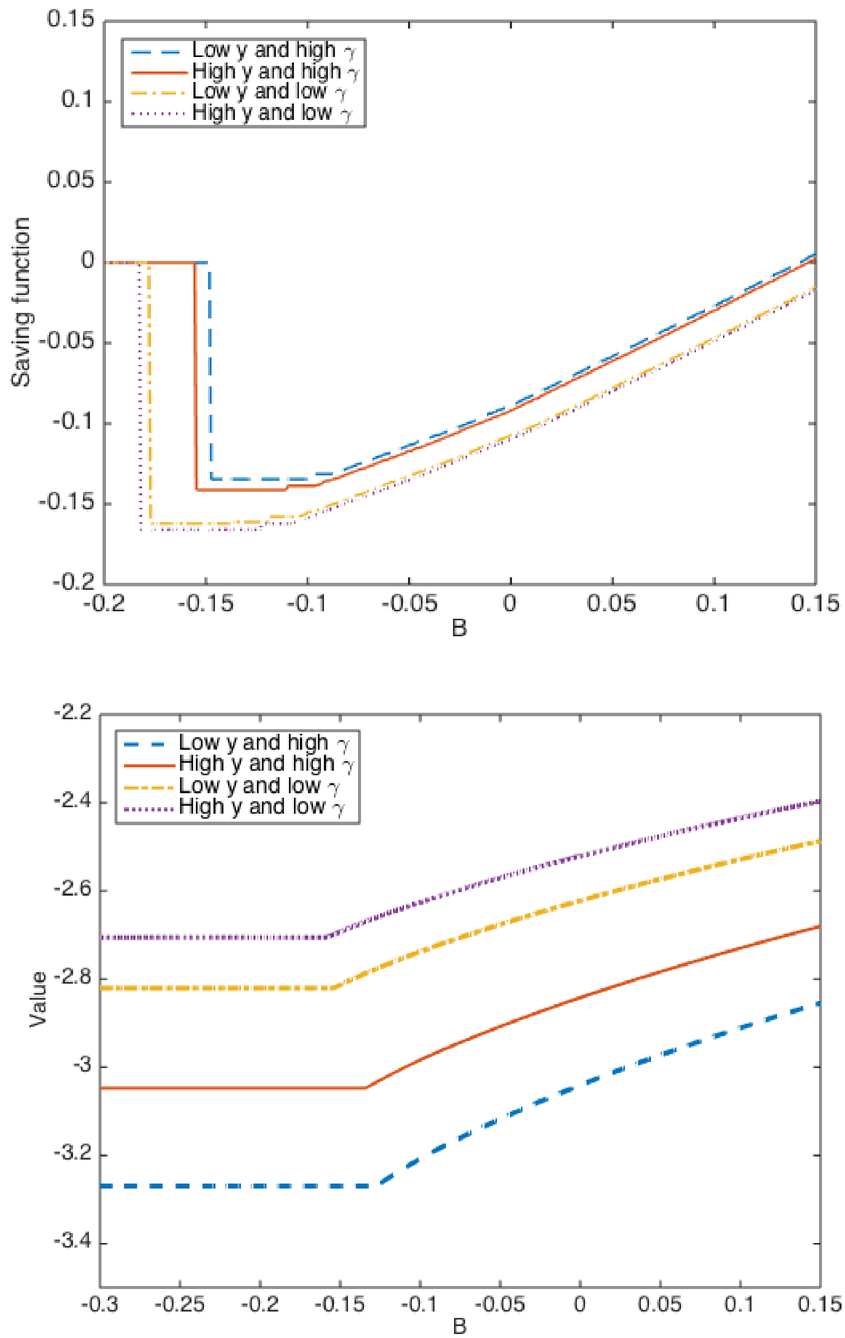


Table 7: Business Cycle Statistics for Argentina

	Default episode x: Q1-2002	std(x)	corr(x, y)	corr(x, r^c)
Interest rate spread (%)	28.60	5.58	-0.88	
Trade balance (% of GDP)	9.90	1.75	-0.64	0.70
Consumption (% deviation from trend)	-16.01	8.59	0.98	-0.89
Output (% deviation from trend)	-14.21	7.78		-0.88
Inequality (% deviation from mean ineq.)	8.6	1.71	-0.23	0.55

these series following Arellano (2008).[11] The trade balance data are a fraction of GDP. We also borrow Arellano (2008)'s spread data, which are defined as the difference between the interest rate in Argentina and the yield of the five-year U.S. treasury bond. The interest rate series is EMBI for Argentina and starts from 1983Q3. The inequality series is the one we constructed to generate a shock process, as explained in the previous section.

Table 12 presents the business-cycle statistics of all the data available up to the default episode that started on December 26, 2001. Consumption and output in the first column show the deviations from the trend, and the other values are in levels in the first quarter of 2002. Relative to the average inequality in the series, in the default episode, inequality increased by 8.6 percent. The second column shows the standard deviations up to the default episode. We find that consumption is more volatile than output. The third and the fourth columns present the correlations of each variable with the output and the interest rate spread, respectively. It has been shown that emerging market economies are characterized by countercyclical spread rates and net exports. Also, their consumption is highly correlated with output. We see similar empirical results for Argentina in column 3. In addition, we show that inequality is countercyclical with output, so the economy has high inequality during recessions. The interest rate spread is negatively correlated with consumption and output, and positively correlated with trade balance. The data show that inequality is positively correlated with the spread, which implies that inequality increases during times of risky borrowing.

4.3.2 Simulation Results

Next, we move on to the business-cycle statistics generated by the benchmark model and evaluate the performance of the model with Argentine data. The upper panel of Table 8

[11] Analysis using HP filtered series (with smoothing parameter 1600) also produces similar results for correlations.

presents the simulation results for the benchmark model, which generates a default probability of 2.80. High volatility of interest rates is a consequence of high default probability. The model also generates large drops in consumption and output during default episodes. Inequality increases by 9.09 percent relative to its mean, which is also close to the increase observed during the default episode (8.60 percent). The model can also generate high volatility in consumption and output. The volatility of inequality is slightly lower than the value observed in the data.

In terms of correlations with output, the simulations can generate a positive correlation with consumption and a negative correlation with the interest rate spread.[12] However, it fails to generate a negative correlation between output and trade balance because the trade balance is quite acyclical in the benchmark model. The reason is that when there are only output shocks, households can consume more than the level of the output during expansions because the government can borrow easily. On the other hand, when there is a recession, borrowing is constrained; therefore, the consumption is less than the output. This generates a countercyclical trade balance over the business cycle in standard models. However, in the benchmark model, defaults can occur not only during the low-output states, but also during high-inequality states. Moreover, shocks to inequality do not alter the total consumption in the economy. So, unlike the standard models, our model generates a weak and positive correlation between the trade balance and output. We do, however, see a positive correlation between the spread and the trade balance. Since the spread reflects the risk due to both inequality and output shocks, it is more correlated with the bad states of the world, in which the government is more likely to face borrowing constraints and experience large trade balances. As we expected, inequality is negatively correlated with output and positively correlated with the spread.

We solve and simulate Model II and Model III, in order to assess the role of output shocks and inequality shocks in the default risk. Figure 4 shows the bond price and interest rate schedules under output shocks, and Figure 5 shows the same plots under inequality shocks. Similar to the benchmark model's results, prices are lower and interest rates are higher in the bad states of the world. The simulation results for Model II and Model III are given in Table 9. We find that the default probability is around 0.12 percent when there are output shocks but 1.77 percent when there are inequality shocks. In Model III, the volatility of interest rate spread is higher, and inequality increases considerably during default episodes.

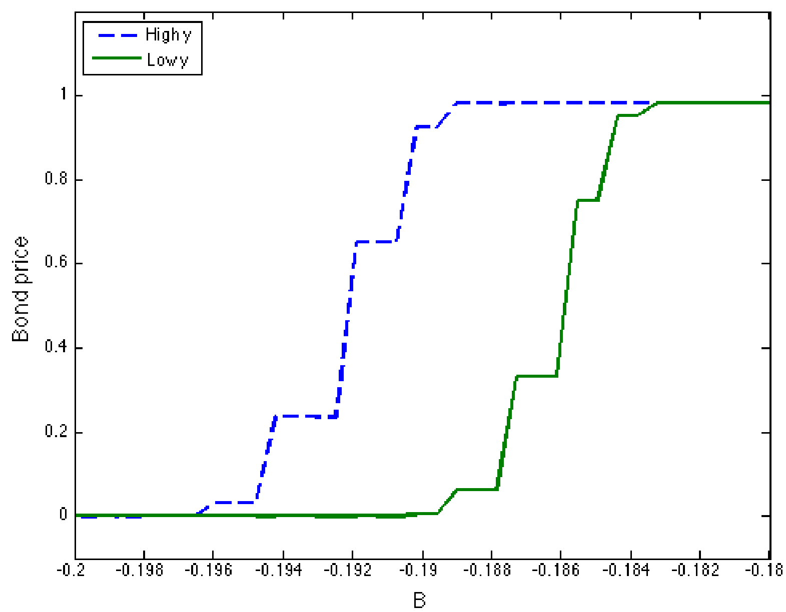
[12] See Neumeyer and Perri (2005), Uribe and Yue (2006) and Aguiar and Gopinath (2007) for the role of countercyclical interest rates in emerging markets.

We obtain a small probability of default when the economy is subject to an adverse output shock because it leads to a small dispersion among households' marginal utilities of consumption. Therefore, the government's borrowing is more about achieving smooth consumption intertemporally. The benefits from default is not as large as in inequality shocks, because the tax burden on the households is similar. However, with an adverse inequality shock, there is a high tax burden, especially on the poor, and default can serve as a better policy to equalize the marginal utilities of consumption across households.

One observation worth mentioning is that the price schedule is quite steep in Model II compared to Model III, which shows that there is a larger range of risky borrowing available to the government. This is consistent with having a higher default rate in Model III. We also find that the default risk in both Model II and Model III is lower than that in the benchmark model. This is strong evidence that shows that the amplification effect comes from the underlying joint shock process. The reason behind this result is the VAR(1) process that we systematically estimated from the Argentine data. Based on the estimated process, it is more likely to have adverse output and inequality shocks together. Moreover, high inequality at time $t-1$ leads to lower output at time t. These characteristics play an important role in altering the expectations of foreign lenders and the government about the future state of the economy. An adverse inequality shock not only amplifies the effect of an adverse output shock today, but also generates a pessimism a the recession with increasing inequality may be long-lasting.[13] As a result, foreign lenders ask for a higher premium, even for smaller levels of debt. This increases the borrowing constraints on the government, and default becomes an optimal decision.

[13] In order to disentangle the effect of inequality on output in the next period, when we generate the Markov process, we assume that $\rho_{y\gamma} = 0$. Under this specification, we find that the probability of default falls to 0.7 percent, even though the economy is subject to both shocks. This result shows that the main channel that generates a high probability of default with inequality shocks is the inequality shocks' effects on future output and the expectations of the agents.

Figure 4: Bond prices and interest rate (Model II)



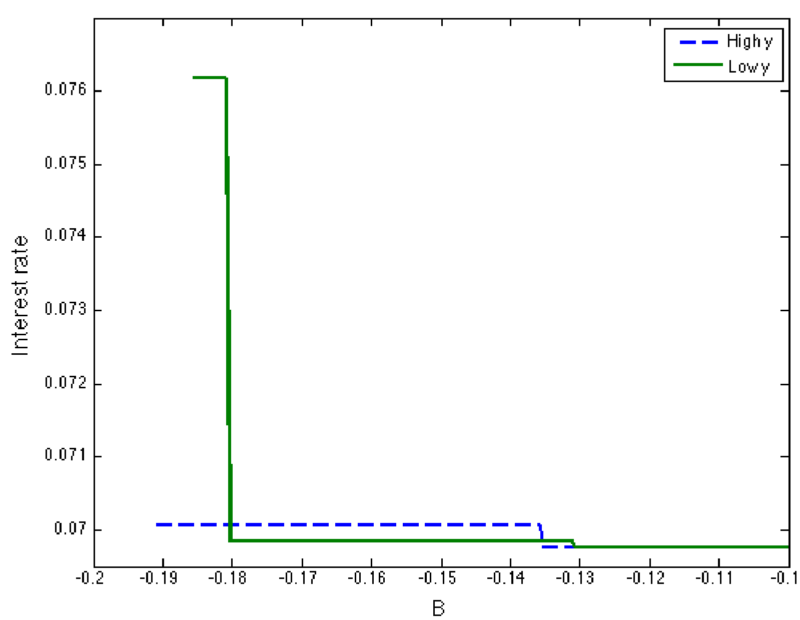
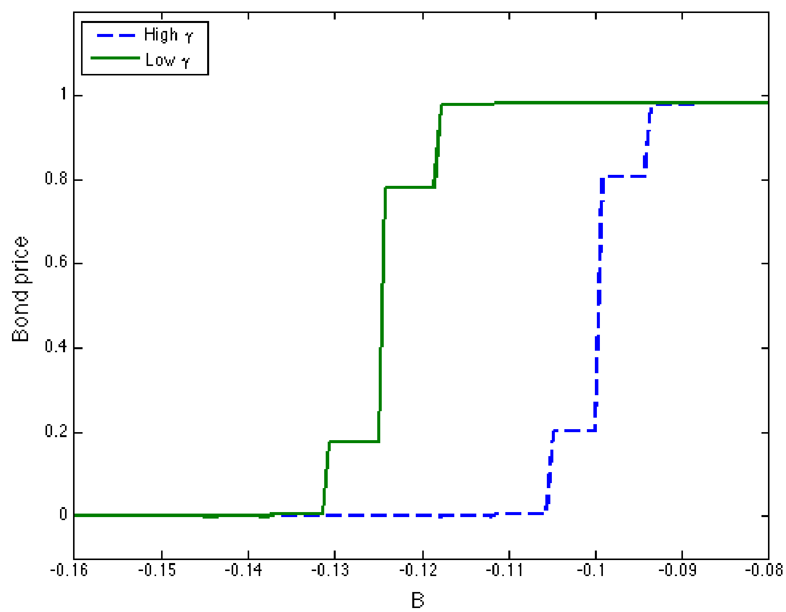


Figure 5: Bond prices and interest rate (Model III)



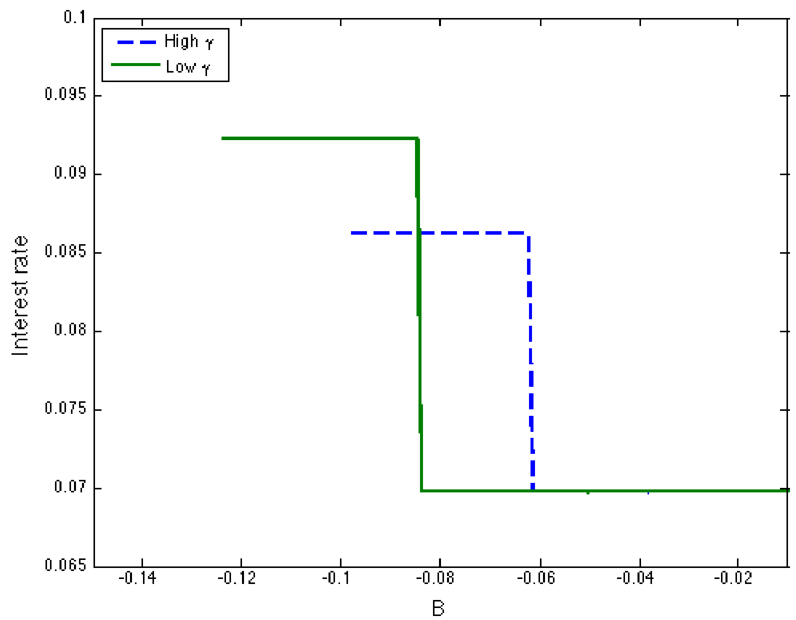


Table 8: **Simulation Results for the Benchmark Model**

	Default episodes	std(x)	corr(x,y)	$corr(x, r^c)$
Model I: Shocks to output and inequality				
Interest rate spread (%)	0.95	0.78	−0.29	-
Trade balance (% of GDP)	−0.00	0.41	0.07	0.06
Total Consumption (% deviation from trend)	−6.99	9.53	0.99	−0.28
Output (% deviation from trend)	−7.12	7.60	–	−0.29
Inequality (% deviation from mean ineq.)	9.09	1.23	−0.18	0.26
Other Statistics				
Mean debt (percent output)	11.44	Mean spread		0.93
Default probability	2.80			

Table 9: **Simulation Results for Model II and Model III**

	Model II		Model III	
	Default episodes	std(x)	Default episodes	std(x)
Interest rate spread (%)	0.19	0.15	1.78	0.53
Trade balance	−0.30	0.12	−0.00	0.40
Total Consumption	−17.16	4.69	−4.95	0.00
Output	−7.89	4.61	−7.75	0.00
Inequality	0.00	0.00	13.63	2.26
Other Statistics				
Mean debt (percent output)	16.22		9.55	
Default probability	0.10		1.69	
Mean Spread	0.12		1.47	

5 Alternative Calibrations

We obtain the main results regarding the effects of inequality shocks using Argentine data. In this section, we calibrate the model for two other emerging economies, Greece and Turkey, as well. Our goal in this exercise is to see whether the model can match the business cycle statistics of other countries. Greece and Turkey have experienced five and seven external debt crisis episodes, respectively.[14]

[14] We count the number of default or restructuring episodes starting from the country's year of independence. For Greece we consider the period between 1827 and 2015, and for Turkey we consider 1923-2015. The data on external debt crisis are from (Reinhart and Rogoff, 2011). An external debt crisis is defined as the failure to meet the principal or interest payment on the due date. The episodes also include instances

Table 10: A-Priori Parameters for Greece and Turkey

Name	Parameters	Description
Risk-free interest rate	$r = 1.7\%$	US 5-year bond quarterly yield
Risk aversion	$\sigma = 2$	
Stochastic structure, Greece	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.91 & -0.11 \\ 0.01 & 0.93 \end{bmatrix}$	Greece GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0001 & -0.0001 \\ -0.0001 & 0.00001 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.01 \end{bmatrix}$	
Stochastic structure, Turkey	$\begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} = \begin{bmatrix} 0.74 & -0.13 \\ -0.00 & 0.98 \end{bmatrix}$	Turkish GDP
	$\begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix} = \begin{bmatrix} 0.0001 & -0.00003 \\ -0.00003 & 0.0002 \end{bmatrix}$	and income inequality
	$\begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} = \begin{bmatrix} 0.04 \\ 0.01 \end{bmatrix}$	

We repeat the similar steps when we estimate the shock processes for each country. Table 10 show the a-priori parameters that we used in the simulations in order to obtain the business cycle statistics for Greece and Turkey. For both countries, we use the same values for the risk-free interest rate and the risk aversion parameter as in the previous sections. The stochastic shock processes come from the VAR estimations based on each country's GDP and income distribution data.[15] For Turkey we find the estimate for $\rho_{y\gamma}$ is negative and it implies that inequality in the previous period reduces the output in the current period. The covariances of the errors are negative for both Turkey and Greece, which implies that it is more likely to receive an adverse inequality shock together with an adverse output shock.

We follow the same calibration strategy. Table 11 shows the calibrated parameter values. We jointly calibrate the discount rate (β), the output cost in autarky (ν) and the probability of reentry (θ) in order to match the default probability, debt service-to-gdp ratio and the trade balance volatility in each country.

The simulation results are given in Table 12. In terms of matching the targets, default probabilities are close to the values in the data for both countries. For Turkey, mean debt is lower than the debt service-to-GDP ratios in the data, which is a common result in

where the rescheduled that ultimately extinguished in terms of less favorable than the original obligation by Reinhart and Rogoff (2011).

[15]The data cover the period between 2000-2012 for Greece and 2002-2010 for Turkey.

Table 11: Calibrated Parameters for Greece and Turkey

Name	Parameters	Calibrated Parameter	Target	Value Target
Greece				
Discount rate	β	0.75	Default probability	2.7 percent
Loss of output in autarky	ν	0.05	Debt service-to-gdp	16.9 percent
Probability of reentry	θ	0.35	Trade balance volatility	2.84
Turkey				
Discount rate	β	0.62	Default probability	7.6 percent
Loss of output in autarky	ν	0.03	Debt service-to-gdp	36.0 percent
Probability of reentry	θ	0.50	Trade balance volatility	3.28

models with bonds with short maturities or no renegotiation for debt restructuring. The trade balance volatility for Greece and Turkey are also smaller than the data. Our model generates lower volatility of trade balance because the inequality shock has opposite effects on the consumptions of poor and rich households. We see that the model can match the main business cycle characteristics for both countries, such as spreads are countercyclical, and consumption is procyclical over the business cycle. Another strength of the model is that consumption volatility is higher than output volatility. Also for both countries, in the simulations the trade balance is positively correlated with spread. Finally, the results show that output is negatively correlated with income inequality.

Table 12: Business Cycle Statistics for Greece and Turkey

	Greece		Turkey	
	Simulation	Data	Simulation	Data
corr(spread, y)	-0.05	-0.34	-0.15	-0.53
corr(spread, tb)	0.01	0.32	0.05	0.43
corr(ineq, y)	-0.20	-0.11	-0.44	-0.62
corr(tb, y)	-0.02	-0.50	0.06	-0.64
corr(tc, y)	0.98	0.42	0.99	0.87
std(tc)/std(y)	3.98/3.29	3.91/1.18	3.70/3.62	3.55/3.49
Targets				
Default probability	1.0%	2.7%	5.4%	7.6%
Debt-to-GDP	23.0%	16.9%	6.2%	36%
std(tb)	0.7	2.8	0.5	3.2

Note: Total consumption and trade balance are denoted by tc and tb, respectively. Income inequality is denoted by ineq in the table.

6 Progressive Income Taxes

In the previous sections, we assume that government distributes the proceeds of the debt payments equally between the households. As mentioned above, these proceeds can function as taxes, when they are negative and they can function as transfers, otherwise. Since these payments are lump sum, the burden (benefit) of taxes (transfers) relative to endowment is quite different across the households. Particularly, the lump sum taxes the burden is on the poor. Therefore, this brings up the question: How would the probability of default change in an economy, if the government could use progressive income taxes to finance the debt when it is costly to borrow?

We impose the following tax regime:

$$T(y^i) = \begin{cases} 0 & B - qB' \geq 0, \\ y^i - \lambda(y^i)^{1-\tau} & B - qB' < 0. \end{cases}$$

As τ increases the tax function becomes more progressive, and when $\tau = 1$, both types of households consume equally. The parameter λ is called the shift parameter and determines the average tax rate. If $B - qB'$ is positive, the government only distributes the proceeds of the debt operations across households as transfers similar to the benchmark model. If $B - qB'$ is negative, then the government uses the revenues from the taxes to finance the debt. The budget constraint of the government for the latter case is given as:

$$T(y^1) + T(y^2) + B - qB' = 0. \tag{18}$$

One can solve for λ using the budget constraint of the government:

$$\begin{aligned} y^1 - \lambda(y^1)^{1-\tau} + y^2 - \lambda(y^2)^{1-\tau} + B - qB' &= 0 \\ y - \lambda[(y^1)^{1-\tau} + (y^2)^{1-\tau}] + B - qB' &= 0 \\ \lambda[(y^1)^{1-\tau} + (y^2)^{1-\tau}] &= y + B - qB' \\ \lambda &= \frac{y + B - qB'}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}. \end{aligned}$$

The disposable incomes are denoted by $\tilde{y}^i$ for each type of household i. When $B - qB' < 0$, we get:

$$\begin{aligned} \tilde{y}^1 &= \lambda(y^1)^{1-\tau}, \\ &= \frac{(y + B - qB')(y^1)^{1-\tau}}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}. \\ \tilde{y}^2 &= \lambda(y^2)^{1-\tau}, \\ &= \frac{(y + B - qB')(y^2)^{1-\tau}}{(y^1)^{1-\tau} + (y^2)^{1-\tau}}. \end{aligned}$$

We can write the budget constraints of the households if the government does not choose to default as:

$$c^1 = \begin{cases} y^1 + \frac{B-qB'}{2} & B - qB' > 0, \\ \frac{(y+B-qB')(y^1)^{1-\tau}}{(y^1)^{1-\tau}+(y^2)^{1-\tau}} & B - qB' \leq 0. \end{cases}$$

$$c^2 = \begin{cases} y^2 + \frac{B-qB'}{2} & B - qB' > 0, \\ \frac{(y+B-qB')(y^2)^{1-\tau}}{(y^1)^{1-\tau}+(y^2)^{1-\tau}} & B - qB' \leq 0. \end{cases}$$

If the government chooses to default, we assume that the progressive taxes are in effect. The budget constraints during autarky are:

$$c^1 = \frac{y^d (y^{d,1})^{1-\tau}}{(y^{d,1})^{1-\tau} + (y^{d,2})^{1-\tau}},$$
$$c^2 = \frac{y^d (y^{d,2})^{1-\tau}}{(y^{d,1})^{1-\tau} + (y^{d,2})^{1-\tau}}.$$

Table 13: Effect of τ_1 on default probability and debt

	$\tau_1 = 0$	$\tau_1 = 0.05$	$\tau_1 = 0.10$	$\tau_1 = 0.15$	$\tau_1 = 0.20$	$\tau_1 = 0.25$	$\tau_1 = 0.30$
Mean debt (% output)	26.1	26.9	27.7	28.1	30.3	31.1	32.4
Probability of default (%)	2.9	2.7	2.5	2.2	1.1	0.9	0.1

We calibrate the model in order to match 3 percent default probability, when $\tau = 0$[16]. We experiment with seven values of $\tau \in \{0, 0.05, 0.10, 0.15, 0.20, 0.25, 0.30\}$ and analyze how the progressivity of the tax system affects the probability of default and debt-to-output ratio. Table 13 shows the results. The model with $\tau_1 = 0$ has mean debt of 26.1 percent. As τ increases, we obtain higher mean debt. Moving from $\tau = 0$ to $\tau = 0.30$, the probability of default decreases from 3 percent to 0.1 percent. The reason is that taxes reduce the dispersion in marginal utilities of consumption between households, by taxing the

[16]The parameters are $\beta = 0.80$, $\theta = 0.20$ and $\nu = 0.05$

rich more than the poor. As the dispersion gets smaller, the government has less incentives to default. Therefore, foreign lenders lend higher levels of debt to the government. Moving from $\tau = 0$ to $\tau = 0.30$, the debt-to-output increases by around 24 percent.

7 Conclusion

This paper studies the role of increasing income inequality in sovereign borrowing and default decisions using a stochastic general equilibrium model in a small open economy with endogenous default risk. To motivate the idea, we analyze the nexus among the Gini index, sovereign bond ratings and GDP per capita using a panel data set. The results show that high inequality lowers the creditworthiness of long-term government bonds significantly. The paper also shows that the inequality is countercyclical over the business cycle for the average country. Next, using a model that belongs to the class of models of Eaton and Gersovitz (1981) and extending it to allow for heterogeneous agents and shocks to the distribution of income, the paper shows analytically that inequality shocks can generate a high probability of default when the markets are incomplete. Using Argentine data, the model predicts a default probability of 2.8 percent and can also match the business-cycle characteristics observed in the data, such as the high volatility of consumption and output, the counter-cyclical interest rates, and positive correlations between the trade balance and interest rates, as well as, inequality and interest rates. Our model's contribution is to highlight the redistributive effects of default as a policy that improves the welfare of the households. We show that progressive income taxes can reduce the default probability and increase the debt-to-output ratio.

Rising income inequality is a general problem that many countries have experienced. Therefore, it is important to understand how inequality induces economic crises and sovereign defaults that last several years and cause large losses. Even though our paper provides a first step toward analyzing the role of income inequality, we abstract from the determinants of income inequality and model it as an exogenous shock. We think that it is also important to study what drives high income inequality and how it affects agents' welfare and a government's decision to default. We leave these issues for future study.

References

Aguiar, Mark and Gita Gopinath, “Defaultable debt, interest rates and the current account,” *Journal of international Economics*, 2006, *69* (1), 64–83.

Aguiar, Mark and Gita Gopinath, “Emerging Market Business Cycles: The Cycle Is the Trend,” *Journal of Political Economy*, 2007, *115* (1), 69–102.

Aguiar, Mark and Manuel Amador, “Sovereign Debt: A Review,” Technical Report, National Bureau of Economic Research 2013.

Amador, Manuel and Mark Aguiar, “Take the Short Route: How to repay and restructure sovereign debt with multiple maturities,” in “2014 Meeting Papers” number 165 Society for Economic Dynamics 2014.

Arellano, Cristina, “Default risk and income fluctuations in emerging economies,” *The American Economic Review*, 2008, pp. 690–712.

Arellano, Cristina and Ananth Ramanarayanan, “Default and the maturity structure in sovereign bonds,” *Journal of Political Economy*, 2012, *120* (2), 187–232.

Broner, Fernando A, Alberto Martin, and Jaume Ventura, “Enforcement problems and secondary markets,” *Journal of the European Economic Association*, 2008, *6* (2-3), 683–694.

Cantor, Richard and Frank Packer, “Determinants and impact of sovereign credit ratings,” *The Journal of Fixed Income*, 1996, *6* (3), 76–91.

Chatterjee, Satyajit and Burcu Eyigungor, “Debt dilution and seniority in a model of defaultable sovereign debt,” 2012.

Chatterjee, Satyajit and Burcu Eyigungor, “Maturity, Indebtedness, and Default Risk,” *The American Economic Review*, 2012, *102* (6), 2674–99.

Cuadra, Gabriel and Horacio Sapriza, “Sovereign default, interest rates and political uncertainty in emerging markets,” *Journal of International Economics*, 2008, *76* (1), 78–88.

Cuadra, Gabriel, Juan M Sanchez, and Horacio Sapriza, “Fiscal policy and default risk in emerging markets,” *Review of Economic Dynamics*, 2010, *13* (2), 452–469.

D'Erasmo, Pablo and Enrique G Mendoza, "Optimal Domestic Sovereign Default," Technical Report, University of Maryland Working Paper 2012.

D'Erasmo, Pablo and Enrique G Mendoza, "Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default," Technical Report, National Bureau of Economic Research 2013.

Dias, Daniel A, Christine Richmond, and T Wang, "Duration of capital market exclusion: An empirical investigation," Technical Report, Working Paper, UCLA 2007.

Eaton, Jonathan and Mark Gersovitz, "Debt with potential repudiation: Theoretical and empirical analysis," *Review of Economic Studies*, 1981, *48* (2), 289–309.

Gelos, R Gaston, Ratna Sahay, and Guido Sandleris, "Sovereign borrowing by developing countries: What determines market access?," *Journal of International Economics*, 2011, *83* (2), 243–254.

Gennaioli, Nicola, Alberto Martin, and Stefano Rossi, "Sovereign default, domestic banks, and financial institutions," *The Journal of Finance*, 2014, *69* (2), 819–866.

González-Rozada, Martín and Eduardo Levy Yeyati, "Global factors and emerging market spreads*," *The Economic Journal*, 2008, *118* (533), 1917–1936.

Hatchondo, Juan Carlos and Leonardo Martinez, "Long-duration bonds and sovereign defaults," *Journal of International Economics*, 2009, *79* (1), 117–125.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza, "The Economics of Sovereign Defaults," *FRB Richmond Economic Quarterly*, 2007, *93* (2), 163–187.

Hatchondo, Juan Carlos, Leonardo Martinez, and Horacio Sapriza, "Heterogeneous borrowers in quantitative Models of sovereign default," *International Economic Review*, 2009, *50* (4), 1129–1151.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L Violante, "Optimal tax progressivity: An analytical framework," Technical Report, National Bureau of Economic Research 2014.

Krueger, Dirk, Fabrizio Perri, Luigi Pistaferri, and Giovanni L Violante, "Cross-sectional facts for macroeconomists," *Review of Economic Dynamics*, 2010, *13* (1), 1–14.

Lane, Philip R and Gian Maria Milesi-Ferretti, "The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries," *Journal of international Economics*, 2001, *55* (2), 263–294.

Lane, Philip R and Gian Maria Milesi-Ferretti, "The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970–2004," *Journal of International Economics*, 2007, *73* (2), 223–250.

Martin, Alberto and J Ventura, "Sovereign Risk and Secondary Markets," *American Economic Review*, 2010, *100* (4), 1523–55.

Neumeyer, Pablo A and Fabrizio Perri, "Business cycles in emerging economies: the role of interest rates," *Journal of monetary Economics*, 2005, *52* (2), 345–380.

Panizza, Ugo, Federico Sturzenegger, and Jeromin Zettelmeyer, "The economics and law of sovereign debt and default," *Journal of Economic Literature*, 2009, pp. 651–698.

Pitchford, Rohan and Mark LJ Wright, "Holdouts in sovereign debt restructuring: A theory of negotiation in a weak contractual environment," *The Review of Economic Studies*, 2011, p. rdr038.

Reinhart, Carmen M, "Default, currency crises, and sovereign credit ratings," *The World Bank Economic Review*, 2002, *16* (2), 151–170.

Reinhart, Carmen M and Kenneth Rogoff, "The Aftermath of Financial Crises," *The American Economic Review*, 2009, *99.*

Reinhart, Carmen M and Kenneth S Rogoff, "From Financial Crash to Debt Crisis," *American Economic Review*, 2011, *101* (5), 1676–1706.

Reinhart, Carmen M, Kenneth Rogoff, and Miguel A Savastano, "Debt Intolerance," *Brookings Papers on Economic Activity*, 2003, *1.*

Solt, Frederick, "Standardizing the world income inequality database*," *Social Science Quarterly*, 2009, *90* (2), 231–242.

Statistics, External Debt, "Guide for Compilers and Users," *International Monetary Fund*, 2003.

Storesletten, Kjetil, Chris I Telmer, and Amir Yaron, "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *Journal of Political Economy*, 2004, *112* (3).

Tauchen, George, "Finite state Markov-chain approximations to univariate and vector autoregressions," *Economics Letters*, 1986, *20* (2), 177–181.

Tomz, Michael and Mark LJ Wright, "Do countries default in bad times?," *Journal of the European Economic Association*, 2007, *5* (2-3), 352–360.

Uribe, Martin and Vivian Z Yue, "Country spreads and emerging countries: Who drives whom?," *Journal of international Economics*, 2006, *69* (1), 6–36.

Wright, Mark LJ, "The theory of sovereign debt and default," *Encyclopedia of Financial Globalization*, 2011.

Yue, Vivian Z, "Sovereign default and debt renegotiation," *Journal of International Economics*, 2010, *80* (2), 176–187.

8 Appendix

8.1 Tables

Table 15: Panel Regressions Explaining Credit Worthiness with Debt Ratios and Inequality using Gini data from The World Bank

Independent Variable	(1)	(2)	(3)
External debt-to-GDP at $t-1$	-0.0214^{**}	-0.0068	-0.0060
	(0.0102)	(0.0067)	(0.0061)
GDP per capita at $t-1$	–	15.3950^{***}	14.7987^{***}
		(2.4341)	(2.3667)
Gini at $t-1$			-0.1762^{**}
			(0.0687)
Year fixed effects	Yes	Yes	Yes
Country fixed effects	Yes	Yes	Yes
No of countries	40	40	40
N	364	364	364

Sample period is 1994-2009. The dependent variable is the credit rating of country i in year t. Estimation is by robust standard errors. Standard errors are reported in parentheses. Per capita GDP is in logs. (***,**,* represent the significance level at 1%, 5% and 10%, respectively.)

Table 14: Fitch Credit Rating Conversion Table

Fitch Rating	Score
AAA	23
AA+	22
AA	21
AA-	20
A+	19
A	18
A-	17
BBB+	16
BBB	15
BBB-	14
BB+	13
BB	12
BB-	11
B+	10
B	9
B-	8
CCC+	7
CCC	6
CCC-	5
CC	4
C	3
DDD	2
D	1
RD	1

Table 16: VAR estimations for different countries

Country	ρ_{yy}	$\rho_{y\gamma}$	$\rho_{\gamma y}$	$\rho_{\gamma\gamma}$	σ_y^2	$\sigma_{y\gamma}$	$\sigma_{\gamma y}$	σ_γ^2
Brazil	0.34	-0.25	0.09	0.64	5.6x10^{-4}	5x10^{-5}	5x10^{-5}	8x10^{-5}
Colombia	0.44	0.09	-0.15	0.33	1.4x10^{-4}	2x10^{-5}	2x10^{-5}	8x10^{-5}
Costa Rica	0.33	-0.07	0.05	0.74	4.5x10^{-5}	-1x10^{-5}	-1x10^{-5}	9x10^{-5}
Dominican Republic	0.26	-0.33	0.07	0.71	6x10^{-4}	-1.3x10^{-5}	-1.3x10^{-5}	9x10^{-5}
Ecuador	0.01	-0.33	0.20	0.82	1.3x10^{-3}	-1.8x10^{-4}	-1.8x10^{-4}	2.3x10^{-4}
Paraguay	-0.74	0.24	-0.05	0.73	4x10^{-4}	-4x10^{-5}	-4x10^{-5}	7x10^{-5}
Uruguay	0.26	-0.33	0.07	0.71	6x10^{-4}	-1.3x10^{-5}	-1.3x10^{-5}	9x10^{-5}
Argentina	0.28	-0.56	0.05	0.79	1.2x10^{-3}	-2x10^{-4}	-2x10^{-4}	1.3x10^{-4}

In this VAR analysis, we assume that log output and the inequality follow a VAR(1) process such that

$$\begin{bmatrix} log(y_t) \\ \gamma_t \end{bmatrix} = \begin{bmatrix} c_y \\ c_\gamma \end{bmatrix} + \begin{bmatrix} \rho_{yy} & \rho_{y\gamma} \\ \rho_{\gamma y} & \rho_{\gamma\gamma} \end{bmatrix} \begin{bmatrix} log(y_{t-1}) \\ \gamma_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

where

$$\boldsymbol{\varepsilon} = \begin{bmatrix} \varepsilon_{yt} \\ \varepsilon_{\gamma t} \end{bmatrix}$$

$$E\left[\boldsymbol{\varepsilon}\right] = \mathbf{0} \quad \text{and} \quad Var\left[\boldsymbol{\varepsilon}\right] = \begin{bmatrix} \sigma_y^2 & \sigma_{y\gamma} \\ \sigma_{\gamma y} & \sigma_\gamma^2 \end{bmatrix}$$

8.2 Proofs of Propositions

8.2.1 Proof of Proposition 1

The proof is similar to Arellano (2008).

First we show that value of repayment is increasing i asset holdings. For all $\{y, \gamma\} \in D(B^2)$,

$$\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2} > \frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y, \gamma)B'}{2},$$
$$\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2} > \frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y, \gamma)B'}{2}.$$

So,

$$u\left(\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma') \geq$$
$$u\left(\frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma').$$

Therefore, for all $\{y, \gamma\} \in D(B^2)$,

$$u\left(\frac{y(1-\gamma)}{2}\right) + u\left(\frac{y(1+\gamma)}{2}\right) + \beta E[\theta v^o(0, y', \gamma') + (1-\theta)v^d(y', \gamma')] >$$
$$u\left(\frac{y(1-\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^2 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma') \geq$$
$$u\left(\frac{y(1-\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + u\left(\frac{y(1+\gamma)}{2} + \frac{B^1 - q(B', y.\gamma)B'}{2}\right) + \beta E v^o(B', y', \gamma').$$

Hence, any pair of $\{y, \gamma\}$ that is in $D(B^2)$, we have $\{y, \gamma\} \in D(B^1)$.
Let $d(B, y', \gamma')$ denote the equilibrium default decision rule. Default probability satisfies

$$\delta(B, y', \gamma') = \int d(B, y', \gamma') f((y', \gamma'), y, \gamma) d(y', \gamma')$$

Since any $\{y, \gamma\} \in D(B^2)$, we have $D(B^2) \subseteq D(B^1)$, if $d(B^2, y', \gamma') = 1$, then $d(B^1, y', \gamma') = 1$. Hence, $\delta(B^1, y, \gamma) \geq \delta(B^2, y, \gamma)$. □

8.2.2 Proof of Proposition 2

The bond price is defined as $q(B', y, \gamma) = \frac{1-\delta(B', y, \gamma)}{1+r}$. Using Proposition 1, we have $B^1 < B^2 \leq 0$ and $\delta(B^2, y, \gamma) < \delta(B^1, y, \gamma)$. Hence, we get $q(B^2, y, \gamma) > q(B^1, y, \gamma)$. □

8.3 Solution Algorithm

To solve the model numerically, we use the discrete state-space method. We discretize the asset space using a finite set of grid points, making sure that the minimum and the maximum points on the grid do not bind when we compute the optimal debt decision. Our solution algorithm for the benchmark model is the following:[17]

1. Guess that the initial price is the reciprocal of the risk-free interest rate, and the initial value function is equal to the autarky value.

2. Given a price $q(B', y, \gamma)$ and $v^o(B, y, \gamma)$, solve for the optimal policy functions and update the value of option given as equation (10) by comparing $v^c(B, y, \gamma)$ and $v^d(y, \gamma)$.

3. Given the price function, compute the default probabilities.

4. Update the price function using equation (9).

5. We simultaneously check whether the initial guesses for price and the value of option are close enough to their updated values. If not, we update the initial values and iterate steps 2-4 until both bond price and the value of option functions converge.

[17] We use the same algorithm to solve the models with a single type of shock. For instance, for Model II, the price function is denoted as $q(B', y)$, and value of option for default or repayment is denoted as $v^o(B, y)$.