Implementation via Codes of Rights*

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Abstract

Implementation of socially acceptable alternatives, described by a *social choice rule*, can be thought of as a design of power distribution in the society whose ”equilibrium outcomes” coincide with the alternatives chosen by the social choice rule at each preference profile of the society. In this paper, we introduce a new institutional framework for implementation which takes the power distribution in the society as its point of departure. The notion of a ”rights structure” introduced by Sertel [29] fits our approach best to formalize the power distribution in the society. We formulate and characterize implementability via rights structures under different specifications. We also identify how implementation via rights structures is related to Nash implementation via mechanisms. In the presence of at least three agents, we find the class of rights structures, implementability via which is equivalent to Nash implementability.

KEYWORDS: Implementation, rights structures, Nash equilibrium, monotonicity, social choice rule.

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1 Introduction

Several institutional real life mechanisms as constitutions, legal codes, rules of coopo-rate culture or social norms aim to rule out unacceptable outcomes and implement only the socially acceptable outcomes under different circumstances. In implementation theory, we search for game forms to implement these socially acceptable outcomes by adhering to widely used equilibrium notions. However, our view is that the resulting game forms are quite different from the institutional mechanisms that we observe. Here, our aim is to analyze a framework for implementation, formulated in a language closer to the real life mechanisms.

To have a better understanding of our motivation, consider any setting where it is given that, at each preference profile of the society a set of outcomes are in some sense socially acceptable and the rest are not. Implementation theory addresses the following question: Can we structure the interaction among individuals so that this interaction results in the socially acceptable outcomes at each preference profile of the society. The implementation problem derives from the principal's lack of ability to observe the players' actual preferences. Initiated with the work of Hurwicz [13], game forms are designed to implement socially acceptable outcomes by adhering to widely used equilibrium notions in different informational settings.

In the last three decades, many successful results are obtained in extending and identifying the set of social choice rules, implementable with this approach.[1] However, a persistent criticism of the theory is that the game forms constructed for the general proofs have unnatural features that take away from the relevance of the theory. Specifically, some sort of an "integer game " or "modulo game" is used to eliminate strategies with unacceptable outcomes from the equlibria. In these games, whenever there is no consensus, agent who announces the highest integer gets to

[1]For instance see, Abreu and Sen [2], Moore and Repullo [20] Danilov [6], Dutta and Sen [9], Palfrey and Srivastava [23], McKelvey [18], Dutta and Sen [10].

be a dictator.[2] As it is argued in detail by Jackson [14], and by Abreu and Matsushima [1], besides being difficult to interpret, there are several technical problems associated with these games.

In order to address the implementation question in a natural language, we propose an explicit specification of the "power distribution" underlying the social interaction. From among several possibilities to represent a power distribution, the notion of a "rights structure" introduced by Sertel [29] seems to fit our approach best. A rights structure, roughly, specifies the power of each coalition to block certain outcomes from being selected, in favor of another outcome. For example, in the course of a presidential election, every voter has the right to vote for a candidate or to abstain. On the other hand, blocking the election of a candidate as the president would typically require a majority of the voters. As another example, consider an institution consulting with a group of experts to undertake a project among several ones. Institution may eliminate a project in favor of another, if a consultant comes up with some evidence, related to his area of expertise, supporting the other project.

More formally, a *rights structure*, Γ, is a triple (S, h, γ) , and it is the object of design for this study. A non-empty set S denotes a collection of (*social*) *states*[3], reflecting the set of all possible situations that the society may end up with, possibly supported by some evidence. An outcome function, h, maps each state to an alternative. The code of rights, γ, associates each ordered distinct pair (s, t) of states, with a family of coalitions, $s \overset{\gamma}{\to} t$, that are entitled to approve the change from state s to state t. Given a preference profile of the society, u, we say a coalition benefits from a change of state s to state t if all the members of this coalition prefers $h(t)$ to $h(s)$. The set of those coalitions that benefits from a change of state s to state t is denoted by $s \overset{u}{\to} t$.

In this setting, the existence of a coalition endowed with the right to move from state s into another state t, conjoined with its willingness to do so means that s cannot

[2]We exemplify a similar construction (in the proof of Proposition 3) to establish a connection between our model and classical implementation.

[3]Social states are objects similar to the states of a finite machine (automata).

be an equilibrium state. Put differently, given a rights structure $\Gamma = (S, h, \gamma)$, a state s is an equilibrium of this rights structure (Γ-equilibrium) at preference profile u, if for each other state t, there is no coalition, K, (i) which is entitled to approve a change from s to t, and (ii) each member of K prefers $h(t)$ to $h(s)$, i.e. $(s \overset{\gamma}{\to} t) \cap (s \overset{u}{\to} t) = \emptyset$.

Given a society and a set of alternatives, a social choice rule (SCR), F, specifies a set of acceptable alternatives at each preference profile. An SCR, F, is implementable via the rights structure, $\Gamma = (S, h, \gamma)$, if at each preference profile, the set of alternatives chosen by F coincides with the equilibrium outcomes of the rights structure at that preference profile.

In the definition of Γ-equilibrium several details of the interaction are left unspecified, as it is the case for many game theoretic equilibrium concepts. The gain from leaving out such details are the simplicity and wider applicability, as for the institutional mechanisms we observe, compared to rather particular designs[4]. Our analysis sheds light on leaving out which of these details are essential for the simplicity of the designed rights structures.

In Section 3, we examine the implementability of an SCR via a rights structure (Γ-implementability) under different specifications. First, considering the most general rights structures, we show that Γ-implementable SCRs are characterized by a slight strengthening of Maskin monotonicity. Further, we observe that any Γ-implementable rule can be implemented in an *individual based* manner. Namely, if a coalition has the right to approve a change of state from s to t, then this coalition must be a singleton.

To motivate our second specification for Γ-implementability, let us recall that in most of game theoretic equilibrium concepts, agents are assumed to be myopic, and the convergence dynamics from a non-equilibrium strategy to an equilibrium strategy are left unspecified. However, in a design environment one can require the designed game forms to be endowed with well defined convergence properties. In the course

[4]We provide a detailed discussion of this point in Section 2, after we formally settle our model. One can also consult Chwe[5] for a similar discussion regarding this tradeoff.

of implementation, as another effect of the appended integer game, general constructions lack such properties. The simplicity of our framework makes it tractable to design rights structures endowed with nice convergence properties and consistent with farsighthed behavior. In Section 3, we also formulate and analyze implementation via such rights structures.

In Section 4, we investigate the relationship between implementation via rights structures and Nash implementation via mechanisms. As for the general Γ-implementability, every Nash implementable SCR is Γ-implementable. Further, in the presence of at least three agents, we characterize the class of rights structures implementability via which is equivalent to Nash implementability. This class can roughly be described as follows: (i) If a coalition is entitled to approve a change of state from s to t, then this coalition must be a singleton (*individual based*), (ii) if an agent is entitled to approve a change of state from s to t as well as from t to w, then the same agent should also be entitled to approve a change from s to w (*individual transitivity*).

Implementation via rights structures proposes a non-strategic framework for implementation. In Section 4, we also formulate a strategic environment for implementation, and show that SCRs implementable in this framework are exactly Γ-implementable SCRs. We name the design object of this environment as *deviation constraint mechanisms* (dc-mechanisms). In a classical mechanism, an agent can deviate from a joint strategy by choosing any strategy from his strategy set, being independent of the joint strategy. In a dc-mechanism, deviation strategies are constraint depending on the joint strategy from which the deviation is to be made. We show that, in the presence of at least three agents, an SCR is implementable via a rights structure if and only if the rule is implementable via a dc-mechanism. This result contributes to have a further understanding of how implementation via rights structures differs from implementation via mechanisms.

One natural specification for the state space of a rights structure is the set of alternatives, where the outcome function can be chosen as the identity map. In this specification, the unique design object is the code of rights. In Section 5, we investigate this simple case of implementation via codes of rights. We show that an SCR rule is implementable via codes of rights if and only if the SCR is *monotonic* and satisfies the *binary consistency* property that we introduce.

The rest of the paper is organized as follows. In Section 6, we formulate a rights structure which implements any Γ- implementable SCR with a minimal state space among the individual based rights structures. In Section 7, we introduce two stage rights structures and provide an application in bargaining theory. We present the related literature in Section 8. We conclude in Section 9.

2 Model

We use A to denote the non-empty, finite alternative set, and N to denote a non-empty finite set of agents. By little abuse of notation, we also denote the number of agents in the society by N. Each non-empty subset of N is called a **coalition**, and denoted generically by K.

For given A and N, for each $i \in N$, u_i denotes the **preference relation**[5] of agent i. For each distinct pair $a, b \in A$, $a \; u_i \; b$ denotes i prefers a to b. A **preference profile** $u = [u_1, \ldots, u_n]$. The collection of all preference profiles is denoted by $\mathcal{P}$. A **social choice rule** (SCR), F, maps each preference profile into a non-empty subset of A, i.e. $F : \mathcal{P} \to 2^A \setminus \emptyset$. An SCR, F, is **unanimous** if for each $a \in A$ we have $F(u) = a$, whenever every agent in the society prefers a to all other alternatives.

In classical implementation, the design object is a **mechanism**, which is a pair (M, g). The **joint strategy space** is $M = \Pi_{i \in N} M_i$, where M_i stands for the **strategy set** of agent i. The **outcome function**, g, maps every joint strategy to an alternative,

[5]A preference relation is a complete, transitive, antisymmetric binary relation on A

i.e. $g : M \to A$. A mechanism, (M, g), combined with a preference profile $u \in \mathcal{P}$, constitutes a normal form game. We denote the **pure strategy Nash equilibria** of this game by $\mathrm{NE}(M, g, u)$. A social choice rule, F, is *Nash implementable* via a mechanism (M, g), if at each preference profile u, alternatives chosen by F coincide with the pure strategy Nash equilibrium outcomes of the game at given u, i.e. for each $u \in \mathcal{P}$, $F(u) = \{g(s) : s \in NE(M, g, u)\}$.

In our framework, the design object is *a rights structure*, Γ, introduced by Sertel [29]. A **rights structure**, Γ, is a triple (S, h, γ). We use S to denote the **state space**, and h, the **outcome function** which maps each state to an alternative, i.e. $h : S \to A$. Let $S \times S$ stands for the set of all ordered pairs (s, t) with $s \neq t$.

Given a state space S, a **code of rights** specifies for each pair $(s, t) \in S \times S$, a family of coalitions denoted by $s \stackrel{\gamma}{\to} t$. We interpret that each coalition in $s \stackrel{\gamma}{\to} t$ is entitled to approve a change from s to t by the code of rights γ (has the right to move from s to t). For additionally given outcome function h and preference profile u, for each $(s, t) \in S \times S$, a coalition K prefers t to s if and only if for each $i \in K, h(t)\ u_i\ h(s)$. This is denoted by $K \in s \stackrel{u}{\to} t$.

In order to define Γ-implementability, first we will specify the Γ-equilibrium notion which plays the role of solution concepts (e.g. Nash) in classical implementation.

Definition 1 *Given a rights structure* $\Gamma = (S, h, \gamma)$*, for each* $u \in \mathcal{P}$*, we say* $s \in S$ *is a* Γ***-equilibrium*** *at* u *if for each* $t \in S$

$$(s \stackrel{\gamma}{\to} t)\ \cap\ (s \stackrel{u}{\to} t) = \emptyset$$

In other words, a state s is a Γ-equilibrium at preference profile u, if there is no other state t and coalition K; (i) which is entitled to approve a change from s to t, and (ii) each member of K prefers $h(t)$ to $h(s)$. We denote the Γ-equilibria set at preference profile u by $E(\Gamma, u)$.

Definition 2 *An SCR,* F*, is* Γ***-implementable*** *if there exists a rights structure,* $\Gamma =$

(S, h, γ)*, such that for each* $u \in \mathcal{P}$*,* $F(u) = h(E(\Gamma, u))$*.*

Being similar to Nash implementability, an SCR F is implementable via the rights structure Γ, if at each preference profile u, alternatives chosen by F coincide with the outcomes of the Γ-equilibrium at u.

Comment: Many game theoretic equilibrium concepts leave several components of the interaction unspecified. Similarly, we leave out the following details of the interaction in the definition of Γ-equilibrium concept.

(1) From a current state, s, typically many coalitions have the right to move. There is no priority ordering specified over these movements.

(2) If a coalition, K, has the right to take a move, K takes this move if and only if the movement is preferable. Issues such as preemtory moves (moving from a state s to t in order to avoid the movement of another coalition to a state w) are ruled out.

(3) An agent can possibly be a member of several coalitions that have the right to move from one state to another. The procedure that agents follow to form a coalition is not specified. Similarly, for a given coalition, there might be more than one preferable movement. The collective decision rule to choose among these movements is left unspecified either.

(4) Coalitions are myopic in their movements, i.e. a state is eliminated in favor of another which could also be eliminated by another coalition.

As it is especially for the cooperative equilibrium notions, the gain from leaving out such details are the simplicity and wider applicability, compared to rather particular designs[6]. As an advantage of design approach, we are able to construct rights structure that are not only simple and similar to institutional real life mechanisms, but also some of the above details can be specified. Our analysis will shed light on leaving out which of these details are essential to implement SCRs via simple

[6] One can consult Chwe[5] for a more detailed discussion along these lines.

rights structures. To further motivate the analysis to follow, we will present several $\Gamma-$implementable rules, and the rights structures that implement these rules..

2.1 Examples

One natural candidate for the state space of a rights structure is the *set of alternatives*, where the outcome function is the identity map. In this setting, a coalition $K \in a \overset{\gamma}{\to} b$ can be interpreted as; if the alternative a is the current status quo, then K can enforce the alternative b as the new status quo. For the following first five examples, the state space S of the designed Γ is the set of alternatives, and the outcome function is the identity map.

Example 1 (Pareto Rule) For each society N, alternative set A, and preference profile u, an alternative $x \in F(u)$ if and only if there is no other alternative y, which *Pareto dominates* x[7]. To see that F is Γ-implementable, let the code of rights be such that, only the entire society, N, has the right to make a movement among any two states. Pareto rule is implementable via this rights structure in an obvious way.

Example 2 (Pareto Rule with Minimal Liberalism) Let $N = \{1, 2, 3\}$ and $A = \{a, b, c\}$. Given a preference profile u, for each alternative $x \neq a$, $x \in F(u)$ if and only if there is no other alternative y, which *Pareto dominates* x, and $a \in F(u)$ if and only if a is not only Pareto efficient, but also agent 1 prefers a to b.

The rights structure, Γ, depicted in in Figure 1 implements F. The code of rights is such that, for each alternative x the entire society, N, has the right to move from x to any other alternative y, and additionally agent 1 has the right to move from a to b. One can easily verify that F is implementable via this rights structure. However, F is not Nash implementable.

Example 3 (Choice with Experts) Consider a decision maker who has a fixed

[7]That is, for each $i \in N$, $y \; u_i \; x$.

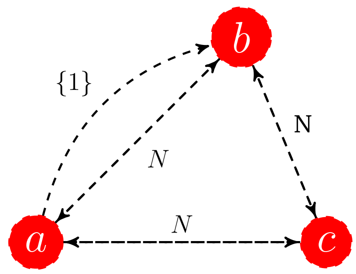


Figure 1: Γ for Pareto Rule with Minimal Liberalism

choice d, and also who can possibly choose among alternatives a, b, and c. Suppose he can consult with two consultants 1 and 2 with expertise on alternatives a and b respectively. Now, given a preference profile u for the experts, decision maker wants to choose a, i.e. $a \in F(u)$, if and only if a is top ranked by the first expert, $b \in F(u)$ if and only if b is top ranked by the second expert, and $c \in F(u)$ if and only if c is top ranked by both of experts. To see that F is Γ-implementable, consider the code of rights, γ, where for each $x \in A$, we have $a \xrightarrow{\gamma} x = \{\{1\}\}$, $b \xrightarrow{\gamma} x = \{\{2\}\}$, and $c \xrightarrow{\gamma} x = \{\{1\}, \{2\}\}$. One can easily verify that, γ implements this choice F.

To see that F is not Nash implementable[8] consider the preference profiles u and u' as specified below, where $F(u) = \{a\}$ and $F(u') = \{b\}$.

u

1	2
a	c
c	b
d	d
b	a

.

u′

1	2
c	b
a	c
d	d
b	a

Suppose, there is a mechanism, (M, g), which implements F in Nash equilib-

[8]Abreu and Sen [2] introduces the virtual implementation notion and show that any SCR is virtually implementable in case of having at least three agents. However, one can easily verify that this rule is not even virtually implementable.

rium. Now, we have $g(NE(M,g,u)) = a$. It follows that there exists $(m_1, m_2) \in NE(M,g,u)$ such that $g(m_1, m_2) = a$. Since a is the worst alternative according to u_2, it should be the case that for every $m_2' \in M_2,\ g(m_1, m_2') = a$. On the other hand, $g(NE(M,g,u')) = b$. It follows that there exists $(p_1, p_2) \in NE(M,g,u)$ such that $g(p_1, p_2) = b$. Since b is the worst alternative according to u_1', it should the case that for every $m_1' \in M_1,\ g(m_1', p_2) = b$. But then, we obtain $g(m_1, p_2) = b$ contradicting for every $m_2' \in\ M_2,\ g(m_1, m_2') = a$.

Example 4 (Walrasian Equilibrium) Consider any 2×2 pure exchange economy where agents have monotonic, continuous and strictly convex preferences over the entire consumption space R^2_+. Given a strictly positive endowment vector ω, let F^w be the rule that chooses the Walrasian equilibrium allocations at each preference profile. Let us consider the code of rights, γ, where for each $a, b \in R^2_+$, $a \overset{\gamma}{\to} b = \{\{1\}, \{2\}\}$ if a, b and ω are collinear or a is not feasible, and $a \overset{\gamma}{\to} b = \emptyset$ otherwise. One can easily verify that F^w implementable via this rights structure.

Example 5 (Majority Rule) Let $N = \{1,2,3\}$ and $A = \{a, b\}$. For each preference profile u, $F(u) = a$ if and only if a is preferred to b by at least two agents. To implement F, consider the rights structure $\Gamma = (S, h, \gamma)$, where $S = \{a^{1,2}, a^{1,3}, a^{2,3}, b^{1,2}, b^{1,3}, b^{2,3}\}$ with h mapping each state $a^{i,j}$ to outcome a and each state $b^{i,j}$ to outcome b. Let γ be such that, for each $i, j \in N$, and $s \in S$, $a^{i,j} \overset{\gamma}{\to} s = \{\{i\}, \{j\}\}$, and $b^{i,j} \overset{\gamma}{\to} s = \{\{i\}, \{j\}\}$. To see that Γ implements F, suppose that the true preference profile is u, where agents 2 and 3 prefer a to b, and agent 1 prefers b to a, so $F(u) = a$.

As it is illustrated in Figure 2, since neither 2 nor 3 prefers b to a at u, $a^{2,3} \in E(\Gamma, u)$. Moreover, for each distinct $i, j \in N$, either i or j prefers a to b at u. It follows that $b^{i,j} \notin E(\Gamma, u)$. Similar reasoning shows that for each $u \in \mathcal{P}$, $F(u) = h(E(\Gamma, u))$.

The rights structures designed for the majority rule as well as the remaining examples have general state spaces and outcome functions. As for the general rights structure, one possible interpretation of a state is a *proposal* for an alternative sup-

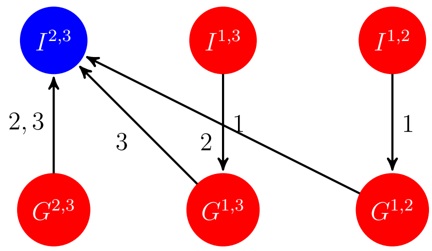


Figure 2: Majority rule

ported by some evidence indicating why this alternative should be acceptable. In such a setting, a coalition $K \in s \overset{\gamma}{\to} t$ can be interpreted as; coalition K can refute the evidence s favoring $h(s)$. Put differently, the nature of the evidence will suggest which coalitions should have the right to move from that state. This is what we will observe for the rights structures designed to implement the remaining examples.

Example 6 (Tops Rule) For each society N, alternative set A, and a given preference profile u, $F(u) = x$ if and only if x is top ranked by at least one of the agents. To see that F is Γ-implementable, let S consists of all alternative, agent pairs. We can interpret a state (x, i) as a claim of; x is top ranked by i. For each $(x, i) \in S$, let $h(x, i) = x$. Let code of rights, γ, be such that for each $(x, i), (y, j) \in S \times S$, if $i \neq j$ then $(x, i) \overset{\gamma}{\to} (y, j) = \emptyset$, and if $i = j$ then $(x, i) \overset{\gamma}{\to} (y, j) = \{\{i\}\}$.

To see that the suggested rights structure implements F, let x be an alternative top ranked by agent i at a given preference profile. Since no agent other than i can move from (x, i), it is an equilibrium state. Moreover, if an alternative y is not top ranked, then from each state (y, j), agent j prefers to move a state where the associated outcome is his top ranked alternative. Hence, no state with outcome y can be an equilibrium state.

Example 7 (Guilty vs. Innocent) Consider a set of agents (e.g jurors) $N = \{1, 2, ...n\}$ who will decide whether a suspect is guilty (G) or innocent (I). Suppose they decide that the suspect is I if and only if at least k agents think he is so. Put differently, if at least k jurors prefer I to G.

To see that for each $k \in \{1, 2, ...n\}$, this rule is Γ-implementable, first consider the preference profiles of the *form* u_I where any k agents prefer I to G and rest prefers G to I. Similarly, consider the preference profiles of the *form* u_G where any $n-k+1$ agents prefer G to I and rest prefers I to G. Let S consist of the preferences of the *form* u_I and of the *form* u_G. Let the outcome function, h, map any state of the form u_I to I, and any state of the form u_G to G. Let the code of rights, γ, entitle any agent i to move from a state of the form u_I to a state of the form u_G if and only if i prefers I to G at that preference profile of the form u_I. It follows that, i must be among the k agents who prefers I to G. Similarly, let γ entitle any agent j to move from a state of the form u_G to a state of the form u_I if and only if j prefers G to I at that u_G. It follows that, j must be among the $n-k+1$ agents who prefers G to I.

For given (S, h) let us see that γ implements F. Suppose u^* is the true preference profile, where $F(u^*) = I$. Now, choose any k agents who prefers I to G at u^*. Next, consider a new preference profile where all the rest prefers G to I. Call this new profile as u'. Note that u is of the form u_I, so we have $u \in S$. For each state v of the form u_G and $i \in N$, we have $\{i\} \in u \xrightarrow{\gamma} v$ only if $I \ u_i \ G$. Hence, $I u_i^* G$, and we obtain $u \in E(\Gamma, u^*)$. On the other hand, consider any state v of the form u_G. To see that $v \notin E(\Gamma, u^*)$, first recall that there are at least k agents who prefers I to G at u^*. It follows that there exists $i \in N$ with $\{i\} \in v \xrightarrow{\gamma} u$, and $I \ u_i^* \ G$. Thus, $F(u^*) = h(E(\Gamma, u^*))$. Symmetric arguments work for $F(u^*) = G$.

3 Monotonicity and Γ-implementation

In classical implementation theory, Maskin [17] shows that any Nash implementable SCR is monotonic, and monotonicity combined with *no veto power* condition is sufficient for Nash implementability in case of having at least three agents. Let us remind this well known monotonicity condition for implementation. Suppose that an alternative a is acceptable at a preference profile u^1 according to the SCR, F, in question.

Then, if a does not fall in anyone's ranking relative to any other alternative in going from profile u^1 to profile u^2, *monotonicity* requires that a also be acceptable at u^2. More formally, an SCR, F, is **monotonic**, if for each $u^1, u^2 \in \mathcal{P}$ and $a \in F(u^1)$, we have $a \in F(u^2)$ whenever for every $i \in N$ and $b \in A$, if $a\ u_i^1\ b$ then $a\ u_i^2\ b$ holds. As an interesting result, it follows from Abreu and Sen [2] that, under a weak domain restriction, if agents are expected utility maximizers, then for each $\epsilon > 0$, there exists an SCR, F_ϵ, which is monotonic and ϵ-close to F.[9]

For the full characterization, Danilov [6] proposes the *essential monotonicity* condition, which strengthens monotonicity by requiring the consequent part of the condition to hold in case of a switch consisting of alternatives which play a kind of essential role in the choice. Danilov [6] shows that *essential monotonicity* is both necessary and sufficient for Nash implementability in case of having at least three agents.

Here, we consider a monotonicity condition slightly stronger than that of Maskin [17], but much weaker than Danilov [6]. We name this condition as *image monotonicity*. We show that an SCR, F, is Γ- implementable if and only if F is image monotonic. Moreover, it will follow from the proof of this proposition that any Γ-implementable rule can be implemented via an **individual based (IB)** rights structure, that is for each distinct state pair (s, t), if a coalition has the right to move from s to t, then that coalition should be a singleton. This would rule out the issue **(3)**, and partially issue **(1)** that we listed as a part of our discussion in Section 2.

Before proceeding to the definition of image monotonicity, let us introduce some useful notation. Let $I(F)$ denote the *image* of F, i.e $I(F) = \{a \in A : a \in F(u) \text{ for some } u \in \mathcal{P}\}$. The **lower contour set of u_i with respect to** $a \in A$, denoted by $\mathrm{L}(u_i, a)$, is the set of alternatives to which a is preferred by agent i, i.e. $\mathrm{L}(u_i, a) = \{b \in A : a\ u_i\ b\}$. By using this definition, (Maskin) monotonicity can be

[9] F_ϵ is ϵ-close to F, if for each $u \in \mathcal{P}$, and $a \in F(u)$, there exists a lottery $p \in F_\epsilon(u)$ which yields outcome a with more than $1 - \epsilon$ probability.

restated as follows: An SCR, F, is **monotonic**, if for each $u^1, u^2 \in \mathcal{P}$ and $a \in F(u^1)$, we have $a \in F(u^2)$ whenever for every $i \in N$, $\mathrm{L}(u_i^1, a) \subseteq \mathrm{L}(u_i^2, a)$.

Image monotonicity: An SCR F is *image monotonic*, if for each $u^1, u^2 \in \mathcal{P}$, and each $a \in F(u^1)$, we have $a \in F(u^2)$ whenever for every $i \in N$,

$$\mathrm{L}(u_i^1, a) \cap I(F) \subseteq \mathrm{L}(u_i^2, a)$$

.

Proposition 1 *Given an SCR F, the following are equivalent;*

i. F is image monotonic

ii. F is $\Gamma - implementable$

iii. F is $\Gamma - implementable$ via an IB rights structure.

To prove Proposition 1 as well as several other results to follow, we will use a particular rights structure which we name as *direct rights structure*. Before proceeding with the proof, we define this rights structure.

Direct rights structures

For a given SCR, F, the direct rights structure $\Gamma^d = (S^d, h^d, \gamma^d)$. The state space, S^d, consists of alternative and preference profile pairs (a, u) with $a \in F(u)$. Put differently S^d is the *graph* of F, i.e. $S^d = \{(a, u) : a \in F(u)\}$. The outcome function h^d maps each $(a, u) \in S^d$ to alternative a. We can interpret a state (a, u) as a proposal for alternative a with supporting proposition u [10].

Given (S^d, h^d), the code of rights, γ^d, will entitle any agent, i, to move from any state (a, u) to another state (b, v) if and only if i prefers a to b at u, i.e. $a \; u_i \; b$. Given γ^d, if i moves from (a, u) to (b, v), this shows that i prefers b to a at the true profile.

[10]This can be thought as an argument of the form, "We should choose a, since the true preference profile (which is not verifiable) is u."

However, if the true profile were to be u as claimed, then there would be no such agent. Put differently, according to γ^d, agents that can move from (a, u) are those who can refute the proposition that supports a.

The idea captured by the direct rights structure can be found in any Maskin [17] type of mechanism. Basically, direct rights structure captures what is captured in a Maskin mechanism without an integer game. Proposition 1 states that this suffices for Γ-implementability.

In Example 5, we constructed a rather artificial rights structure to implement the majority rule, F. In the following example, we tailor the the direct rights structure to implement F, which also renders the previous construction.

Example 8 (Majority Rule) Let $N = \{1, 2, 3\}$ and $A = \{a, b\}$. For each preference profile u, $F(u) = a$ if and only if a is preferred to b by at least two agents. Notice that there are four profiles $\{u^0, u^3, u^2, u^1\}$ where a is chosen by F, since a is preferred to b by N, $\{1, 2\}$, $\{1, 3\}$, and $\{2, 3\}$ respectively. Similarly let $\{v^0, v^3, v^2, v^1\}$ be the profiles where the roles of a and b are changed, so b is chosen by F.

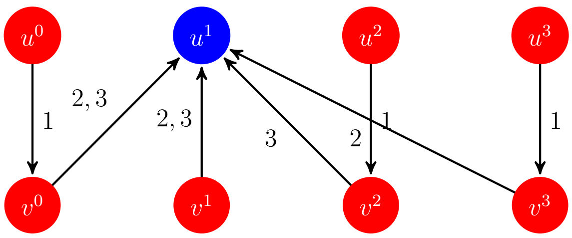


Figure 3: Majority rule

Consider the direct rights structure $\Gamma^d = (S^d, h^d, \gamma^d)$ tailored for this rule. Namely, we have $S^d = \{u^m, v^m\}_{m=0}^3$, where the outcome function h^d maps each u^m to a and each v^m to b. Now, the code of rights, γ^d, entitles any agent $\{i\}$ to move from any state u^m to another state v^r if and only if $a \; u_i^m \; b$.

To see that Γ^d implements F, suppose the true preference profile is u^1, so we have

$F(u^1) = a$. Now, consider the state u^1. Only agent 1 prefers to move from u^1 to a state with outcome b, but by the design of γ^d, 1 does not have the right to move. Hence, $u^m \in E(\Gamma^d, u^m)$.

On the other hand for each v^r since at least two agents prefer b to a at v^r, there necessarily exists $i \in \{2,3\}$ such that $b\ v_i^r\ a$ and $a\ u_i^1\ b$. By the design of γ^d, this means i is entitled to move from (b, v^r) to (a, u^m). Since $a\ u_i^1\ b$, it follows that $(b, v^r) \notin E(\Gamma^d, u^1)$. Thus, we obtain $h^d(E(\Gamma^d, u^1)) = a$.

We prove Proposition 1 by using direct structures and following a similar line of simple reasoning as used in this example.

Proof of Proposition 1.

$(i \Rightarrow iii)$ Let F be an image monotonic social choice rule. Let Γ be the direct rights structure which is IB. Formally for each $(a, u), (b, v) \in S^d \times S^d$ and $i \in N$ we have,

$$\{i\} \in (a, u) \xrightarrow{\gamma^d} (b, v) \text{ if and only if } a\ u_i\ b.$$

We will show that for each $u^2 \in \mathcal{P}$ and $a \in A$, we have $a \in F(u^2)$ if and only if there exists $(a, u^1) \in E(\Gamma^d, u^2)$. Given $u^2 \in \mathcal{P}$, suppose $a \in F(u^2)$. We will show that $(a, u^2) \in E(\Gamma^d, u^2)$. Now, for each $(b, v) \in S^d$ and agent $\{i\} \in (a, u^2) \xrightarrow{\gamma^d} (b, v)$, by the design of γ^d, we have $a\ u_i^2\ b$. Thus, we obtain $(a, u^2) \in E(\Gamma^d, u^2)$.

Conversely, suppose there exists $(a, u^1) \in E(\Gamma^d, u^2)$. Note that, for each $b \in I(F)$, there exists a preference profile v such that $(b, v) \in S^d$. Since we have $(a, u^1) \in E(\Gamma^d, u^2)$, this means for each $i \in N$ and $b \in I(F)$, if $a\ u_i^1\ b$ then $a\ u_i^2\ b$. That is, $\mathrm{L}(u_i^1, a) \cap I(F) \subseteq \mathrm{L}(u_i^2, a)$. Since $a \in F(u^1)$, it follows from *image monotonicity* that $a \in F(u^2)$.

$(iii \Rightarrow ii)$ Obvious.

$(ii \Rightarrow i)$ Let F be a Γ-implementable social choice rule, and let $\Gamma = (S, h, \gamma)$ be a rights structure which implements F. Let $u^1, u^2 \in \mathcal{P}$ be such that, $a \in F(u^1)$ and suppose for every $i \in N$, we have $\mathrm{L}(u_i^1, a) \cap I(F) \subseteq \mathrm{L}(u_i^2, a)$. Now, we will show that $a \in F(u^2)$. Suppose $a \notin F(u^2)$. Let $s \in E(\Gamma, u^1)$ with $h(s) = a$. Since $a \notin F(u^2)$, $s \notin E(\Gamma, u^2)$. It follows that, there exists $t \in S$ and a coalition K, such that $h(t) = b$ for some $b \neq a$, where $K \in s \overset{\gamma}{\to} t \cap a \overset{u^2}{\to} b$.

We first claim that $b \in I(F)$. To see this, consider the preference profile u^b where every agent top ranks b. Since $t \in S$ with $h(t) = b$, there is no agent that prefers a change of state from t to another state. Thus, $t \in E(\Gamma, u^b)$. Since $F(u^b) = E(\Gamma, u^b)$, we obtain $b \in F(u^b)$.

Now, since for every $i \in K$, $\mathrm{L}(u_i^1, a) \cap I(F) \subseteq \mathrm{L}(u_i^2, a)$ and $b \; u_i^2 \; a$, we obtain $K \in a \overset{u^1}{\to} b$. But, $K \in s \overset{\gamma}{\to} t$ as well, contradicting that $s \in E(\Gamma, u^1)$. ■

Let F be an SCR and Γ be a rights structure that implements F. In general, starting from any non-equilibrium state it may not be possible to reach an equilibrium state. Put differently, as depicted in Figure 4, the only possible movement from a non-equilibrium state might direct into a cycle consisting of only the non-equilibrium states. In such a case, one can argue that equilibrium state would prevail, only if the society has that state as the current status quo. To avoid such a restrictive conclusion, one can require the following robustness condition to be satisfied by a rights structure: For each given preference profile u, each $s_0 \notin E(\Gamma, u)$, and state $s \in E(\Gamma, u)$, there exists a *path* (an array of states) $s_1, \ldots, s_m$ connecting s_0 to s. That is; for each $k \in \{0, \ldots, m-1\}$, one has $(s_{k-1} \overset{\gamma}{\to} s_k) \cap (s_{k-1} \overset{u}{\to} s_k) \neq \emptyset$, and $(s_m \overset{\gamma}{\to} s) \cap (s_m \overset{u}{\to} s) \neq \emptyset$. One can easily show that if F is *unanimous* , then the direct rights structure will satisfy this property. To see this, for a given preference profile u, let $(a, u') \notin E(\Gamma, u)$, it follows that there exists $b \in A$ and $i \in N$ such that $a \; u_i' \; b$ and $b \; u_i \; a$. If $b \in F(u)$, then we are done. If not, let u^b be a preference profile where b is top ranked by each agent. Since F is unanimous, $F(u^b) = b$. Further, we know that $\{i\} \in (a, u') \overset{\gamma}{\to} (b, u^b)$ and $b \; u_i \; a$. Now, let $(c, u'') \in E(\Gamma, u)$. Note that any agent has the right to move from (b, u^b) to (c, u''). Now, we have to show that at least one agent prefers this movement. Since F is image monotonic and unanimous, F is

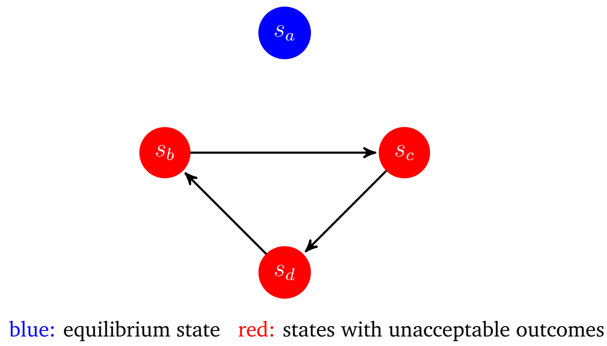


Figure 4:

also Pareto efficient. Hence there exists $j \in N$ with $c \; u_j \; b$.

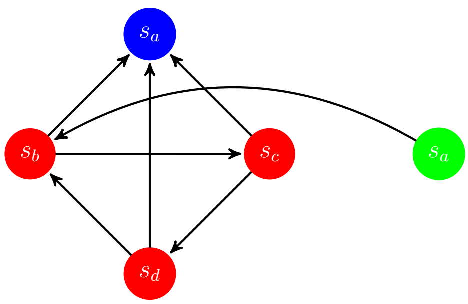
3.1 Γ-implementation with External Stability

Most of the game theoretic equilibrium concepts are *myopic* in the sense that; a joint strategy is eliminated from the equilibria, whenever a player prefers an unilateral deviation, similarly, a joint strategy is an equilibrium, whenever there is no preferable unilateral deviation. However, one can question the myopia of an agent eliminating a strategy in favor of another which could also be eliminated. Several studies in the game theory literature addresses this question. As an incomplete list we can count the stability discussions of von Neumann [30], Aumann and Maschler [4], ”social decision systems” by Rubinstein [26], ”social situations” by Greenberg [12], and ”largest consistent set ” by Chwe [5]. However, none of these discussions are in a design framework.

Similarly Γ-equilibrium is not immune to this criticism. According to the formulation of Γ-equilibrium, agents use their rights to approve a change from any state to another, whenever latter outcome is better, regardless of whether it is an equilibrium state or not. Departing from this point on, we formulate a rather stringent stability condition. We require the rights structure implementing an SCR to satisfy the following property: Even if agents only use their rights to move from any non-equilibrium state with an unacceptable outcome to an equilibrium state, whenever the equilibrium outcome is preferable.

External Stability: A rights structure, $\Gamma = (S, h, \gamma)$, is *externally stable* if for each $u \in \mathcal{P}$, and $s \notin E(\Gamma, u)$ with $h(s) \notin F(u)$, there exists $t \in E(\Gamma, u)$ and $i \in N$ such that;

$$\{i\} \in (s \xrightarrow{\gamma} t) \cap (s \xrightarrow{u} t)$$



blue: equilibrium state red: states with unacceptable outcomes green: a non-eq. state with an acceptable outcome

Figure 5: An externally stable Γ

In the analysis to follow, we provide a characterization of SCRs which are implementable via such rights structures. The following strengthening of image monotonicity, which is similar to the well known *independence of irrelevant alternatives* [3], characterizes this class of SCRs.

Winner Monotonicity: An SCR, F, is *winner monotonic*, if for each $u^1, u^2 \in \mathcal{P}$ and $a \in F(u^1)$, we have $a \in F(u^2)$ whenever for every $i \in N$,

$$\mathrm{L}(u_i^1, a) \cap F(u^2) \subseteq \mathrm{L}(u_i^2, a)$$

Proposition 2 *An SCR, F, is implementable via an externally stable rights structure if and only if F is* winner monotonic.

Proof. $(\Rightarrow)$ Let F be implementable via an externally stable rights structure Γ. Let $u^1, u^2 \in \mathcal{P}$ such that $a \in F(u^1)$. Since $a \in F(u^1)$, there exists $s \in E(\Gamma, u^1)$ s.t $h(s) = a$.

Suppose for every $i \in N$, we have $\mathrm{L}(u_i^1, a) \cap F(u^2) \subseteq \mathrm{L}(u_i^2, a)$. It follows that for every $i \in N$ and $b \in F(u^2)$, if $a\ u_i^1\ b$ then $a\ u_i^2\ b$ as well. Put differently, we have $a \overset{u^2}{\to} b \subseteq a \overset{u^1}{\to} b$. Now, since $s \in E(\Gamma, u^1)$, for each $b \in F(u^2)$ and $t \in S$ with $h(t) = b$, we have $(s \overset{\gamma}{\to} t) \cap (a \overset{u^2}{\to} b) = \emptyset$. It follows from the external stability that we have $s \in E(\Gamma, u^2)$, hence $a \in F(u^2)$.

($\Leftarrow$) Let F be winner monotonic. Let $\Gamma = (S, h, \gamma)$ be the direct rights structure. From Proposition 1, we know that Γ implements F. Now, we will show that Γ is externally stable.

Let $u^1 \in \mathcal{P}$ and $a \notin F(u^1)$. If $a \notin I(F)$, then there is no state $s \in S$ with $h(s) = a$, so we are done. Suppose $a \in I(F)$ and let $(a, v) \in S$. We claim that, there exists $i \in N$ and $b \in F(u^1)$ s.t $b\ u_i^1\ a$ and $a\ v_i\ b$. If not, for each $i \in N$, we have $\mathrm{L}(v_i, a) \cap F(u^1) \subseteq \mathrm{L}(u_i^1, a)$. It follows from winner monotonicity that $a \in F(u^1)$, a contradiction. Now, for such $i \in N$ and $b \in F(u^1)$ by the design of γ, we have $\{i\} \in (a, v) \overset{\gamma}{\to} (b, u^1)$. It follows that Γ is externally stable. ■

4 Rights Structures underlying the Nash Implementability

In this section, first we introduce two simple properties pertaining to rights structures. Then, we show that the class of Nash implementable SCRs coincide with the class of Γ-implementable SCRs if the rights structure used to implement an SCR satisfies these two properties.

(IB) A rights structure, $\Gamma = (S, h, \gamma)$, is **individual based** if for each distinct $s, t \in S$, $s \overset{\gamma}{\to} t$ is either empty or consists of only single agents.

(IT) A rights structure, $\Gamma = (S, h, \gamma)$, is **individually transitive** if for each distinct $s, t, w \in S$, and $i \in N$, if $\{i\} \in (s \overset{\gamma}{\to} t) \cap (t \overset{\gamma}{\to} w)$, then $\{i\} \in s \overset{\gamma}{\to} w$.

Proposition 3 *Given $N \geq 3$, an SCR, F, is Nash-implementable if and only if F is implementable via an individual based and individually transitive rights structure.*

Proof. ($\Rightarrow$) Let F be an SCR which is Nash-implementable via a mechanism (M, g). Let $\Gamma = (S, h, \gamma)$ be such that $S = M$ and $h = g$. Define γ such that, for each $s, t \in S$, if $s_{-i} = t_{-i}$ for some $i \in N$, then $\{i\} \in s \stackrel{\gamma}{\rightarrow} t$. If s is different from t by two or more components, then $s \stackrel{\gamma}{\rightarrow} t = \emptyset$. Clearly Γ is IB and IT. Moreover, one can easily verify that $NE(M, g, u) = E(\Gamma, u)$.

($\Leftarrow$) Suppose there is an IB and IT rights structure $\Gamma = (S, h, \gamma)$ which implements F. If S consists of only two states s, s', then let us consider another rights structure with the only difference that there is an additional state s'' such that; the outcome is same as s, and the agents who has the right to take a move from/to s'' are specified as same as γ specifies for s. Notice that this new rights structure with three states would implement F as well. Hence we can w.l.o.g. assume that S contains at least three different states.

Next, we define the mechanism to implement F under Nash equilibrium. For each $i \in N$, let $M_i = S \times N \times \mathcal{Z}_+$ where $N = \{1, \ldots, n\}$ stands for the set of agents ordered accordingly, and $\mathcal{Z}_+$ stands for the set of positive integers. Let the outcome function, $g : M \rightarrow A$, be such that for each $m \in M = \Pi_{i \in N} M_i$;

(1) If everybody announces the same state s, then we have $g(m) = h(s)$.

(2) If everybody but i announces the state s, and i announces the state s' such that $\{i\} \in s \stackrel{\gamma}{\rightarrow} s'$, then $g(m) = h(s')$. If $\{i\} \notin s \stackrel{\gamma}{\rightarrow} s'$, then $g(m) = h(s)$.

(3) If there are at least three different states announced, let i be the agent with the minimum index among the ones who announce the highest integer. Let agent j be the agent whom is indicated by agent i. Let s'' be the state announced by agent j, we have $g(m) = h(s'')$.

Next, consider the mechanism (M, g). For each $u \in \mathcal{P}$ and $s \in E(\Gamma, u)$, consider any strategy profile, m, such that for every $i \in N$, $m_{i,1} = s$. Since $s \in E(\Gamma, u)$, we have for every $s' \neq s$ and $i \in N$, if $h(s)\ u_i\ h(s')$ then $i \notin s \xrightarrow{\gamma} s'$. It follows that $m \in NE(M, g, u)$.

Next we show that for each $m \in NE(M, g, u)$, we have $g(m) \in h(E(\Gamma, u))$. By contradiction, suppose there exists $m \in M$ such that $g(m) \notin h(E(\Gamma, u))$. Assuming $g(m) = h(s)$ for some $s \in S$, this equivalently means $s \notin E(\Gamma, u)$. Combined with IB, it follows that there exists $s' \in S$ and $i \in N$ such that $i \in s \xrightarrow{\gamma} s'$ and $h(s)\ u_i\ h(s')$. We obtain the contradiction by showing that there exists $m'_i \in M_i$ such that $g(m'_i, m_{-i}) = h(s')$. For brevity, from now on let $t \in \mathcal{Z}_+$ be such that for each $j \in N,\ t > m_{j,3}$.

Case 1: Suppose $g(m)$ is realized by condition (1), then let m'_i be such that $m'_{i,1} = s'$.

Case 2: Suppose $g(m)$ is realized by condition (2). Let $\bar{s}$ stand for the state announced by everyone except a single agent.

i) If $m_{i,1} = s$, then for every $j \neq i$, $m(j, 1) = \bar{s}$. Since $m \in NE(M, g, u)$ with $g(m) = h(s)$, we have $i \in \bar{s} \xrightarrow{\gamma} s$. Moreover, we know that $i \in s \xrightarrow{\gamma} s'$. Since γ is IT, we obtain $i \in \bar{s} \xrightarrow{\gamma} s'$. Now, let m'_i be such that $m'_{i,1} = s'$. Notice that we have $g(m'_i, m_{-i}) = h(s')$.

ii) If $m_{i,1} = \bar{s}$ and $\bar{s} = s'$, since $N \geq 3$ and $g(m)$ is realized by condition (2), there exists $k \in N$ such that $m_{k,1} = s'$. Since there are at least three states, let $m'_i = [s'', k, t]$ where $s'' \notin \{s, s'\}$.

iii) If $m_{i,1} = \bar{s} \notin \{s, s'\}$, then let $m'_i = [s', i, t]$.

Case 3: Suppose $g(m)$ is realized by condition (3).

i) If $s' \notin \{m_{j,1}\}_{j \in N}$, then let $m'_i = [s', i, t]$.

ii) If there exists $j \in N$ such that $m_{j,1} = s'$, then let $m'_i = [m_{i,1}, j, t]$.

Thus, we reach the contradiction that $m \notin NE(M, g, u)$. ∎

We know from Proposition 1 that each Γ-implementable SCR is implementable via an *individual based* code of rights. This observation combined with Proposition 3 shows that; what distinguishes Nash-implementable SCRs from the rest is the underlying rights structure being IT. Let us turn back to Examples 5-7, where we have shown that these rules are Γ-implementable. Now, one can easily verify that rights structure that we use in each example is IB and IT. Hence, it follows from Proposition 3 that these rules are Nash implementable.

We learn from Proposition 3 that, not only that each Nash implementable SCR is Γ-implementable, but what should be the additional restrictions on the rights structures to obtain Nash implementability. In what follows we are motivated by the dual question: How can we slacken the description of a mechanism to implement Γ-implementable SCRs.

4.1 Deviation Constraint Mechanisms

Implementation via general rights structures proposes a non-strategic framework for implementation. In this section, we first formulate a strategic environment for implementation. Then, we show that an SCR, F, is implementable in this environment if and only if F is Γ-implementable. We name the design object of this environment as *deviation constraint mechanisms* (dc-mechanisms). In a classical mechanism, an agent can deviate from a joint strategy by choosing any strategy from his strategy set, being independent of the joint strategy from which the deviation is to be made. Dc-mechanisms are different from usual mechanisms on this front. Namely, in a dc-mechanism, deviation strategies are constraint depending on the joint strategy from which the deviation is to be made.

Formally, a **deviation constraint mechanism** (dc-mechanism) is a triple $(M, \mathcal{D}, g)$. As usual, the joint strategy space $M = \Pi_{i\in N} M_i$ where M_i stands for the strategy set of agent i. The outcome function g maps every joint strategy to an alternative, i.e. $g : M \to A$. For each agent i, a constraint function, $\mathcal{D}_i$, maps each joint strategy m to a subset of M_i, i.e. $\mathcal{D}_i : M \to M_i$. In a dc-mechanism, if an agent i would deviate from strategy m, he is constraint to choose his strategy from $\mathcal{D}_i(m)$.

Given a preference profile u, a joint strategy m is an equilibrium of the dc-mechanism, $(M, \mathcal{D}, g)$, at u if and only if for each $i \in N$ and $m'_i \in \mathcal{D}_i(m)$, $g(m)\ u_i\ g(m'_i, m_{-i})$. We denote the equilibria of $(M, \mathcal{D}, g)$ at u, by $\mathbf{E}(M, \mathcal{D}, g, u)$.

Proposition 4 *Given $N \geq 3$, an SCR, F, is Γ-implementable if and only if there exists a dc-mechanism, $(M, \mathcal{D}, g)$, such that for each $u \in \mathcal{P}$, $F(u) = \mathbf{E}(M, \mathcal{D}, g, u)$.*

The construction of the dc-mechanism, used to prove Proposition 4, has many similarities to the construction in the proof of Proposition 3. Once the dc-mechanism that we use to obtain the result is described, we will omit the similar parts which can be found in the proof of Proposition 3.

Proof. $(\Leftarrow)$ This part follows from a reasoning similar to the one we used in the proof of Proposition 3.

$(\Rightarrow)$ Suppose F is implementable via a rights structure (S, h, γ) Assume w.l.o.g. (by Proposition 1) that this rights structure is individual based. Let $\mathcal{S}^{\mathcal{A}}$ denote the collection of all finite arrays of distinct states $(s_1, \ldots, s_k)$ from S. For each $i \in N$, let $M_i = \mathcal{S}^{\mathcal{A}} \times N \times \mathcal{Z}_+$ where $N = \{1, \ldots, n\}$ stands for the set of agents ordered accordingly and $\mathcal{Z}_+$ stands for the set of positive integers.

First, let us specify the deviation constraints. For each $m \in M$,

(DC1) If everybody announce the same single state then each agent can deviate to a strategy where he announces a single state, i.e. for each $i \in N$, $\mathcal{D}_i(m) = S \times N \times \mathcal{Z}_+$.

(DC2) If everybody but i announces the same single state and i announces another single state, then i can deviate to any strategy, i.e. $\mathcal{D}_i(m) = M_i$. Any other agent $j \neq i$, can deviate to a strategy where j announces a single state, i.e. for each $j \in N \setminus \{i\}$, $\mathcal{D}_j(m) = S \times N \times \mathcal{Z}_+$.

We pose no deviation constraints other than DC1 and DC2, so for each other $m \in M$ and for each $i \in N$, $\mathcal{D}_i(m) = M_i$.

We define the outcome function $g : M \to A$ such that;

(1) If everybody announces the same single state s, then we have $g(m) = h(s)$.

(2) If everybody but i announces the single state s and i announces the state array $\{s_1, \ldots, s_k\}$ such that $\{i\} \in (s \overset{\gamma}{\to} s_1) \cap (s_1 \overset{\gamma}{\to} s_2) \cdots \cap (s_{k-1} \overset{\gamma}{\to} s_k)$, then $g(m) = h(s_k)$. If $\{i\} \notin (s \overset{\gamma}{\to} s_1) \cap (s_1 \overset{\gamma}{\to} s_2) \cdots \cap (s_{k-1} \overset{\gamma}{\to} s_k)$, then $g(m) = h(s)$.

(3) Otherwise, let i be the agent with the minimum index among the ones who announce the highest integer. Let agent j be the agent whom is indicated by agent i. Let s_k be the last state in the state array announced by agent j. We have, $g(m) = h(s_k)$.

Next, consider the dc-mechanism $(M, \mathcal{D}, g)$. For each $u \in \mathcal{P}$ and $s \in E(\Gamma, u)$, consider any strategy profile, m, such that for every $i \in N$, $m_{i,1} = s$. Recall that each $i \in N$ can deviate from m by only announcing a single state, i.e. $D_i(m) = S \times N \times \mathcal{Z}_+$. Since $s \in E(\Gamma, u)$, we have for every $s' \neq s$ and $i \in N$, if $h(s)\ u_i\ h(s')$ then $\{i\} \notin s \overset{\gamma}{\to} s'$. It follows that $m \in \mathbf{E}(M, D, g, u)$.

Next, we show that for each $m \in \mathbf{E}(M, \mathcal{D}, g, u)$, we have $g(m) \in h(E(\Gamma, u))$. We show this by similar reasoning as we have in the proof of Proposition 3, so we will not replicate those in here. Yet, there is only one case where we use a different line of reasoning to obtain the contradiction, where IT condition was used in the proof of Proposition 3.

Case 2: Suppose $g(m)$ is realized by condition (2).

i) If $m_{i,1} = s_1, \ldots, s_k, s$ and for every $j \neq i$, $m(j, 1) = \bar{s}$. Since $m \in \mathbf{E}(M, D, g, u)$ with $g(m) = h(s)$, we have $\{i\} \in (\bar{s} \overset{\gamma}{\to} s_1)\ \cap \cdots \cap\ (s_k \overset{\gamma}{\to} s)$. Moreover, we know that $\{i\} \in s \overset{\gamma}{\to} s'$. Now, let m'_i be such that $m'_{i,1} = s_1, \ldots, s_k, s'$. Notice that by (2) $m'_i \in \mathcal{D}_i(m)$, and we have $g(m'_i, m_{-i}) = h(s')$. ■

5 γ-implementation

One natural candidate for the state space of a rights structure is the *set of alternatives*, where the outcome function is the identity map. In this setting the unique design object is the code of rights. In this section, we analyze this simple case of implementation via rights structures, which we name as γ-implementation.

We obtain the following definition for γ-equilibrium if we simply specify the alternative set A as the state space and identity function as the outcome function in the definition of Γ-equilibrium.

Definition 3 *For each $u \in \mathcal{P}$, we say $a \in A$ is a γ-**equilibrium** at u if for each $b \in A$, $(a \overset{\gamma}{\to} b) \cap (a \overset{u}{\to} b) = \emptyset$.*

Put differently, an alternative a is a γ -equilibrium at a given preference profile u, if there is no coalition, K, such that i) K is entitled to approve a change from a to another alternative b, and ii) each member of K prefers b to a. We denote γ -equilibria at a preference profile u by $E(\gamma, u)$.

Definition 4 *We say an SCR, F, is γ-**implementable** if there exists a code of rights, γ, such that for each $u \in \mathcal{P}$, $F(u) = E(\gamma, u)$.*

5.1 Results

In this section, we provide a full characterization of γ-implementable SCRs. [11] It follows from Proposition 1 that an SCR should satisfy Maskin monotonicity to be γ-implementable. However, one can easily verify that Maskin monotonicity is not sufficient for γ-implementability. To obtain a full characterization of γ-implementable SCRs, we introduce a new property pertaining to SCRs, named as *binary consistency*.

[11] Yildiz [31] shows that unanimous $(A, \gamma)-$implementable SCRs are characterized in terms of a non-trivial strengthening of monotonicity.

To introduce binary consistency, we need an auxiliary definition. Consider any $u \in \mathcal{P}$ and $a, b \in A$ such that b is not Pareto dominated by a at u, i.e. for some $i \in N$, $b\ u_i\ a$. Let $\mathbf{u^{ab}}$ stand for the preference profile obtained from u, such that the only difference is: For each $c \in A \setminus \{a, b\}$ and $i \in N$, we have $a\ u_i^{ab}\ c$ and $b\ u_i^{ab}\ c$. Note that we continue to have $\{i \in N : b\ u_i\ a\} = \{i \in N : b\ u_i^{ab}\ a\}$.

Definition 5 *An SCR, F, satisfies* ***binary consistency*** *if for each $u \in \mathcal{P}$ and $a \in A$, we have $a \in F(u)$ whenever for each $b \in A$ that is not Pareto dominated by a at u, one has $a \in F(u^{ab})$.*

Proposition 5 *An SCR, F, is γ-implementable if and only if F satisfies Maskin monotonicity and binary consistency.*

Proof. ($\Rightarrow$) Let F be an SCR implementable via a code of rights γ. It follows from Proposition 1 that F is monotonic. To see that F satisfies binary consistency, consider any profile $u \in \mathcal{P}$, and let $a \in A$ be such that for each $b \in A$ that is not Pareto dominated by a at u, we have $a \in F(u^{ab})$. We show that $a \in F(u)$. Suppose not, i.e. $a \notin F(u)$. Since $F(u) = E(\gamma, u)$, there exists an alternative $b \neq a$ and a coalition $K \in a \overset{\gamma}{\to} b \cap a \overset{u}{\to} b$. Since, $\{i \in N : b\ u_i\ a\} = \{i \in N : b\ u_i^{ab}\ a\}$, we get $K \in a \overset{u^{ab}}{\to} b$. It follows that $a \notin E(\gamma, u^{ab})$. Hence we obtain the contradiction $a \notin F(u^{ab})$.

($\Leftarrow$) Let F be an SCR that satisfies Maskin monotonicity and binary consistency. For each pair $a, b \in A$, a coalition K is *(a, b)-blocking* if for each $u \in \mathcal{P}$, we have $a \notin F(u)$ whenever for each $i \in K$, $b\ u_i\ a$ Now, design the code of rights, γ, such that for each $a, b \in A$, any coalition $K \in a \overset{\gamma}{\to} b$ if and only if K is (a, b)-blocking. Next, we show that γ implements F. Let $u \in \mathcal{P}$, we first show that if $a \in F(u)$, then $a \in E(\gamma, u)$. To see this, since $a \in F(u)$, there is no (a, b)-blocking coalition $K \in a \overset{u}{\to} b$. It follows that $a \in E(\gamma, u)$.

Next, we show that if $a \in E(\gamma, u)$, then $a \in F(u)$. Suppose not, i.e. $a \notin F(u)$. Since F satisfies binary consistency, there should exist $b \in A$, not Pareto dominated by a at u, such that $a \notin F(u^{ab})$. Let $K = \{i \in N : b\ u_i\ a\}$. Next, we argue that K is an (a,b)-blocking coalition. To see this, let u' be any preference profile such that for each $i \in K$, $b\ u'_i\ a$. We have to show that $a \notin F(u')$. Suppose not, i.e. $a \in F(u')$. Now, by the construction of u^{ab}, $K = \{i \in N : b\ u_i^{ab}\ a\}$. Moreover, for each $c \in A \setminus \{a,b\}$ and $i \in N$, we have $a\ u_i^{ab}\ c$. It follows from monotonicity that $a \in F(u^{ab})$, leading to a contradiction. Hence, we conclude that K is an (a,b)-blocking coalition. Since $K \in a \overset{u}{\to} b$, we have $a \notin E(\gamma, u)$. But this contradicts to our initial supposition. It follows that, if $a \in E(\gamma, u)$, then $a \in F(u)$. ■

In particular, it follows from the proof of Proposition 5 that a SCR is γ-implementable if and only if it is implementable via the blocking coalitions as introduced in the proof.

6 Γ-implementation with minimal state spaces

Implementation via codes of rights (γ-implementation) proposes a descriptionally simple environment for implementation. However, the rules which are γ-implementable have to satisfy a stringent monotonicity condition. On the other hand, for Γ-implementation, direct rights structures are not only sufficient for Γ-implementing any image monotonic rule but also these rights structures have a natural interpretation. One can question the simplicity of the rights structure on the basis of its number of states. For illustration, let us turn back to the example of a majority rule (Example 9). Recall that u^1 and v^1 were standing for the profiles where every agent top ranks a and every agent top ranks b respectively. Now let us remove u^1 and v^1 from the state space S and keep h and γ as they are on $S \setminus \{u^1, v^1\}$. It is easy to notice that if the true profile is say u^1 then $E(\Gamma, u^1) = \{u^2, u^3, u^4\}$, so we obtain that this new rights structure implements the simple majority rule as well.[12] However, apparently there are six states in this rights structure versus eight in the original one. This brings the question of whether one can describe a class of rights structures which

[12]In fact, this reduces to the Γ we define in Example 5.

would implement an image monotonic rule with the minimum possible number of states.

Next, we introduce the *critical rights structures* and show that if an SCR is implementable via a rights structure then the state space of this rights structure should be at least as large as that of the critical rights structure associated with the given social choice rule.

Critical rights structures

A critical rights structure $\Gamma^c = (S^c, h^c, \gamma^c)$. The state space, S^c, consists of alternative and *critical preference profile* pairs (a, u). Below, we will provide two different descriptions of critical profiles.[13]

(1) For each $a \in A$, we say a preference profile u is *critical for* a, if any agent i reverses his preference between a and b for any $b \in L(a, u_i)$, then F no longer chooses a in the new profile.

(2) For each $a \in A$, let us first define a binary relation $\succeq_a$ over the set of preference profiles. For each $u, v \in \mathcal{P}$, we say $v \succeq_a u$ if and only if v is an $a-$*monotonic transformation* of u. Namely, if for each $i \in N$, we have $\mathrm{L}(u_i, a) \subseteq \mathrm{L}(v_i, a)$. We say a preference profile u is *critical for* a if u is minimal with respect to $\succeq_a$, i.e. there is no preference profile v such that $u \succeq_a v$.

Let $\Lambda(F, a)$ stands for the family of *a-critical profiles*.

Remark 1 *Since F is image monotonic, for each $a \in I(F)$ we have $\Lambda(F, a) \neq \emptyset$. To see this, let us use the second formulation of critical profiles. Since $a \in I(F)$, there exists a preference profile u, such that $a \in F(u)$. If u' is an a-monotonic transformation of u, then it follows from image monotonicity that $a \in F(u')$ as well. Since A is a finite set there exists $v \in \mathcal{P}$ such that v is minimal with respect to $\succeq_a$ and for each u' which is an $a-$ Maskin monotonic transformation of u we have $u' \succeq_a v$.*

[13]The notion of critical profiles is used by Koray [16], Dogan and Koray [8] to classify Maskin monotonic SCRs.

Now, for a given image monotonic SCR F, we define the critical state space S^c as,

$$S^c = \{(a,u): \ u \in \Lambda(F,a)\}$$

Let h^c and γ^c be the restrictions of h and γ of the direct rights structure to S^c. Namely, h^c maps each $(a,u) \in S^c$ to alternative a and the code of rights, γ^c, entitle any coalition K to approve a change from any state (a,u) to another state (b,v) if and only if there is $i \in K$ such that $a \ u_i \ b$.

Proposition 6 *If an individual based rights structure, $\Gamma = (S,h,\gamma)$, implements F, then the number of states in S is at least equal to the number of states in S^c, i.e. $|S| \geq |S^c|$.*

Proof. Let F be a Γ-implementable social choice rule.

Step 1: We will show that F is implementable via Γ^c.

It follows from Proposition 1 that, F is implementable via the direct rights structure $\Gamma^d = (S^d, h^d, \gamma^d)$. We know that $S^c \subseteq S^d$ and h^c coincides with h^d over S^c. Moreover, γ^c coincides with γ^d, over $S^c \times S^c$. We will show that for each $u \in \mathcal{P}$, $h^d(E(\Gamma^d,u)) = h^c(E(\Gamma^c,u))$.

Let $(a,u) \in E(\Gamma^d,u)$. This means $a \in F(u)$, so there exists an $a - critical$ profile u', such that $u \succeq_a u'$. We will show that $(a,u') \in E(\Gamma^c,u)$.

Suppose not, i.e. there exists $(b,v) \in S^c$, and $i \in N$ where $\{i\} \in (a,u') \overset{\gamma}{\to} (b,v)$ and $b \ u_i \ a$. By the design of γ^c, we can have $\{i\} \in (a,u') \overset{\gamma^c}{\to} (b,v)$ only if $a \ u'_i \ b$. Since $u \succeq_a u'$, it follows $a \ u_i \ b$ as well. But this contradicts $(a,u) \in E(\Gamma^d,u)$.

For the other direction, let $(a,u'') \in E(\Gamma^c,u)$. Notice, it is not necessarily true that $u \succeq_a u''$. Next, we will show that $(a,u'') \in E(\Gamma^d,u)$.

Suppose not, i.e. there exists $(b,v) \in S^d$, and $i \in N$ such that $\{i\} \in (a,u'') \overset{\gamma^d}{\to} (b,v)$ and $b \ u_i \ a$. By the design of γ^d, we can have $\{i\} \in (a,u'') \overset{\gamma^d}{\to} (b,v)$ only if $a \ u''_i \ b$. Moreover, $(b,v) \in S^d$ implies $b \in F(v)$, so there exists a $b-$critical profile v' with $v \succeq_b v'$. Hence we have $(b.v') \in S^c$. But now, $\{i\} \in (a,u'') \overset{\gamma^c}{\to} (b,v')$ and $b \ u_i \ a$. This contradicts $(a, u'') \in E(\Gamma^c, u)$.

Step 2: Let $\Gamma = (S, h, \gamma)$ be any any rights structure which implements F. We will show that $|S| \geq |S^c|$.

It is clear that $|S| \geq |I(F)|$. Moreover, for each distinct $(a, u), (a, u') \in S^c$ there exist $s, s' \in S$ such that $s \in E(\Gamma, u)$ and $s' \in E(\Gamma, u')$, with $h(s) = h(s') = a$. Next, we will show that indeed s and s' are distinct.

Suppose not, i.e. $s = s'$. Since $u \neq u'$ there exists $i \in N$ and $b \in A \setminus \{a\}$ s.t we have w.l.o.g $a\ u_i\ b$ and $b\ u'_i\ a$. Consider the profile v which is a copy of u with the exception that we have $b\ v_i\ a$. Since u is $a - critical$, we have $a \notin F(v)$. Since $F(v) = E(\Gamma, v)$ where γ is individual based, this is possible only if we have $\{i\} \in s \xrightarrow{\gamma} t$ for some $t \in S$ with $h(t) = b$. But since we also have $h(s)\ u'_i\ h(t)$, this contradicts $s \in E(\Gamma, u')$. So far, we have shown that for each $a \in A$ and distinct $(a, u), (a, u') \in S^c$, there exist distinct $s, s' \in S$ with $h(s) = h(s') = a$, it follows that $|S| \geq |S^c|$. ∎

7 Two Stage Rights Structures with an application to Nash Bargaining

For all of the preceding analysis, we assumed that there is only one stage to obtain the equilibrium outcome of a rights structure. In this section, we formulate implementation of SCRs via two stage rights structures. Then, by using the formulation of Rubinstein, Safra, and Thomson [27] (RST from now on), we show that Nash bargaining rule is implementable via a two stage rights structure.

A two stage rights structure, Γ^2, is a quadruple $(S, h, \gamma^1, \gamma^2)$. At the first stage, the rights structure $\Gamma^1 = (S, h, \gamma^1)$ is active, and non-equilibrium states according to Γ^1 are eliminated. At the second stage, the rights structure $\Gamma^2 = (E(\Gamma^1, u), h, \gamma^2)$ is active, and $E(\Gamma^2, u)$ is obtained.

More formally, given a preference profile, u, a state s is an **equilibrium of** Γ^2 **at** u, denoted by $s \in E(\Gamma^2, u)$, if the following holds:

(1) $s \in E(S, h, \gamma^1)$, and

(2) for each $t \in E(S, h, \gamma^1)$, $(s \overset{\gamma^2}{\to} t) \cap (s \overset{u}{\to} t) = \emptyset$.

An SCR, F, is **implementable via a two stage rights structure**, if there exists Γ^2, such that for each $u \in \mathcal{P}$, $F(u) = E(\Gamma^2, u)$.

Bargaining Problem and Nash Solution:

Let us first provide the classical formulation of a bargaining problem. A bargaining problem consists of the *feasible set*, U, and a *disagreement point*, d. Each element of U is an utility pair for two agents. The utilities are *von Neumann-Morgenstern* utilities, in that they are extended to lotteries from deterministic allocations, by satisfying expected utility assumptions (*independence* and *continuity*). A bargaining solution is a function which assigns a unique pair of utility levels to each problem $< U, d >$, where U is compact, convex, and contains a point that provides a higher utility level for both agents, compared to d. RSD argues that $< U, d >$ is a condensed form of a problem $< X, D, \succeq_1, \succeq_2 >$, where X is a set of feasible alternatives, **described in physical terms**, D is the disagreement alternative, and $\succeq_1, \succeq_2$ are the preferences defined on the space of lotteries over deterministic prizes $X \cup D$. Let us assume that X is compact, convex, and contains an alternative that is preferable to D for both agents.

Next, in this alternative-preference based setting, we introduce the formulation of RSD for the Nash bargaining solution. To do so, first, let us introduce a piece of notation. For each $x \in X$ and $p \in [0, 1]$, px stands for the lottery that yields x with probability p, and yields d with probability $1 - p$.

Definition 6 *An* ***ordinal-Nash solution outcome*** *for the bargaining problem* $< X, D, \succeq_1, \succeq_2 >$ *is an alternative* y^**, such that for each* $p \in [0, 1]$ *and for each* $x \in X$

and i, if $px \succ_i y^*$, then $py^* \succeq_j x$.

RST shows that (in Proposition 1), if preferences $\succeq_1$ and $\succeq_2$ are expected utility preferences, then ordinal Nash solution is well defined and coincides with the classical (utility) Nash solution [22].

For a given pair (X, D), let $\succeq_1, \succeq_2$ be a pair of expected utility preferences. Now, by using the above formulation of RSD, we can define the **Nash bargaining rule** as an SCR, i.e the Nash bargaining rule, F, maps each pair $\succeq_1, \succeq_2$ to the ordinal-Nash solution outcome for the problem $< X, D, \succeq_1, \succeq_2 >$.

Proposition 7 *The Nash bargaining rule, F, is implementable via a two stage rights structure, $\Gamma^2 = (S, h, \gamma^1, \gamma^2)$.*

Proof. For each $p \in (0, 1)$ and $x, y \in X$, let $x, (px, y), (p^2x, y) \in S$, where $h(x) = x$, $h(px, y) = px$, and $h(p^2x, y) = p^2x$. Next, define γ^1 such that for each $p \in (0, 1)$ and $x, y \in X$,

$$(px, y) \overset{\gamma^1}{\to} (p^2y, x) = \{\{1\}, \{2\}\} \text{ and } (p^2y, x) \overset{\gamma^1}{\to} y = \{\{1\}, \{2\}\}$$

No other movement is allowed by γ^1. Finally, define γ^2 such that for each $p \in (0, 1)$ and $x, y \in X$;

$$y \overset{\gamma^2}{\to} (px, y) = \{\{1\}, \{2\}\} \text{ and } (px, y) \overset{\gamma^2}{\to} x = \{\{1\}, \{2\}\}$$

No other movement is allowed by γ^2. For any preference profile $\succeq = (\succeq_1, \succeq_2)$, let $F(\succeq_1, \succeq_2) = y^*$. First, we show that $y^* \in E(\Gamma^2, \succeq)$. To see this, note that in the first stage, for each $x \in X$, no one has the right to move from x. Hence, y^* is not eliminated in the first stage. For the second stage, suppose there exists $p \in (0, 1)$, $x \in X$, and $i \in \{1, 2\}$, such that $i \in y \overset{\gamma^2}{\to} (px, y^*)$ and $px \succeq_i y^*$. Assume w.l.o.g. that $i = 1$. Now, by the definition ordinal Nash solution, $y^* \succeq_2 px$. Since $\succeq_2$ is an expected utility preference, it follows that $py^* \succeq_2 p^2x$. Since $\{2\} \in (px, y^*) \overset{\gamma^1}{\to} (p^2y^*, x)$, at first stage (px, y^*) is eliminated. Hence, $y^* \in E(\Gamma^2, \succeq)$. Next, we show that for each $s \in S$, if $s \neq y^*$, then $s \notin E(\Gamma^2, \succeq)$.

Case 1: Suppose $s = (p^2x, y)$ for some $p \in (0,1)$ and $x, y \in X$. Since for each $i \in \{1,2\}$, $\{i\} \in (p^2x, y) \overset{\gamma^1}{\to} x$ and $x \succ_i p^2x$, at the first stage, (p^2x, y) is eliminated in favor of x.

Case 2: Suppose $s = (px, y)$ for some $p \in (0,1)$ and $x, y \in X$. For each $i \in \{1,2\}$, $\{i\} \in (px, y) \overset{\gamma^2}{\to} x$ and $x \succ_i px$. Since x is not eliminated at the first stage, (px, y) is eliminated in favor of x at the second stage.

Case 3: Suppose $s = y$ for some $y \in X \setminus \{y^*\}$. Since $y \neq F(\succeq)$, there exists $x \in X$ and $p \in (0,1)$, such that w.lo.g. $px \succ_1 y$ and $x \succ_2 py$. It follows that both $px \succ_1 p^2y$ and $px \succ_2 p^2y$. Hence, (px, y) is not eliminated at the first stage. Since $\{1\} \in y \overset{\gamma^2}{\to} (px, y)$ and $px \succ_1 y$, at the second stage, y is eliminated. ■

8 Related Literature

The idea that "rights" should be incorporated into social choice theory originates from the seminal work of Sen [28] on the impossibility of a Paretian liberal. The notion of a "rights structures", we use as a design object in this study, is a slightly simplified version of a "Rechtsstaat" formulated by Sertel [29]. A Rechtsstaat is defined as a triplet of maps representing *ability*, *benefit* and *code of rights* for a given state space. An ability map associates with each distinct state pair (s, t) a collection of coalitions, each of which is meant to be (cognitively or technologically) able to transform state s into state t. The benefit map and code of rights are defined similarly. In this setting, the transformation of state s into state t requires the existence of two coalitions, one which is able to achieve this change and one which approves it. Sertel[29] analyzes the efficiency properties of the equilibria defined in this setting. More recently, Ben and Sugden [19] proposes the notion of a *game in transition function form* as a generalization of the ”effectivity functions” due to Moulin and Peleg [21]. The concept and the analysis is similar to that of Sertel [29].

Following the work of Sen [28], as another approach, Moulin and Peleg [21] proposed the notion of an ”effectivity function” (see also Rosenthal [25]) as a representation of the power distribution induced from a game form. Later on, Peleg and Winter [24] carried the notion of an effectivity function to implementation framework. They argue that a certain game form implementing an SCR under Nash equilibrium does not mean that this game form is a natural mechanism for implementing that rule. With this motivation, they formulate the so-called ”constitutional implementation”, which requires the Nash implementation of a given SCR via a game form which induces the same effectivity function. Here, we adopt a different approach in order to implement an SCR, we propose explicitly designing the rights structure which represents the power distribution in the society.

The notion of *social equilibria* introduced by Debreu [7] is closest to our definition of *deviation constraint mechanisms*. Debreu [7] analyzes a strategic environment, where the strategy choice of an agent is not entirely free but, the strategies of all the other agents determine the subset to which his strategy selection is restricted. Put differently, given a joint strategy, deviation strategies of an agent is restricted depending on the strategies of the others. Our formulation of dc-mechanism differs from this formulation, since given a joint strategy, deviation strategies of an agent is restricted depending not only on the strategies of the others, but also on that of himself.

More recently, Glazer and Rubinstein [11] analyzes a model of mechanism design with boundedly rational agents. They present a persuasion situation as a leader-follower relation. In this model, the persuasion rule and its frame is determined by the leader. As it is the case in a dc-mechanism, the strategies that the follower can choose are restricted depending on the way in which the persuasion rule is framed by the leader.

9 Conclusion

We have taken a persistent criticism of implementation theory, bringing out that the game forms used in the general constructions contain unnatural features as an appended ”integer game ”. These features takes away from the relevance of the theory to the real life mechanisms. Implementation via rights structures offers a framework for implementation, formulated in a language closer to institutional mechanisms. We have shown that, in this framework we can implement SCRs by explicitly designing simple rights structures. Furthermore, our results relating Γ-implementation to Nash implementation contribute to have a better understanding of several complications related to classical implementation.

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