Free to Cruise: Designing a Market for Tradable Taxicab Cruising Rights*

David Seymour†

November 24, 2014

Abstract

In metropolitan areas, taxicab regulation often has local exclusivity, regulations that prevent taxis that are affiliated with one city from picking up passengers in another city. When there are multiple cities in close proximity, the empty return trips that occur after taxis drive passengers from one city to another results in inefficiency. This inefficiency can be eliminated by a single metropolitan-level affiliation and letting all affiliated taxis pick up passengers in any city; however, under this regulation each incentives cause an allocation of taxis across locations that is not socially optimal. We develop a regulatory exchange market that maintains the separate affiliations, but allows taxi drivers the ability to exchange the right to pick up passengers in each others' affiliated location. The regulatory exchange market will be preferred to both local exclusive cruising regulation and metropolitan-level regulation. Further, the regulatory exchange market achieves the first best in a large class of situations.

*I am grateful to my advisors, Dilip Mookherjee, Juan Ortner, and Michael Manove, for their advice, encouragement, and support. I would also like to thank Sambuddha Ghosh, Gautam Bose, and Albert Ma for their helpful comments.

†Correspondence: Department of Economics, Boston University. email: dtseymou@bu.edu

1 Introduction

The taxi industry is characterized by entry regulations, price restrictions, and, when there are multiple regulatory locations in close proximity, local exclusivity. Local exclusivity occurs in areas with multiple regulatory locations that each have regulations requiring taxicabs be affiliated with the regulatory location in order to provide service. These regulations prevent taxis from simultaneously being affiliated with multiple regulatory locations. Taxis that drive passengers from their own affiliated location to a location with a different affiliation have to return to the location they are affiliated with in order to be able to pick up passengers. When empty return trips from different regulatory locations overlap there are costs of duplication, resulting in inefficiency.

Local exclusive cruising often occurs in metropolitan areas where there are multiple regulators in close proximity, such as the Boston metropolitan area. The Boston metropolitan area has a number of cities that border each other. Local regulations in these cities prevent taxis that are affiliated with other locations from picking up passengers in the city. Despite this, taxi drivers frequently drive passengers from one city to another. After dropping off the passenger, these taxis are required to return empty to their affiliated city in order to be able to pick up passengers.

Local exclusive cruising regulation also occurs when a single regulator divides a city up into multiple affiliations. This is done in Los Angeles and New York City. In Los Angeles, the city is subdivided into five zones. Taxi companies are licensed to provide service in specific zones, with some companies being able to pick up passengers across multiple zones. New York City recently introduced boro taxis, taxis that are only able to operate in the lower demand areas of the city. Yellow taxis, are are able to operate in the entire city. The boro taxis were introduced because of most of the taxi service was provided in Manhattan, leaving too little vacancy in the other boroughs.

In other cities, such as Chicago and Houston, registered taxis are able to pick up passengers throughout the city. These metropolitan-level regulations also have sources of inefficiency. Since affiliated taxis can move freely between locations, taxi drivers will direct their search in a manner which grants them the highest expected revenue. The location that has the highest expected revenue may not be the most socially desirable location for taxis to be searching for passengers, creating the potential for too many taxis to be in one location and two few taxis to be in another. The flow of taxis to the high-value location creates inefficiency by limiting the ability of the regulator to choose price and vacancy levels that are ideal for all of the locations.

These differing sources of inefficiency may cause either type of regulation to be preferred. In certain instances, local exclusive cruising regulation will be the preferred regulation and in other instances metropolitan-level regulation will be the preferred regulation. As established in my prior workSeymour (2014), there is a trade-off between inefficiency caused by duplicate return trips under local exclusive cruising regulation, and the inefficiency caused by loosing the flexibility to choose price and vacancy levels that meet the needs of the localities under metropolitan-level regulation. Local exclusive cruising regulation will tend to be preferred when there are few trips between locations and the localities are different, while metropolitan-level regulation will tend to be preferred when there are a large number of trips between locations and the locations are similar.

Looking at the source of inefficiency under local exclusive cruising regulation suggests a way to deal with the problem. Under local exclusive cruising regulation, returning taxis from different regulatory affiliations drive by passengers on their return trip and are unable to pick these passengers up. One may envision a regulatory structure that allows these drivers to temporarily exchange the right to pick up passengers in each other's regulatory affiliation. If two taxicabs, each in the others affiliated location, are about to make a return trip, then the taxicabs can temporarily exchange for the right to pick up passengers affiliated with the other location. By allowing taxis to exchange the right to pick up passengers, there is the potential to eliminate some of the costs of duplicate trips while still giving regulators flexibility to choose policies that meet the particularities of their localities.

In this paper we study the consequence of a proposed exchange market that allows the regulator to choose prices and the total number of taxis affiliated with each location. Taxis would be able to pick up passengers in their affiliated location but have to exchange for the right to pick up passengers in locations that they are not affiliated with. When there are two taxis that are affiliated with different locations and are in each other's affiliated location, the taxis would be able to trade for the right to be able to temporarily pick up passengers in each other's affiliated location. This exchange gives taxi drivers the opportunity to search for passengers in the other location instead of being required to return to their original location.

This setup gives the regulator the ability to choose service levels for the individual location but eliminates the duplicate return trips that are prevalent under local exclusive cruising regulation. The regulatory exchange market can achieve the first best, provided that at the first best the "no-subsidy condition" is satisfied. The no-subsidy condition is satisfied if the expected revenue at each location exceeds the cost of operating a taxi. When this condition is satisfied, profit from one of the locations is not necessary to subsidize losses at the other location and the first best can be achieved using a regulatory exchange market. The first best can be achieved by using prices such that, at the first best outcome, taxi drivers are indifferent between searching for passengers in each of the locations.

When the no-subsidy condition is not satisfied the regulatory exchange market may not be able to achieve the first best. Even when it does not achieve the first best, it is still worth comparing to the two most common types of regulation; local exclusive cruising regulations and metropolitan-level regulations. We find that, even when the first does not satisfy the no-subsidy condition, the regulatory exchange market is preferred to both local exclusive cruising regulation and metropolitan-level regulation. The regulatory exchange market can be used to eliminate the duplicate costs under local exclusive cruising regulation, and can be used to give the regulator more flexibility in choosing price and vacancy levels than under metropolitan-level regulation.

We model the regulatory exchange market by assuming that there is a market maker that chooses a regulatory exchange price. By participating in the exchange, drivers temporarily gain the right to pick up passengers in a location that they are not affiliated with. Depending on whether the price is positive or negative, they either make a payment or receive a payment. By participating in the regulatory exchange, taxis also temporarily forgo the opportunity to pick up passengers in their own regulatory affiliation. Under the regulatory exchange market, the market maker chooses regulatory exchange prices such that the regulatory exchange prices represent a payment from drivers affiliated with one location to drivers affiliated with another for the right to pick up passenger in each others' affiliated locations.

We choose to focus on a market because the regulatory exchange market would likely preform well under conditions that are more general than the model that we study, for instance when demand varies over time, using a market to exchange regulatory locations ensures that minimum service levels are met. Under a regulatory exchange market, regulators will have less ability to manipulate the regulatory exchange price than they would if the exchange prices were independently set by the regulator. Under the regulatory exchange market, the drivers' net revenue will typically depend on the price and vacancy rates in the location in question, limiting the ability for the regulators to manipulate the exchange prices.

The paper proceeds as follows. In Section 2, we look at the taxi industry and the relevant literature. In Section 3, we develop the model that allows us to look at local exclusive cruising regulation, metropolitan-level regulation, and the regulatory exchange market. In Section 4, we define a Local Exclusive Cruising Equilibrium, a Metropolitan-Level Equilibrium, and the Regulatory Exchange Market Equilibrium. In Section 5, we show the results; we characterize the first best from a joint regulator's perspective and look at the efficiency properties of the three types of equilibria, focusing our attention on the Regulatory Exchange Market Equilibrium. Section 6 concludes.

2 Background / Literature Review

Despite the lack of natural barriers to entry, the taxicab industry is highly regulated. In most major cities in North America and Europe, it is subject to price, quantity, and service quality regulation. The regulator chooses a regulated fare schedule that taxi drivers are required to charge. The fare schedule typically includes a fixed fee for entering the taxi, a fee per distance traveled, and a fee based on the amount of time spent idle. In order to limit the quantity, the regulator requires that taxicabs have medallions in order to be able to pick up passengers. By limiting the supply of medallions, the regulator limits the total number of taxicabs.

Taxicab regulation is often justified because there are issues with bargaining. The bargaining process will not necessarily lead to the efficient vacancy rate. When passengers and taxis bargain, they do not contract over the vacancy rate; therefore, the price that they bargain to, and the corresponding vacancy rates that occur with entry may not be efficient. In addition, price communication is costly, particularly in areas where taxis are contracted by signaling vacant taxis on the street. Taxis will often obstruct traffic while bargaining with potential passengers, creating a negative externality for other drivers. By choosing a regulated price, the regulator eliminates bargaining issues.

Bargaining issues alone, however, are insufficient to justify the quantity restrictions. If bargaining issues were the only reason for the regulation, taxi drivers would make zero profit under the surplus maximizing price and quantity (Arnott, 1996). The regulator could implement the efficient outcome by choosing the efficient price and allowing entry into the taxi market. In practice, medallion prices are typically positive; therefore, the medallions restrict entry. This suggests there are other factors beyond bargaining that are also motivating the choice of the regulatory structure.

The two most plausible explanations for choosing regulations that give taxi drivers positive profits are that there is an externality caused by taxi travel and that the regulator has a preference towards driver profit. The externality from taxi travel is due to the general externalities caused by driving, and any additional congestion caused by searching for, picking up, and dropping off passengers. If the externality is sufficiently high, the regulator will prefer a higher price. This will cause drivers' profit and the medallion price to be positive.

The regulator may place a higher weight on the driver component of surplus because the sale of medallions acts as a source of revenue and because of regulatory capture by medallion holders. New medallions are often introduced through an auction mechanism, providing a source of revenue for the local regulators. Regulators that are concerned about generating income may give preference to driver profit because the regulator is able to capture some of the profit as revenue through the auctioning of medallions.[1] Regulatory capture by medallion owners could also be a source of a regulatory preference towards driver revenue. Medallion owners typically lobby local governments against increases in the number of medallions. When the regulator places a sufficiently high weighting on driver profit, the regulator may prefer a price that generates positive profits for taxi drivers and a positive medallion price.

We explicitly take into account these justifications for regulation in our model.

[1]In Boston, between 1991 and 2001 medallions auctioned for an average price of $180,000 each. Over that time period the city was able to auction 225 medallions.

We assume that the regulator choose a price for each location, so bargaining issues between the driver and passenger are eliminated. We model the externality and regulatory preference towards driver profit, allowing the externality to differ between locations. Both of these factors have been discussed in the literature. The inclusion of a traffic externality has been formally modeled (Yang et al., 2005), suggesting a justification for restricting the quantity of taxis. A regulatory preference towards profit has been discussed heuristically as a justification for restricting entry in the taxi industry (Flores-Guri, 2005). We believe that these elements are vital to include in the model, because they lead to positive medallion prices and a restrictions on entry.

Models of the taxi market differ from standard economic models because the demand for taxi travel depends on the vacancy rate; as the vacancy rate increases, the expected time it takes to find a taxi decreases and the service becomes more appealing. Since demand depends on vacancy rate, the total quantity feeds back into the demand equation through the vacancy rate. The classic models of the taxi industry were modeled with this is mind (Douglas, 1972; Orr, 1969; De Vany, 1975). Like most models in literature, we use an extension of the Douglas cruising model (Douglas, 1972). The Douglas cruising model looks at taxi demand as a function of the price and vacancy rate. Instead of solving for the optimal price and number of medallions, the model looks at the optimal price and vacancy levels and determines the medallions from the prices, vacancy rate, and quantity demanded at the price and vacancy rates. We extend Douglas's framework to incorporate multiple locations and the strategic choice of search behavior by taxi drivers.

We use a network model of the taxi industry similar to Yang et al. (2002). Each location has demand for travel to every other location. The demand for trips to other locations depends on the price and location specific vacancy levels. They assume the price is the same in all locations and that taxi drivers search for passengers in a manner that minimizes the time it takes for the driver to find a passenger. In contrast, we allow the price to differ between locations and assume that taxi drivers search for passengers in a strategic fashion, choosing the search behavior in a manner that maximizes their expected discounted profit.

Flores-Guri (2005) examines the efficiency of local exclusive cruising regulations in a partial equilibrium framework. He considers a single location where, for any price and vacancy level, a fixed proportion of passengers demand trips to the other location. Under local exclusive cruising, taxis that drive to the other location are required to return to their affiliated location before being able to pick up passengers. Under metropolitan-level regulation, the return trips are eliminated. When looking at metropolitan-level regulation the model does not take into account the differences between the locations. The optimal prices and vacancy rates under metropolitan-level regulation may not be sustainable because taxis have an incentive to locate to the higher-value location.

In prior work (Seymour, 2014), I have looked at a dynamic network model of the taxi industry that makes it possible to compare local exclusive cruising regulation to metropolitan-level regulation. He considers an environment where locations can differ based on demand, preference towards driver profit, and the external cost of taxi traffic. Under metropolitan-level regulation, drivers search for passengers, taking into account the expected profit that they receive when they operate in each location. He shows that when locations have different characteristics, local exclusive cruising regulation will be preferred to metropolitan-level regulation despite the duplicate costs that occur under local exclusive cruising regulation. This paper extends the results in Seymour (2014), by creating a market-based mechanism that allows taxi drivers the opportunity to temporarily exchange regulatory locations. In the process, we develop a general framework that allows us to compare the regulatory exchange market, local exclusive cruising regulation, and metropolitan-level regulation.

3 Model

We assume that there are two locations, $\mathcal{L} = \{1, 2\}$ with index i. There are a continuum of taxis. Each taxi is affiliated with one of the two locations, denoted by a. Over time, taxis move between locations. A taxi's movement will be influenced by the search behavior of the taxi's driver and the location preferences of the passenger. These in turn will depend on the prices, number of taxis, and regulatory environment chosen by the regulator.

Taxis are able to pick up passengers in their own regulatory location; however, to be able to pick up passengers in the other location, taxis must acquire the right to pick up passengers by participating in the regulatory exchange. When participating in the regulatory exchange, drivers either incur a cost or receive a payment and are temporarily able to pick up passengers in the location that they are not affiliated with. By choosing the appropriate prices, we can use the regulatory exchange to model local exclusive cruising, metropolitan-level regulation, and the regulatory exchange market.

3.1 Taxi Flow

We assume that time is broken up into discrete periods. At the start of each period, location i has n_i taxicabs, with $n_{i,a}$ taxicabs affiliated with location a. When taxis are not in their affiliated location, they make a decision of whether to participate in the regulatory exchange. For taxicabs to be able to search for passengers at location i, the taxi must be affiliated with the location or they must participate in the regulatory exchange. Taxis that search for passengers either become occupied or remain vacant. Taxis at location i can also choose to transition to the other location without searching for passengers.

The number of occupied taxicabs that pick up passengers in location i is given by Q_i. Based on the preferences of the passenger, an occupied taxi either drives the passenger to the same location or to the other location. The number of taxicabs that drive passengers to the same location is denoted by q_i, and the number of taxicabs that drive passengers to the other location is denoted by $\tilde{q}_i$; therefore, the total number of occupied taxicabs originating from location i is

$$Q_i = q_i + \tilde{q}_i \tag{1}$$

Occupied taxis take until the end of the period to drive passengers to their destination, regardless of whether they drive the passengers to the same location or to a different one. Taxis that drive passengers to the same location start the next period in the same location, and taxis that drive passengers to the other location start the next period in the other location.

The number of taxis that remain vacant after searching for passengers is given by V_i. Vacant taxis either remain in the same location or relocate to the other location. The number of vacant taxicabs that remain in the same location is v_i and the number of vacant taxicabs that relocate to the other location is $\tilde{v}_i$; therefore, the total number of taxis from in location i that remain vacant is

$$V_i = v_i + \tilde{v}_i \tag{2}$$

The search direction determines the location where the taxis will be located in the next period if they do not find passengers. Taxis that chose to search for passengers while remaining in the same location and remain vacant will remain in the same location in the next period, while taxis that chose to search for passengers while relocating to the other location and remain vacant will end up in the other location in the next period.

Taxis can also transition to the other location without searching for customers. The number of transitioning taxicabs is given by T_i. We assume that it takes until the start of the next period to arrive at the other location when transitioning. In a given period, taxis must either become occupied, remain vacant, or transition; therefore, the total number of taxis satisfies

$$n_i = Q_i + V_i + T_i \tag{3}$$

Of these taxis,

$$\tilde{q}_i + \tilde{v}_i + T_i$$

will transition from location i to location j. The number of taxis at location i at the start of the next period is

$$n_i' = v_i + q_i + \tilde{v}_j + \tilde{q}_j + T_j \tag{4}$$

We will focus our attention to stationary Markov equilibria; therefore, in equilibrium we want to look at the situation where the number of taxis remains constant over time. For the number of taxis to remain constant over time, we require that the number of taxis that travel from location i to location j is equal to the number of taxis that travel from location j to location i. The flow of taxis is equal when

$$T_1 + \tilde{q}_1 + \tilde{v}_1 = T_2 + \tilde{q}_2 + \tilde{v}_2 \tag{5}$$

In a similar fashion, we can look at the number of taxis that are affiliated with each location. We assume that there are $Q_{i,a}$ occupied taxis at location i that are affiliated with location a, of these $q_{i,a}$ are driving passengers to the same location and $\tilde{q}_{i,a}$ are driving passengers to the other location. Likewise, there are $V_{i,a}$ searching taxis that are affiliated with location a that do not find a passenger in location i. Of these taxis, $v_{i,a}$ remain in the same location and $\tilde{v}_i$ relocate. Finally, $T_{i,a}$ taxis in location i that are affiliated with location a choose to transition to the other location without searching for passengers. Equation (1) through (5) will have analog statements where we look at the number of taxis associated with each regulatory affiliation.

In particular, the number of taxis of each regulator remains constant when, for $r \in \{1, 2\}$

$$T_{1,r} + \tilde{q}_{1,r} + \tilde{v}_{1,r} = T_{1,r} + \tilde{q}_{2,r} + \tilde{v}_{2,r} \tag{6}$$

This allows us to define a steady-state distribution of taxis.

A steady-state distribution of taxis is a distribution of taxis such that the total number of taxis involved in each task remains constant over time. Formally, an outcome is a steady-state distribution of taxis if the following variables remain constant over time.

$$(v_{i,a}, \tilde{v}_{i,a}, q_{i,a}, \tilde{q}_{i,a}, T_{i,a})_{i\in\{1,2\}, r\in\{1,2\}}$$

This mean that the location specific aggregate quantity and vacancy rates will also remain constant. For these values to remain constant, equation (6) will necessarily be satisfied. Alternatively when equation (6), is satisfied and the taxis affiliated with each location are playing the same mixed strategies, the outcome will have a steady-state distribution.

3.2 Demand

Demand for taxi travel in location i depends on the location specific price, p_i, and the location specific vacancy rate, V_i.[2] We extend the Douglas cruising model to account

[2]Consumers care about the vacancy rate rather than the number of searching taxicabs. Since the discrete model is a continuous time approximation, Q_i taxis will be occupied at any given time; therefore, in the analogous continuous time model, the vacancy rate is given by the number of

for two locations and the possibility of trips between locations. We represent demand for trips from location i as

$$Q_i\left(p_i, V_i\right)$$

Demand can be decomposed into demand for trips to the same location, $q_i\left(p_i, V_i\right)$, and demand for trips to the other location $\tilde{q}_i\left(p_i, V_i\right)$. We assume that taxis cannot refuse passengers; therefore, the total demand for trips originating from location i satisfies

$$Q_i\left(p_i, V_i\right) = q_i\left(p_i, V_i\right) + \tilde{q}_i\left(p_i, V_i\right)$$

We assume that q_i and $\tilde{q}_i$ are continuous in their arguments. Further, we assume $Q_i(p_i, 0) = 0$. We assume an increase in price and a decrease in vacancy lowers the quantity of trips demanded to both the same location and to the other location

$$\frac{\partial q_i}{\partial p_i} < 0, \qquad \frac{\partial \tilde{q}_i}{\partial p_i} < 0, \qquad \frac{\partial q_i}{\partial v_i} > 0, \qquad \frac{\partial \tilde{q}_i}{\partial v_i} > 0$$

We also assume that for any (p_i, V_i) and (p_i', V_i') such that

$$Q(p_i, V_i) = Q(p_i', V_i')$$

we have

$$q(p_i, V_i) = q(p_i', V_i') \qquad q(p_i, V_i) = q(p_i', V_i')$$

This assumption simplifies the proof of many of the results. In particular, it leads to a simpler representation of the first best, thereby allowing us to state a simpler

searching taxicabs that do not have a passenger.

definition of the no-subsidy condition. In the appendix, we state the more complicated version of the results that do not rely on this assumption.[3]

3.3 Driver Strategies

Taxi drivers seek to maximize their expected discounted profit. Taxis are able to freely pick up passengers in their affiliated location; however, to be able to pick up passengers in the other location, taxis must exchange for the right to pick up passengers. Upon participating in the regulatory exchange, the taxi driver may either incur a cost or receive a payment. Over time taxis flow according to their search behavior and whether they acquire passengers.

At the beginning of each period, taxis in their own regulatory jurisdiction choose between one of three actions; searching for a passenger and remaining in the same location if unoccupied, ν, searching for a passenger and moving to the other location if unoccupied, $\tilde{\nu}$, and transitioning to the other location without searching for a passenger, T. Taxis that are not in their affiliated location have to choose whether to exchange regulatory affiliation. If they change regulatory affiliation, they have a choice between, remaining in the same location if unoccupied, ν_m, searching for a passenger and moving to the other location if unoccupied, $\tilde{\nu}_m$, and transitioning to the other location without searching for a passenger, T_m. Otherwise, if they choose not to participate in the regulatory exchange, they have to return to their affiliated location without searching for passengers, T.

[3]This simplifying assumption is weaker than the assumption that the demand for trips is proportional to the aggregate quantity demanded. For instance, for low quantities, the proportion of trips between locations could be low and could increase as the total number of trips increases. The assumption says that the proportion of trips between locations must be the same only when the number of trips between locations is the same, but that proportion can differ as the total number of trips differs.

Drivers at location i that are affiliated with regulator i have a choice set

$$S_{i,i} = \{\nu, \tilde{\nu}, T\}$$

and drivers at location i affiliated with location j have a choice set

$$S_{i,j} = \{T, \nu_m, \tilde{\nu}_m, T_m\}$$

We assume that drivers can choose strategies based on their location, and the number of taxis at each location that are affiliated with each regulator; therefore, the state space is

$$(\boldsymbol{n_1}, \boldsymbol{n_2}, i)$$

where $\boldsymbol{n_k} = (n_{k,1}, n_{k,2})$ is a vector of the number of taxis in location k of each regulatory affiliation, and i is the location of the taxi.

Given the state, drivers affiliated with location a choose a mixed strategy σ^r. Given σ^r the probability that a taxi in location i and affiliated with location a choose an action s is $\sigma^r(s; \boldsymbol{n_1}, \boldsymbol{n_2}, i)$. We assume that all taxis with the same affiliation choose the same mixed strategy. Using these strategies, we can determine the number of taxis choosing a particular strategy and the number of taxis with each affiliation choosing a particular strategy. Define n_i^s as the total number of drivers in location i choosing strategy s or s_m, . Then

$$n_{i,a}^s = \sum_{s \in \{S_{i,a} \cap \{s, s_m\}\}} n_{i,a}\, \sigma^r(s; \boldsymbol{n_1}, \boldsymbol{n_2}, i)$$

with

$$n_i^s = \sum_{a \in \{1,2\}} n_{i,a}^s$$

In a given period, the total number of taxicabs that are searching for passengers in location i is:

$$n_i^{\nu_{tot}} = n_i^{\nu} + n_i^{\tilde{\nu}}$$

Since all searching taxis in location i must either find a passenger or remain vacant, the total number of searching taxicabs must satisfy

$$n_i^{\nu_{tot}} = Q_i + V_i \tag{7}$$

The previous equation, combined with the demand equation,

$$Q_i = Q_i(p_i, V_i) \tag{8}$$

determine the aggregate quantity and vacancy rates. Since Q_i is continuous and increasing in V_i, for a given price p_i, the total number of taxicabs that remain vacant and become occupied can be found by solving the system of equations implied by equations (7) and (8). The quantity and vacancy rates in location i are uniquely determined by the number of searching taxis and regulated price in location i.

We assume that all taxis in location i that are searching for passengers have the same probability of finding a passenger, regardless of their regulatory affiliation and the direction of their search. Given this, we can determine the number of taxis that remain vacant at the end of the period that remain in the same location and that relocate to the other location:

$$v_i = \frac{n_i^{\nu}}{n_i^{\nu_{tot}}} V_i \qquad \tilde{v}_i = \frac{n_i^{\tilde{\nu}}}{n_i^{\nu_{tot}}} V_i$$

Of the vacant taxis, the number affiliated with each regulator that remain in the same location and relocate to the other location are given by:

$$v_{i,a} = \frac{n_{i,a}^{\nu}}{n_i^{\nu_{tot}}} V_i \qquad \tilde{v}_{i,a} = \frac{n_{i,a}^{\tilde{\nu}}}{n_i^{\nu_{tot}}} V_i$$

The number of taxis that drive a passenger from location i to the same location and the number that drive a passenger to the other location are given by

$$q_i = q_i(p_i, V_i) \qquad \tilde{q}_i = \tilde{q}_i(p_i, V_i)$$

Of these taxis, the number affiliated with each regulator that drive a passenger to the same location and to the other location are given by

$$q_{i,a} = \frac{n_{i,a}^{\nu_{tot}}}{n_i^{\nu_{tot}}} q_i \qquad \tilde{q}_{i,a} = \frac{n_{i,a}^{\nu_{tot}}}{n_i^{\nu_{tot}}} \tilde{q}_i$$

For a given state and a given set of strategies, we can determine the vacancy rates, the quantities, and the transitioning behavior of the taxis at a given point in time

$$(v_{i,a}, \tilde{v}_{i,a}, q_i^r, \tilde{q}_{i,a}, T_{i,a})_{i \in \{1,2\}, r \in \{1,2\}} \tag{9}$$

For a given initial $\boldsymbol{n_1}, \boldsymbol{n_2}$, a steady-state distribution is induced by the strategies $(\sigma^r)_{r \in \{1,2\}}$ if the steady-state flow equation

$$T_{1,r} + \tilde{q}_{1,r} + \tilde{v}_{1,r} = T_{1,r} + \tilde{q}_{2,r} + \tilde{v}_{2,r}$$

is satisfied for $r \in \{1, 2\}$

3.4 Driver Payoff

Taxi drivers' payoff depend on the strategies that they choose. At the start of each period, taxis in location i affiliated with regulator j pay a cost τ_i in order to participate in the regulatory exchange. By participating in the regulatory exchange, drivers are able to search for passengers in their unaffiliated location and forgo the opportunity to pick up passengers in their affiliated location. τ_i is the cost of acquiring the right to be able to search for passengers in location i.

The value of τ_i can either be positive or negative. When the value of τ_i is negative, drivers in in location i receive a payment for temporarily switching which location they can pick up passengers in. Taxis that search for passengers and become occupied receive the location specific price, p_i. In each period, regardless of the strategy choice, drivers incur a cost, c, of operating the taxi. Drivers discount the future at a rate δ. To be willing to provide service taxis need a non-negative expected discounted profit.

We look to determine the expected discounted profit that drivers make given their strategy choice. We start by determining the expected revenue that a driver receives when choosing an action s in a given state. If a searching driver finds a passenger, he receives the location specific price p_i. If the searching driver does not pick up a passenger, then he receives zero revenue. Alternatively, if the driver transitions to the other location, he does not pick up a passenger and, as a result, does not receive any revenue. Given this, the expected revenue of a taxicab choosing a strategy s at location i is

$$R_i^s = \begin{cases} \frac{Q_i}{Q_i+V_i} p_i & \text{if } s \in \{\nu, \tilde{\nu}\} \text{ or } s \in \{\nu_m, \tilde{\nu}_m\} \\ 0 & \text{if } s = T \text{ or } s \in \{T, \tilde{T}_m\} \end{cases}$$

When vacant, the single period revenue is the same whether the driver chooses to stay in the same location or relocate to the other; therefore, we define

$$R_i^{\nu} = \frac{Q_i}{Q_i + V_i} p_i$$

as the expected revenue that drivers at location i receive when searching for passengers.

Under the regulatory exchange, taxis in their own affiliated location neither incur a cost nor receive a benefit. Taxis in location i affiliated with regulator j must pay a cost τ_i in order to be able to search for a passenger for a single period. Taxis are able to choose whether they participate in the regulatory exchange. Taxis that do not participate in the regulatory exchange, have to transition back to the original location. Taxis that participate in the regulatory exchange can search for passengers while remaining in the same location, search for passengers while moving back to their affiliated location, or transition back without searching for passengers. The driver's single period expected profit is given by

$$\pi_{i,i}^{s_i} = \begin{cases} R_i^{s_i} - c & \text{if } s_i \in \{\nu, \tilde{\nu}\} \\ -c & \text{if } s_i = T \end{cases}, \qquad \pi_{i,j}^{s_i} = \begin{cases} R_i^{s_i} - c - \tau_i & \text{if } s_i \in \{\nu_m, \tilde{\nu}_m, T_m\} \\ -c & \text{if } s_i = T \end{cases}$$

In addition to affecting the expected revenue, the choice of strategies also affects the transition probabilities. When searching for passengers, drivers pick up a passenger that is going to the same location with probability

$$\frac{q_i}{Q_i + V_i}$$

They pick up a passenger going to the other location with probability

$$\frac{\tilde{q}_i}{Q_i + V_i}$$

Searching taxis remain vacant and locate in the direction they are searching with the remaining probability:

$$\frac{V}{Q_i + V_i}$$

Taxis that choose to transition end up in the other location with certainty.

Defining $\rho_{ik}^{s_i}$ as the probability of transitioning from location i to location k when choosing the strategy s_i, the above statements imply that the transition probabilities are:

$$\rho_{ii}^{\nu} = \frac{V_i + q_i}{V_i + Q_i} \qquad \rho_{ij}^{\nu} = \frac{\tilde{q}_i}{V_i + Q_i}$$

$$\rho_{ii}^{\tilde{\nu}} = \frac{q_i}{V_i + Q_i} \qquad \rho_{ij}^{\tilde{\nu}} = \frac{V_i + \tilde{q}_i}{V_i + Q_i}$$

$$\rho_{ii}^{T} = 0 \qquad \rho_{ij}^{T} = 1$$

where $i, \in \{1, 2\}$ and $j \neq i$. The transition probabilities will be the same regardless of whether or not drivers participate in the regulatory exchange.

Given a particular set of strategies chosen by the other drivers σ' and a current state $(\boldsymbol{n_1}, \boldsymbol{n_2}, i)$, we can determine the Bellman equation for a taxi at location i facing prices p_i and τ_i:

$$U_{i,a}(\sigma'; \boldsymbol{n_1}, \boldsymbol{n_2}) = \max_{s_i \in S_{i,a}} \left\{ \pi_{i,a}^{s_i} + \delta \sum_{k \in \{1,2\}} \rho_{ik}^{s_i} \, U_{k,r}(\sigma'; \boldsymbol{n_1'}, \boldsymbol{n_2'}) \right\} \tag{10}$$

The maximizing strategy set at each location is the set of the strategy that maximizes equation (10).

Since taxi drivers have the option of whether to offer services or not, we require individual rationality. In order to be willing to continue to operate, that taxis must have a non-negative expected discounted profit at any given point in time. For individual rationality to be satisfied, the bellman equations for each of the locations must be non-negative:

$$U_{i,a}(\sigma'; n_1, n_2) \geq 0$$

for $i \in \{1, 2\}$.

For an outcome to be an equilibrium, we require that for any state that occurs with positive probability, taxis are playing best responses and have a non-negative expected discounted profit. Given an initial set of taxis in each location, an equilibrium is an individually rational set of stationary Markov strategies such that at every state that occurs with positive probability, every strategy that is played with positive probability maximizes the Bellman equation.

We focus our attention to symmetric stationary equilibria. For a given value of N_1 and N_2, a stationary Markov equilibrium occurs, the strategies are incentive compatible, the steady-state flow equation is satisfied, and strategies are individually rational. Formally we define a stationary Markov equilibrium as:

Definition *Steady-State Equilibrium:* Let N_1 and N_2 be the number of taxis affiliated with each location, and let p_1 and p_2 be the prices at the locations. A steady-state equilibrium is a set of mixed strategies such that

1. $N_i = n_{1,i} + n_{2,i}$ for $i \in \{1, 2\}$

2. The set of strategies that is played in states $(\boldsymbol{n_1}, \boldsymbol{n_2}, i)$ that occur with positive

probability satisfies

$$\arg\max_{s_i \in S_i^r} \left\{ \pi_{i,r}^{s_i} + \delta \sum_{k \in \{1,2\}} \rho_{ik}^{s_i} \, U_k^r(\sigma_-; \boldsymbol{n_1'}, \boldsymbol{n_2'}) \right\}$$

3. For all $i \in \{1,2\}$ and $a \in \{1,2\}$ where $n_i^r > 0$

$$U_i^r \geq 0$$

4. For all $a \in \{1,2\}$

$$T_1^r + \tilde{q}_1^r + \tilde{\nu}_1^r = T_2^r + \tilde{q}_2^r + \tilde{\nu}_2^r \tag{11}$$

In a steady-state equilibrium, the flow condition is satisfied for each regulatory affiliation; therefore, the total number of taxis with each affiliation will remain constant over time. Since, in any given state, all taxis with the same affiliation choose the same mixed strategies, the same number of taxis will search for passengers in every period. As a result, the vacancy rates and quantities will also stay constant.

3.5 Regulator's Objective

To model the regulator's preferences, we assume that there is a single regulator that seeks to maximize the joint weighted surplus of the two locations. We use the joint regulator to represent a single regulator in a large city, such as New York or Los Angeles, that is divided up in to multiple affiliations. we We can also use the joint regulator to represent multiple regulatory jurisdictions in close proximity, such as in the greater Boston metropolitan area, by assuming that transfers between regulators are feasible and that information is symmetric. The joint regulator takes into account the consumer surplus, the driver profit, and the location specific external cost of taxi traffic. We allow the joint regulator to place a higher weighting on driver profit than it does on other components of the surplus.

The number of vacant taxis affects the benefits consumers receive and the costs taxi drivers incur. The consumer surplus from trips originating at location i is given by

$$CS_i = \int_0^{Q_i} p_i\left(Q, v_i\right) dQ - p_i\, Q_i$$

The consumer surplus depends on both the price and the vacancy rate. Increasing the vacancy rate causes consumers' willingness to pay to increase, thereby increasing consumer surplus. The producer surplus generated from taxis at location i is given by

$$PS_i = p_i\, Q_i - c\, n_i$$

Taxis receive revenue from becoming occupied and driving passengers to their desired locations. They incur costs whenever they drive passengers to their desired locations, remain vacant after searching for passengers, or transition to the other location.

The traffic externality is a location dependent constant external cost per unit of time spent driving. Each location has a constant marginal external cost ϕ_i of driving. The external cost is the same whether the taxi remains vacant, becomes occupied, or transitions to the other location without searching for passengers; therefore, we represent the total the total external cost is

$$\Phi_i(V_i, Q_i, T_i) = \phi_i\ \left(V_i + Q_i + T_j\right) = \phi_i\, n_i$$

By representing the externality in this fashion, we assume that location i incurs the entire external cost of trips originating from location i and traveling to the other location. Since we are looking at steady-state values, this representation is equivalent to one where we assume that half of the externality is incurred in each location. We choose the former representation for simplicity. By representing the externality in this way, we also assume that the unit cost of the externality is independent of the number of taxis on the road, thereby ignoring the externality that taxis place on other taxis.

The regulator may place a greater weight on drivers profit than she does on other components of the surplus; the consumer surplus and the externality. To model the regulator's preference over driver profit, we assume that the regulator places a weight of ω on driver profit. We assume that $\omega \geq 1$ so that the regulator either maximizes total surplus or gives greater weighting to driver profit. When $\omega = 1$, the regulator maximizes total surplus. When $\omega > 1$, the regulator places a greater weight on driver profit then she does on consumer surplus and the external cost components of total surplus.

Given the location specific externalities and preferences towards driver profit, we represent the weighted total surplus by: [4]

$$\begin{aligned} WTS &= \sum_{i \in \{1,2\}} \left(CS_i + \omega PS_i - \Phi_i \right) \\ &= \sum_{i \in \{1,2\}} \left(\int_0^{Q_i} p_i \left(Q, v_i \right) dQ - c\, n_i - \phi_i\, n_i + (\omega - 1)(p_i\, Q_i - n_i\, c_i) \right) \end{aligned} \tag{13}$$

Given this setup, we can characterize an environment by the demand functions, the externalities, the shared preference towards driver profit, the shared marginal

[4]We can allow the preference over driver profit to differ by location. When there are multiple regulators, we can assume that the transferable component of the surplus is the driver revenue and represent the joint weighted total surplus by

$$WTS = \sum_{i \in \{1,2\}} \left(PS_i + \frac{1}{\omega_i} CS_i - \frac{1}{\omega_i} \Phi_i \right) \tag{12}$$

This is done in Seymour (2014). Assuming this general structure does not compromise any of the major results.

cost, and the shared discount rate.

$$\{(q_i(\,), \tilde{q}_i(\,), \phi_i, \omega, c, \delta)\}_{i\in\{1,2\}}$$

For a given environment, prices and medallion levels, and a regulatory exchange price. These choices induce an equilibrium.

4 Regulatory Arrangements

The regulatory exchange setup allows us to model local exclusive cruising regulation, metropolitan-level regulation, and the regulatory exchange market. Under each type of regulation, the equilibrium need to satisfy additional conditions. Under the regulatory exchange market, the number of taxis participating in the regulatory exchange must be the same in both location and the regulatory exchange payment made by drivers in one location must be equal to the payment received by drivers in the other location. Local exclusive cruising regulation can be characterized by an infinite regulatory exchange price for both locations. Under metropolitan-level regulation, we can assume that taxis are all affiliated with the same location and have no cost of acquiring the right to pick up passengers in the other location. When service is provide to both locations and there are trips between locations, these regulatory frameworks will lead to a different equilibrium.

Under local exclusive cruising regulation each taxi has an affiliated location. The regulator chooses the total number of taxis that are affiliated with each of the two locations. Taxis are able to pick up passengers in their affiliated location, but are prevented from picking up passengers in the other location. When the regulatory exchange price is set to $\tau_i = \infty$ for $i \in \{1, 2\}$, whenever drivers drive passengers to the other affiliated location, they will not find it worthwhile to participate in the regulatory exchange and will transition back to their affiliated location. Since an infinite price makes it too costly to search for passengers in the other affiliated location, local exclusive cruising regulation can be characterized as an equilibrium with an infinite exchange price.

Definition *Equilibrium Under Local Exclusive Cruising Regulation:* An Equilibrium Under Local Exclusive Cruising Regulation is a stationary Markov equilibrium in a setting with $\tau_i = \infty$ for all $i \in \{1, 2\}$.

Alternatively, under metropolitan-level regulation, there is a single affiliation for the entire metropolitan area. All drivers with that affiliation are free to pick up passengers in either location. Without loss of generality, we can represent this situation by requiring that all the taxis are affiliated with location 1 and there is a zero cost of picking up passengers in the other location. As a result, taxis that are affiliated with the location are free to pick up passengers in both locations. We define a Metropolitan-level equilibrium as:

Definition *Equilibrium Under Metropolitan-Level Regulation:* An Equilibrium Under Metropolitan-level regulation is a stationary Markov equilibrium in a setting with $\tau_i = 0$ for all $i \in \{1, 2\}$ and $N_2 = 0$.

Under the regulatory exchange market, we assume that there is a market maker that chooses exchange prices so that the prices create a market for the right to search for passengers in the other affiliated location. An equilibrium will be a market if and only if the regulatory exchange prices represent a payment from drivers affiliated with one of the locations to drivers affiliated with the other location, and the number of drivers that pay for the right to search for passengers equals the number of drivers that receive the payment. The outcome will represent a payment when, in equilibrium, the regulatory exchange prices satisfy $\tau^1 = -\tau^2$. The quantity supplied will be equal to the quantity demanded when the number of taxis that participate in the regulatory exchange at each of the two locations is equal. We define a Regulatory Exchange market as:

Definition *Regulatory Exchange Market Equilibrium:* A Regulatory Exchange Market is a stationary Markov equilibrium with a setting with $\tau^1 = -\tau^2$ and $n_1^e = n_2^e$.

Under a regulatory exchange market, when τ_i is positive, τ_j will be negative. The drivers affiliated with location i will be the sellers and the drivers affiliated with location j will be the buyers. Sellers receive the payment, τ_i and acquire the right to search for passengers in their unaffiliated location for a single period. They forgo the right to search for passengers in their affiliated location for the same amount of time. Buyers make a payment of τ_i and receive the right to search for passenger in their unaffiliated location for a single period, forgoing the right to pick up passengers in their affiliated location for a single period.

We can see from the definitions, that a regulatory exchange market is different from equilibrium under local exclusive cruising regulation and equilibrium under metropolitan-level regulation. Under local exclusive cruising regulation, the fee for the regulatory exchange is positive in both location; therefore, the regulatory exchange prices are not payments from drivers affiliated with one location to drivers affiliated with the other location. Under metropolitan-level regulation, taxis are affiliated with location 1; therefore, when service is provided to both locations, there will be a positive number of taxis affiliated with location 1 participating in the regulatory exchange, and none from location 2.

5 Results

5.1 First Best

We characterize the first best stationary outcome from the regulator's perspective. Under the first best, we assume that the regulator is able to assign search behavior to the taxicabs without regard to their incentives and does not have to ensure that drivers make non-negative expected discounted profit. In this sense, the first best ignores both the incentive compatibility and the individual rationality constraints. Since the regulator is not bound by incentive compatibility and individual rationality, under the first best we can disregard the affiliations of the taxicabs.

The regulator chooses price levels and the total number of taxicabs at the two locations, and assigns behaviors to the taxicabs. We assume the regulator assigns n_i taxis to each of the two locations. At any point in time, the regulator chooses whether a given taxi searches for passengers while remaining in the same location if vacant, searches for passengers while relocating when vacant, or transitions without searching for passengers. Without loss of generality, we can say that the regulator assigns probabilities that taxis at location i are assigned actions v, $\tilde{v}$, and T. We assume that the regulator assigns all taxis at location i the same probability of choosing a given action. We denote the probability of a taxi at location i being assigned an action s by $\sigma_i(s)$. Given the choice of prices, number of taxis in each location, and taxi actions, we can determine the number of taxis that search for passengers, the number that remain vacant, and the number that become occupied. The choice of prices, number of taxis, and taxi actions also determines the flow of taxis between locations.

The total number of taxis at location i that search for passengers is determined by the number of taxis at location i and the probability of being assigned either ν or $\tilde{\nu}$. The number of searching taxicabs at location i and the price, p_i, determine the number of taxis in location i that become occupied and the number that remain vacant. The taxis that become occupied drive to their passengers desired location. Of the Q_i occupied taxis, q_i remain in the same location and $\tilde{q}_i$ relocate. Of the vacant taxis, v_i stay in the same location and $\tilde{v}_i$ relocate. The proportion of the V_i taxis that remain in the same location is the same as the proportion of searching taxis that are assigned to search for passengers and remain in the same location when vacant. The proportion of the V_i taxis that relocate is the same as the proportion of searching taxis that are assigned to search for passengers and relocate when vacant.

The regulator's problem is to choose prices, the number of taxicabs at each location, and taxi actions in order to maximize the joint weighted total surplus:

$$\max_{\{p_i,n_i,\sigma_i\}_{i\in\{1,2\}}} \sum_{i\in\{1,2\}} \left(\int_0^{Q_i} p_i\,(Q,v_i)\,dQ - c\,n_i - \phi_i\,n_i + (\omega-1)(p_i\,Q_i - n_i\,c_i) \right)$$

subject to the steady state flow condition:

$$\tilde{v}_1 + \tilde{q}_1 + T_1 = \tilde{v}_2 + \tilde{q}_2 + T_2$$

The solution to the first-best looks at an outcome in terms of the number of taxis assigned to each location, the prices, and the taxi actions; however, since Douglas (1972), the taxicab literature has looked at equilibria in terms of prices and aggregate vacancy levels. We show that we can represent the first best as an optimization problem in terms of price and vacancy rates, subject to a flow condition. The proof of this and subsequent results are provided in the appendix.

Proposition 1. *The first best has no transitioning; therefore, the planner's problem can be stated as a solution to the surplus maximizing problem with respect to price* *and aggregate vacancy levels:*

$$\max_{\{p_i, V_i\}_{i \in \{1,2\}}} \sum_{i \in \{1,2\}} \left(\int_0^{Q_i} p_i\,(Q, v_i)\, dQ - c\, n_i - \phi_i\, n_i + (\omega - 1)(p_i\, Q_i - n_i\, c_i) \right)$$

subject to

$$V_i \geq \tilde{q}_j - \tilde{q}_i \qquad\qquad i \in \{1,2\} \tag{14}$$

This result characterizes the first best in terms of the choice of prices and aggregate vacancy rates for each location. Under the first best, there will be zero transitioning because it is better for taxis to be vacant than it is for taxis to transition. When taxis are vacant instead of transitioning, the value of taxi services to consumers increases. Whenever there is transitioning, the regulator can increase weighted total surplus by reassigning taxis that transition to taxis that search for passengers. The regulator increases the price so that the same quantity will be demanded, and chooses search behavior such that the same number of taxis relocate. This increases the consumers' value while leaving drivers' cost and the external cost unchanged. This causes total surplus to increase. Since the price increases, driver profit also increases and, as a result, the weighted total surplus increases.

Since there is zero transitioning, the steady-state flow equation can be stated as:

$$\tilde{v}_i - \tilde{v}_j = \tilde{q}_j - \tilde{q}_i \tag{15}$$

The net flow of vacant taxis from location i to location j equals the net flow of occupied taxis from location j to location i. Whenever the steady-state flow equation is satisfied for some choice of actions, σ, where there is zero transitioning, equation (14) will also be satisfied. Conversely, whenever (14) is satisfied, there will be sufficient vacancy to satisfy the flow equation for appropriately chosen actions, σ. The maximum net flow rate of vacant taxis from location i to location j is given by V_i. This occurs when all taxis at location i are assigned to search while relocating and all taxis at location j are assigned to search while remaining in the same location. Net flow rates of vacant taxis of less than V_i from location i to location j can be obtained by assigning actions such that taxis at location i choose to search while remaining in the same location with a positive probability or taxis at location j choose to relocate with positive probability. By choosing the appropriate driver actions, any flow rate that satisfies equation (14) will also satisfy the flow equation.

Since the joint weighted total surplus depends on the price, the vacancy level, and the corresponding quantities, any set of actions that induce the same price and vacancy levels are payoff equivalent. This allows us to look at the first-best problem in terms of the price and the vacancy rates, and use equation (14) to determine whether the prices and vacancy levels satisfy the steady-state flow equality. This allows us to represent the first best as the solution to the weighted surplus maximization problem with respect to price and vacancy.

Characterizing the problem in terms of prices and vacancy levels is a more intuitive way to represent the first best optimization problem because the interpretation of the first order conditions are more intuitive then the first order conditions corresponding to the price, total number of taxis, and assignment of search behavior. Under the price and vacancy characterization, when the flow inequality does not bind at the first best, the problem can be solved by looking at the optimal price and vacancy levels for each location. When the flow inequality does not bind, the first order conditions with respect to the quantity and vacancy rate at location i are:

$$\omega p_i = \omega c + \phi_i - (\omega - 1)\frac{\partial p_i}{\partial Q_i} Q_i \qquad\qquad \int_0^{Q_i} \frac{\partial p_i}{\partial V_i} dQ = \omega c + \phi_i$$

When the flow inequality does not bind, the optimal price depends on the cost, the per-unit external cost of driving, the preference towards driver profit, and the demand curve. By producing an additional unit of taxi service, drivers receive a payment p_i. The drivers incurs a cost c of for the additional occupied trip. There is an externality from the additional trip that is place on non-market participants. By producing an additional unit of taxi service the price falls; therefore, taxi drivers incur an inframarginal loss and passengers receive an inframarginal benefit. Since the weighing on driver profit is higher than the weighting on consumer surplus, the regulator puts more weighting on the inframarginal loss than on the inframarginal benefit.

By increasing the vacancy rate, the benefit that passengers receive increases. The drivers incur the cost of operating the taxi, and there is an external cost that is incurred by non-market participants. The vacancy rate is chosen such that the weighted benefit of increasing the vacancy is equal to the weighted cost of increasing the vacancy. The total increase in the willingness to pay from an additional unit of vacancy will be equal to the marginal cost weighted by the preference to driver profit plus the marginal external cost. When the flow inequality for location j binds, increasing V_i or decreasing p_i increases the number of trips to location j. This requires that either p_j decreases or that V_j increases; therefore, the first order conditions are more complicated.

We define a condition, the no-subsidy condition, which is a sufficient condition for the first best to be obtainable using a regulatory exchange market. For a given set of price and vacancy levels, the no subsidy condition will be satisfied when the expected revenue in each location exceeds the cost of operating the taxi:

Definition *No Subsidy Condition*

An outcome satisfies the no subsidy condition so long as the expected revenue at each location is sufficient to cover the expected cost:

$$R_i^\nu \geq c \qquad\qquad i \in \{1,2\}$$

The no subsidy condition will be satisfied whenever, for each location, the total revenue from taxi service exceeds the cost of being vacant.

$$p_i\, Q_i \geq c\,(Q_i + V_i) \qquad\qquad i \in \{1,2\}$$

When, for a given set of price and vacancy levels, $R_i^\nu < c$ for at least one of the two locations, the efficient outcome can not be realized without a subsidy. The subsidy could either occur indirectly, as a result of profit from the other location being used to subsidize losses, or directly, through a subsidy from the regulator. Alternatively, when $R_i^\nu \geq c$ for $i \in \{1,2\}$, neither type of subsidy is necessary to satisfy the individual rationality constraint.

We will show that if the efficient outcome satisfies the no-subsidy condition, it can be achieved using a regulatory exchange market. Given this, one would hope that the efficient outcome always satisfies the no-subsidy condition. Unfortunately, as the following result shows, this is not the case; when there is no externality imposed by taxi traffic and the regulator does not place any weighting on driver profit the efficient outcome will not satisfy the no subsidy condition.

Proposition 2. *For a given environment where $\phi_i = 0$ and $\omega_i = 1$ for all $i \in \{1,2\}$, the fist best will not satisfy the no subsidy condition.*

When there is no externality or preference towards driver profit, the first order conditions imply that the the ideal price in absence of the flow equation is $p_i = c$. If $\tilde{q}_i < \tilde{q}_j$, decreasing the price in location i will cause $\tilde{q}_i$ to increase. If the price in location i is above the cost, lowering the price will increase the surplus and will still maintain the flow inequality. As a result when there is no preference towards driver profit, $p_i \leq c$ for at least one of the two regions. Due to vacancy, the expected revenue will be below the price and the no subsidy condition will not be satisfied.

Alternatively, when locations are sufficiently similar and the preference towards driver profit is sufficiently high, the No Subsidy Condition will be satisfied at the first best. In particular, if the locations are identical then, provided there are some price and vacancy levels under which profits are positive, for a sufficiently large preference towards driver profit, the profit considerations will lead the regulator to prefer price and vacancy levels that are close to the profit maximizing price and vacancy levels.

5.2 Local Exclusive Cruising and Metropolitan-Level Regulation

We examine the properties of local exclusive cruising and metropolitan-level regulation. We characterize situations where we can achieve the first best under each of the equilibrium concepts. In certain situations, local exclusive cruising regulation will achieve the first best but metropolitan-level regulation will not. In other situations, the reverse will occur; metropolitan-level regulation will achieve the first best but local exclusive cruising will not. This shows that there are instances where each type of regulation is preferred to the other.

In this section, we assume that the first best has $Q_i > 0$ for $i \in \{1, 2\}$. This means that the demand is sufficiently high such that it is worth providing service to both locations under the first best. Under the alternative forms of regulation, we allow for the possibility that the optimal under a particular type of regulation may be to only provide service to one of the two locations. We also assume that the No Subsidy Condition is satisfied at the first best. In both of the following results, if the No Subsidy Condition is violated at the first best, then the first best will not be achievable.

Under local exclusive cruising regulation, taxis that drive passengers from their affiliated location to the other location are forced to transition back to their affiliated location. Since the first best has zero transitioning, if the demand function has a positive number of trips to the other location whenever the total number of trips is positive, local exclusive cruising regulation will not be able to achieve the first best. Alternatively, when the demand at each location has no trips to the other location for any price and vacancy level, the first best is achievable. The no-subsidy condition is required and ensures that each location generates enough revenue to cover its costs.

Proposition 3.

1. *If, for some $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) > 0$ whenever $Q_i(p_i, V_i) > 0$, then the first best is not achievable under local exclusive cruising regulation.*

2. *If, for $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) = 0$ for all p_i and V_i, then the first best is achievable under local exclusive cruising regulation.*

When the No-Subsidy Condition is not satisfied for a particular set of price and vacancy levels, the corresponding outcome will not be achievable under local exclusive cruising regulation. As a result, when the first best does not satisfy the No-Subsidy Condition, it will not be achievable under local exclusive cruising regulation. At location where the expected revenue is less than the cost, the regulator will not be able to choose price and vacancy levels corresponding to the first best because it violates the individual rationality constraint for taxis affiliated with that location.

Under metropolitan-level regulation, the situation is more complex. For our characterization results we look at environments where the locations are symmetric, and we look at situations where the demand for trips to the other location is zero for all prices and vacancy levels. When the locations are symmetric, metropolitan level regulation will achieve the first best. When the demand for trips to the other location is zero, the first best may or may not be achievable under metropolitan level regulation.

When, for a particular environment, the first best is achievable under metropolitan level regulation, we can perturb the demand function to find an environment where the first best is not achievable.

Proposition 4.

1. *If the two locations have the same demand and the same external cost, metropolitan level regulation achieves the first best.*

2. *If, for $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) = 0$ for all p_i and V_i then metropolitan level regulation may or may not achieve the first best. The set of environments where it does not achieve the first best is dense.*

When the locations are symmetric, there exists first best allocations that have the same price and vacancy in each location. By choosing these prices and setting the number of medallions equal to the total number of taxicabs at the first best, the allocation can be achieved under metropolitan level regulation. When the prices and number of taxis in each location is the same as under the first best, the quantity of trips from each location will be the same, and will be the same as under the first best. The resulting single-period profit in each location will be also the same; therefore, taxi drivers will prefer to search for passengers but will be indifferent between remaining in the same location and moving to the other location. Since the locations are symmetric and the prices and vacancy rates are the same, the number of occupied trips between locations will be the same. When the drivers at each of the two locations choose the same search behavior, such as when all taxis choose to search while remaining in the same location when they remain unoccupied, the number of vacant taxis moving from one location to another will also be the same. This means that the flow of taxis will remain the same, and the outcome will be an equilibrium.

Alternatively, when in general environments where there is no demand for trips to the other locations, the first best may not be achievable. When there are no trips demanded to the other locations, allocations where the expected revenue differs and where service is provided to both locations will not be an equilibrium. When the expected revenue is different, the location with the higher expected revenue will generate higher profit; therefore, drivers in the low-value location will prefer to either search and relocate when they do not find passengers or transition without relocating. In either case, since no occupied taxis move from the high-value location to the low-value location, there will be an excess flow of taxis to the high-value location. When, for a given environment, the profit levels at the first best are the same for some allocation, we can slightly perturb the environment so that the first best has different profit levels.

Collectively, propositions (3) and (4) imply that there are instance where each type of regulation is strictly preferred to the other. In certain instances metropolitan-level regulation will be preferred to local exclusive cruising regulation, and in other insistences local exclusive cruising regulation will be preferred to metropolitan-level regulation. Neither type of equilibrium will always achieve the first best, even when we restrict our attention to situations where the no subsidy condition is satisfied at the first best.

5.3 Regulatory Exchange Market

We now look at the types of outcomes that can be implemented using a regulatory exchange market. We characterize the types of equilibria that exist under the regulatory exchange market. We then show that so long as the first best satisfies the no subsidy condition, it can be achieved using the regulatory exchange market. Finally, we show that even when the first best does not satisfy the no subsidy condition, the regulatory exchange market will be preferred to local exclusive cruising regulation and metropolitan level regulation.

Under the regulatory exchange market, the equilibria that are obtainable fall into two cases; the case where taxis are indifferent between moving between locations and the case where taxis prefer one of the two locations. Taxis will be indifferent between searching in each of the locations when τ_i satisfies

$$\tau_i = R_i^\nu - R_j^\nu \tag{16}$$

When this condition is satisfied, the single period expected profit from searching for a passenger is

$$\pi_{i,i}^\nu = R_i^\nu(p_i, V_i) - c \qquad\qquad \pi_{j,i}^\nu = R_i^\nu(p_i, V_i) - c$$

For drivers affiliated with a specific location, the single-period expected profit from searching for passengers is the same regardless of location. When the single-period profit is the same in each of the locations, drivers would prefer to search for passengers rather than transition to the other location; however, they are indifferent between searching for passengers while remaining in the same location and searching for passengers while relocating to the other location.

For the outcome to be individually rational, the expected discounted profit must be positive. Since the expected single period profit for a taxi affiliated with location i is

$$\pi_{i,k}^\nu = R_i^\nu - c$$

regardless of location k, the outcome will be individual rational if and only if the no-subsidy condition is satisfied.

When the no-subsidy condition is satisfied at the first best, the joint regulator can choose the τ_i from equation (16). Drivers receive the same single-period profit regardless either location; therefore, they will be indifferent between which direction the direct their search. As a result, the strategies that induce the first best will be incentive compatible. The no-subsidy condition ensures that the discounted expected profit is non-negative; therefore, the outcome will satisfy the individual rationality constraint. By assigning the appropriate number of taxis to each of the locations, the outcome is a Regulatory Exchange Market Equilibrium.

Proposition 5. *If the efficient outcome satisfies the no subsidy condition, then it can be realized using the unrestricted regulatory exchange market.*

There are also equilibria where the profit levels are different. When

$$\tau_i > R_i^{\nu} - R_j^{\nu}$$

location j is more appealing than location i for taxis affiliated with each of the locations. Depending on the regulatory exchange fee, taxis at location i will either search while relocating when they remain unoccupied or they will transition. This allows equilibria with transitioning and equilibria where the no subsidy condition is violated. Due to the regulatory exchange price, taxi affiliated with location i could make sufficient profit in location j to cover their cost in location i. This allows us to find regulatory exchange markets where the no subsidy condition is violated, but the outcome achieves the first best.

We now look at look at how the regulatory exchange market compares to local exclusive cruising regulations and metropolitan-level regulations. We see that the regulatory exchange market will always be preferred to local exclusive cruising regulation and metropolitan-level regulation. It will be preferred to local exclusive cruising regulation because we can choose a regulatory exchange market equilibrium such that the same quantities are chosen but some of the duplicate trips are eliminated. It will be preferred to a metropolitan-level equilibrium because any metropolitan-level equilibrium can be mimicked as a regulatory exchange market with a regulatory exchange price of zero.

Proposition 6. *The regulatory exchange market is preferred to local exclusive cruising regulation. This preference will be strict provided for some* $i \in \{1, 2\}$*,* $\tilde{q}_i(p_i, V_i) > 0$ *whenever* $Q(p_i, V_i) > 0$*.*

Under local exclusive cruising regulation, the infinite regulatory exchange price means that taxis will transition back to the original location. The regulatory exchange market is preferred to local exclusive cruising is that we can choose the same price and vacancy levels that are achieved under local exclusive cruising regulation and choose the lowest possible level of transitioning that can maintain the price and vacancy rates. When there trips to the other location, this will decrease the total level of transitioning. This the cost of providing the same level of service decreases; therefore, the drivers make higher profit and the externality declines.

The regulatory exchange price is used to ensure that this outcome can be maintained as a part of an equilibrium. When there is no transitioning, the regulatory exchange price can be set equal to

$$\tau_i = R_i^\nu - R_j^\nu$$

When there is no transitioning, strategies that satisfy the flow equation will be incentive compatible. The outcome will be individually rational, since taxis affiliated with each of the two locations have the single-period expected profit in both locations as they did in their affiliated location under local exclusive cruising regulation.

If it is possible to satisfy the flow condition with the same price and vacancy levels and no transitioning taxicabs, then the outcome can be realized by giving taxis incentives to choose the appropriate direction for their search. We allocate $Q_i + V_i$ taxis to be affiliated with each location and set the price such that, when in the other location, the expected profit is the same as the taxis receive in their own location

$(\tau_i = R_i^\nu - R_j^\nu)$. Under these prices, drivers are indifferent between searching while remaining in the same location and searching while relocating; therefore, it is incentive compatible for taxis to choose the desired quantity. The outcome will be individually rational because, under local exclusive cruising taxis needed to cover the cost of the return trip; therefore, the expected revenue exceeds the cost at each location.

When it is not possible to eliminate all of the duplicate trips, it is necessary for taxis to transition from one location to the other. By limiting the amount of transitioning to the lowest amount possible, drivers will only transition at one of the two locations. To ensure that taxis have an incentive to transition, we need to choose a transfer fee such that some taxis will want to transition. By choosing the appropriate fee, taxis at the lower single-period profit location will be indifferent between searching for passengers while relocating when vacant and transitioning to the other location without searching for passengers. As a result, the flow of taxis will be incentive compatible. The drivers affiliated with both locations will have a higher profit than under local exclusive cruising regulation because they always have the option of transitioning back to their affiliated location and receiving the same expected profit as they did under local exclusive cruising regulation. This means that the individual rationality condition is satisfied.

Proposition 7. *Every metropolitan level regulation equilibrium is payoff equivalent to a regulatory exchange market equilibrium. As a result the regulatory exchange market is preferred to metropolitan level regulation.*

The metropolitan-level equilibrium will satisfy the flow equation, will be incentive compatible, and will be individually rational; however, since all taxis are affiliated with location 1, the outcome will not be a regulatory exchange market equilibrium. We can maintain the same quantity while reallocating some of the taxis to affiliation 2 such that the number of taxis that change affiliations is the same in both locations. This means that any metropolitan-level equilibrium can be be achieved as a regulatory exchange equilibrium.

The regulatory exchange market gives the regulator more flexibility than under metropolitan-level regulation. The regulatory exchange price allows the regulator to maintain price and vacancy rates that cannot be maintained under metropolitan-level regulation because of incentive compatibility issues.

6 Conclusion

In this paper, we use a dynamic network model of the taxicab industry to model taxi regulation. We develop a framework that allows to look at a situation where taxis have regulatory affiliations, but are able to use a regulatory exchange mechanism to temporarily be able to pick up passengers in the other location. Using the regulatory exchange mechanism framework, we are able to look at the two most prevalent types of regulation, local exclusive cruising regulation and metropolitan level regulation. We are also able to use the regulatory exchange mechanism to model a regulatory exchange market; a market that allows taxis to temporarily exchange the right to pick up passengers in each other's affiliated location.

We characterized the first best and showed that sometimes the efficient outcome requires that the losses from driving in one location be subsidized by profit in another location. We show that the two most prevalent types of regulation are inefficient, even when the no-subsidy condition is satisfied. Alternatively, the regulatory exchange market achieves the first best provided that, at the first best, the no subsidy condition is satisfied. Further, the regulator exchange market is preferred to local exclusive cruising regulation and metropolitan level regulation, regardless of whether the no subsidy condition is satisfied.

References

Arnott, R. (1996). Taxi travel should be subsidized. *Journal of Urban Economics*, 40(3):316–333.

Cairns, R. D. and Liston-Heyes, C. (1996). Competition and regulation in the taxi industry. *Journal of Public Economics*, 59(1):1–15.

De Vany, A. S. (1975). Capacity utilization under alternative regulatory restraints: an analysis of taxi markets. *The Journal of Political Economy*, pages 83–94.

Douglas, G. W. (1972). Price regulation and optimal service standards: The taxicab industry. *Journal of Transport Economics and Policy*, pages 116–127.

Flores-Guri, D. (2005). Local exclusive cruising regulation and efficiency in taxicab markets. *Journal of Transport Economics and Policy*, pages 155–166.

Orr, D. (1969). The" taxicab problem": A proposed solution. *The Journal of Political Economy*, 77(1):141–147.

Seymour, D. (2014). Combining local regulations in the taxicab industry. Unpublished manuscript.

Yang, H., Wong, S. C., and Wong, K. (2002). Demand–supply equilibrium of taxi services in a network under competition and regulation. *Transportation Research Part B: Methodological*, 36(9):799–819.

Yang, H., Ye, M., Tang, W. H., and Wong, S. C. (2005). Regulating taxi services in the presence of congestion externality. *Transportation Research Part A: Policy and Practice*, 39(1):17–40.

Appendix: Proofs

Proposition 1. *The first best has no transitioning; therefore, the planner's problem can be stated as a solution to the surplus maximizing problem with respect to price and aggregate vacancy levels:*

$$\max_{\{p_i, V_i\}_{i \in \{1,2\}}} \sum_{i \in \{1,2\}} \left(\int_0^{Q_i} p_i\,(Q, v_i)\, dQ - c\, n_i - \phi_i\, n_i + (\omega - 1)(p_i\, Q_i - n_i\, c_i) \right)$$

subject to

$$V_i \geq \tilde{q}_j - \tilde{q}_i \qquad\qquad i \in \{1, 2\}$$

Proof. We start by showing that $T_i = 0$ for all $i \in \{1, 2\}$. Let $\{p_i^p, n_i^p, \sigma_i^p\}_{i \in \{1,2\}}$ be a feasible solution to the planner's problem with equilibrium values

$$\{q_i^p, \tilde{q}_i^p, v_i^p, \tilde{v}_i^p, T_i^p\}_{i \in \{1,2\}}$$

where $T_i > 0$ for some $i \in \{1, 2\}$.

We show that we can find $\{p_i', n_i', \sigma'\}_{i \in \{1,2\}}$ with

$$\{q_i', \tilde{q}_i', v_i', \tilde{v}_i', T_i'\}_{i \in \{1,2\}}$$

such that the joint weighted total surplus is greater.

Choosing $\tilde{v}_i' = \tilde{v}_i^p + T_i^p$, $V_i' = v_i^p + \tilde{v}_i'$, $T_i' = 0$, and $p_i' = p_i(Q_i^p, V_i')$, and let the rest of the variables keep their original values. This can be induced by choosing

$$\sigma_i'(\tilde{\nu}) = \frac{v_i'}{V_i} \qquad\qquad \sigma_i'(\tilde{\nu}) = \frac{\tilde{v}_i'}{V_i} \qquad\qquad \sigma_i'(T) = 0$$

while leaving the rest of the strategies unchanged, $\sigma_j' = \sigma_j^p$.

The steady-state flow condition is satisfied since

$$\tilde{q}_i' + \tilde{v}_i' + T_i' = \tilde{q}_i^p + \tilde{v}_i^p + T_i^p$$
$$= \tilde{q}_j^p + \tilde{v}_j^p + T_j^p = \tilde{q}_j' + \tilde{v}_j' + T_j'$$

Then $Q_i' = Q_i^p$ and $p_i(Q_i, V_i') > p_i(Q_i, V_i)$; therefore,

$$WTS' - WTS = \sum_{i\in\{1,2\}} \int_0^{Q_i^p} p_i(Q, V_i')\, dQ - \int_0^{Q_i^p} p_i(Q, V_i^p)\, dQ \geq 0$$

Therefore, the joint weighted total surplus increases by choosing $T_i = 0$ and the surplus maximizing outcome has $T_i = 0$ for all $i \in \{1, 2\}$.

Now we show that the constraints are equivalent. Assume that an optimal set of $\{p_i, n_i, \sigma_i\}_{i\in\{1,2\}}$ satisfies

$$\tilde{v}_i + \tilde{q}_i + T_i = \tilde{v}_j + \tilde{q}_j + T_j$$

Then, since $T_i = 0$ it satisfies

$$\tilde{v}_i + \tilde{q}_i = \tilde{v}_j + \tilde{q}_j$$

So

$$V_i \geq \tilde{v}_i = \tilde{v}_j + \tilde{q}_j - \tilde{q}_i \geq \tilde{q}_j - \tilde{q}_i$$

As a result, any payoff that can be obtained in the price-strategy representation can be obtained in the price-vacancy representation.

Alternatively, let $\{p_i, V_i\}_{i\in\{1,2\}}$ be a set of equations that satisfy the constraints in the proposition (with $T_i = 0$). Without loss of generality we can assume

$$\tilde{q}_i + V_i \leq \tilde{q}_j + V_j$$

The regulator can choose

$$\begin{array}{lll} n_i = V_i + Q_i & & \\ \sigma_i(T) = 0 & \sigma_i(v) = 0 & \sigma_i(\tilde{v}) = 1 \\ \sigma_j(T) = 0 & \sigma_j(v) = 1 - \dfrac{\tilde{q}_i + V_i - \tilde{q}_j}{V_j} & \sigma_j(\tilde{v}) = \dfrac{\tilde{q}_i + V_i - \tilde{q}_j}{V_j} \end{array}$$

For these strategies,

$$\begin{array}{lll} T_i = 0 & v_i = 0 & \tilde{v}_i = V_i \\ T_j = 0 & v_j = \tilde{q}_j + V_j - \tilde{q}_i - V_i & \tilde{v}_j = \tilde{q}_i + V_i - \tilde{q}_j \end{array}$$

Therefore,

$$\begin{aligned} \tilde{v}_j + \tilde{q}_j &= \tilde{q}_i + V_i - \tilde{q}_j + \tilde{q}_j \\ &= \tilde{q}_i + V_i \\ &= \tilde{q}_i + \tilde{v}_i \end{aligned}$$

Any other strategies that gives the same $\{p_i, V_i, Q_i\}_{i \in \{1,2\}}$ will have the same total surplus. As a result, any payoff that can be obtained in the price-vacancy representation can also be obtained in the strategy framework.

$\square$

Proposition 2. *For a given environment where $\phi_i = 0$ and $\omega_i = 1$ for all $i \in \{1, 2\}$, the fist best will not satisfy the no subsidy condition.*

Assume that $\{p_i, V_i\}_{i \in \{1,2\}}$ satisfies the No-Subsidy condition. We show that the price and vacancy rates do not maximize weighted total surplus. By assumption, $Q_i > 0$ for $i \in \{1, 2\}$. Assume, without loss of generality, that $\tilde{q}_i \leq \tilde{q}_j$ with $p_i > c$. Then taking $p'_j = p_j$, $V'_i = V_i$, V'_j. Since

$$\frac{\partial}{\partial Q_i} WTS = p_i - c$$

By choosing $p'_i < p$ such that p'_i is sufficiently close to p_i, $p'_i > c$, $\tilde{q}_j - \tilde{q}_i(p'_i, V_i) \leq \tilde{q}_j - \tilde{q}_i(p_i, V_i) \leq V_i$ $\tilde{q}_i(p'_i, V_i) - \tilde{q}_j < V_j$; therefore, by choosing the appropriate p'_i the weighted total surplus increases and the flow inequality constraint is still satisfied. This means that any choice of $\{p_i, V_i\}_{i \in \{1,2\}}$ that satisfies the No Subsidy condition is not first best; therefore, the first best does not satisfy the No Subsidy condition.

Proposition 3.

1. *If, for some $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) > 0$ whenever $Q_i(p_i, V_i) > 0$, then the first best is not achievable under local exclusive cruising regulation.*

2. *If, for $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) = 0$ for all p_i and V_i, then the first best is achievable under local exclusive cruising regulation.*

Proof. **(1)** Assume $\tilde{q}_i(p_i, V_i) > 0$ for all p_i, V_i such that $Q_i(p_i, V_i) > 0$. Then under local exclusive cruising regulation any strategies that give $\left(p_i^{fb}, V_i^{fb}\right)$, have $\tilde{q}_i > 0$; therefore, $\tilde{q}_{i,i} = \tilde{q}_i$ and $\tilde{q}_{i,i} = T_{j,i}$. Since under the first best, $T_j = 0$, the first best can not be achieved under local exclusive cruising regulation.

(2) Assume that, for $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) = 0$ for all p_i, V_i. Then choosing $p_i = p_i^{fb}$ and $N_i = Q_i^{fb} + V_i^{fb}$ for $i \in \{1, 2\}$ induces the first best under local exclusive cruising regulation.

Let $s_{i,a}$ be a strategy that a taxi in location i with affiliation a plays. By Lemma 3, on page 67, $s_{i,i} = \nu$, $s_{i,j} = T$ incentive compatible. The number of searching taxis at location i is $N_i = Q_i^{fb} + V_i^{fb}$; therefore, since $p_i = p_i^{fb}$, $Q_i = Q_i^{fb}$ and $V_i = V_i^{fb}$. Since $s_{i,i} = \nu$ and $n_{i,j} = 0$, $T_i = 0$; therefore, $T_i = T_i^{fb}$.

By Lemma 3, the discounted payoff is

$$U_{i,i} = \frac{R_i^{\nu} - c}{1 - \delta} \geq 0$$

the individual rationality condition is satisfied.

Since $\tilde{q}_i = 0$, $\tilde{q}_{i,i} = 0$. Since $s_{i,i} = \nu$, $\tilde{v}_{i,i} = 0$ and $T_{i,i} = 0$. Since $n_{j,i} = 0$, $\tilde{q}_{j,i} = 0$, $\tilde{v}_{j,i} = 0$ and $T_{j,i} = 0$. Therefore,

$$\tilde{q}_{i,i} + \tilde{v}_{i,i} + T_{i,i} = 0 = \tilde{q}_{j,i} + \tilde{v}_{j,i} + T_{j,i}$$

and steady-state flow equation is satisfied.

□

Proposition 4.

1. *If the two locations have the same demand and the same external cost, metropolitan level regulation achieves the first best.*

2. *If, for $i \in \{1, 2\}$, $\tilde{q}_i(p_i, V_i) = 0$ for all p_i and V_i then metropolitan level regulation may or may not achieve the first best. The set of environments where it does not achieve the first best is dense.*

Proof. **(1:)**

Choose $(p_i^{fb}, Q_i^{fb}, V_i^{fb}, T_i^{fb})$ as first best price, quantity, vacancy rates, and transitioning levels.

Choose $p_i = p_i^{fb}$, $\tau_2 = 0$, and

$$N_1 = 2Q_1 + 2V_1$$

We show the outcome with

$$n_{1,1} = Q_1 + V_1 \qquad\qquad n_{2,1} = Q_2 + V_2$$

and

$$s_{1,1} = \nu \qquad\qquad s_{2,1} = \nu$$

is an equilibrium.

IC: Since $\pi^{\nu}_{i,1} = R^{\nu}_1 - c$ and $\pi^{\tilde{\nu}}_{i,1} = R^{\nu}_1 - c$, and $\pi^{T}_{k,1} = -c$, the payoff from choosing $s_{i,1} = \nu$ is equal to the payoff from choosing $s_{i,1} = \tilde{\nu}$ and exceeds the payoff from choosing T in each location; therefore, the strategies are incentive compatible.

IR: Since $\pi^{\nu}_{1,1} = \pi^{\nu}_{2,1}$, the expected payoff is

$$U_{i,1} = \frac{\pi_{i,1}}{1-\delta}$$

Flow equation: Since $\tilde{q}_{1,1} = \tilde{q}_{2,1}$,$\tilde{v}_{i,1} = 0$, and $T_{i,1} = 0$:

$$\tilde{q}_{1,1} + \tilde{v}_{1,1} + T_{1,1} = \tilde{q}_{2,1} + \tilde{v}_{2,1} + T_{2,1}$$

(2:)

We first show that, under these assumptions, any outcome with service to both locations under metropolitan level regulation must have $\pi^{\nu}_1 = \pi^{\nu}_2$. If $\pi^{\nu}_i > \pi^{\nu}_j$, then incentive compatibility requires taxis at location j choose $s_{j,1} = \tilde{\nu}$ or $s_{j,1} = T$, and taxis at location i will choose $s_{i,1} = \nu$. This means

$$\tilde{q}_{i,1} + \tilde{v}_{i,1} + \tilde{T}i, 1 = 0$$

but

$$\tilde{q}_{j,1} + \tilde{v}_{j,1} + \tilde{T}j, 1 > 0$$

so the outcome is not an equilibrium; therefore, the metropolitan level regulation must have $\pi^{\nu}_{1,2} = \pi^{\nu}_{1,2}$.

Let Π_i^{fb} be the set of profit levels that are obtainable under the fist-best. If $\Pi_1 \cap \Pi_2$ is a single point, multiply the willingness to pay for vacancy above V by a factor of $\frac{1}{1+\epsilon}$ and multiply the quantity demanded by $1+\epsilon$. The first best vacancy rate will remain the same and while the first best quality will increase; therefore, the first best profit will no longer be equal and he first best is not achieved in some neighborhood of the original environment.

If there is a set of points such that the profit is equal, choose first best outcomes such that $\pi_1^{fb} \neq pi_2^{fb}$. Increase the demand at the chosen first best values by ϵ_i at the first best price and vacancy rates, such that the new demand function satisfies the demand assumptions in a and only differs from the old demand function in a neighborhood of the price and vacancy rates. Then the new first best is in this neighborhood. By choosing ϵ_i and the neighborhood sufficiently small the profit level at the new first best is arbitrarily close to π_i^{fb}; therefore, the first best is not achieved in some neighborhood of the original environment.

□

Proposition 5. *If the efficient outcome satisfies the no subsidy condition, then it can be realized using the unrestricted regulatory exchange market.*

Proof. Since the efficient outcome has $T_i = 0$ for $i \in \{1, 2\}$,

$$\begin{aligned} V_i \geq \tilde{v}_i &= \tilde{q}_j + \tilde{v}_j - \tilde{q}_i \\ &\geq \tilde{q}_j - \tilde{q}_i \end{aligned}$$

for all $i \in \{1,2\}$; therefore, since no subsidy condition is satisfied; therefore, we apply the result from Lemma 1 on page 56. The outcome is an equilibrium with $N_i = Q_i + V_i$, $\tau_i = R_i^\nu - R_j^\nu$, and $p_i = p_i^{fb}$. □

Proposition 6. *The regulatory exchange market is preferred to local exclusive cruising regulation. This preference will be strict provided for some* $i \in \{1,2\}$, $\tilde{q}_i(p_i, V_i) > 0$ *whenever* $Q(p_i, V_i) > 0$.

Proof. Let $\{p_i^{LEC}, V_i^{LEC}\}_{i\in\{1,2\}}$ be the equilibrium price and vacancy rates with associated with the weighted total surplus maximizing local exclusive cruising equilibrium. By Lemma 3 the local exclusive cruising strategies are $s_{i,i} = \nu$ and $s_{i,j} = \tilde{\nu}$.

We look at two cases:

1: $V_i^{LEC} \geq \tilde{q}_j^{LEC} - \tilde{q}_i^{LEC}$ for all $i \in \{1,2\}$

2: $V_i^{LEC} < \tilde{q}_j^{LEC} - \tilde{q}_i^{LEC}$ for some $i \in \{1,2\}$

In both case we find an allocation that has the same price and vacancy rate, but a lower level of transitioning when one location has $\tilde{q}_i^{LEC} > 0$.

Case 1 We use Lemma 1. We show that the two conditions of the Lemma are satisfied. By assumption, $V_i^{LEC} \geq \tilde{q}_j^{LEC} - \tilde{q}_i^{LEC}$ for all $i \in \{1,2\}$, so we need to show that the no subsidy condition is satisfied.

Since $U_{i,i}^{LEC} > 0$

$$\frac{R_i^\nu}{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})} - \frac{c}{1-\delta} > 0$$

since

$$\begin{aligned} 1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii}) &= 1 - \delta + (1-\rho_{ii})(\delta - \delta^2) \\ &\geq 1 - \delta \end{aligned}$$

$R_i^\nu \geq c$ for $i \in \{1, 2\}$, so the No Subsidy condition is satisfied. By Lemma 1, $p_i = p_i^{LEC}$, $N_i = Q_i^{LEC} + V_i^{LEC}$, $\tau_i = R_i^\nu - R_j^\nu$ form a regulatory exchange market.

The difference in surplus is

$$WTS^{REM} - WTS^{LEC} = \sum_{i \in \{1,2\}} (\omega c\, T_i + \phi_i\, T_i)$$

which is positive whenever $T_i > 0$ for some $i \in \{1, 2\}$.

Case 2 Let

$$V_i^{LEC} < \tilde{q}_j^{LEC} - \tilde{q}_i^{LEC}$$

We use Lemma 2 to show that there exists a regulatory exchange market with $p_k = p_k^{LEC}$, $V_k = V_k^{LEC}$, and $Q_k = Q_k^{LEC}$ for all $k \in \{1, 2\}$, and $T_i = \tilde{q}_j^{LEC} - \tilde{q}_i^{LEC} - V_i^{LEC}$ and $T_j = 0$.

We need to show the two revenue conditions hold. Since $U_{j,i} \geq 0$,

$$\frac{\delta\, R_i^\nu}{1 - \delta\, \rho_{ii} - \delta^2(1 - \rho_{ii})} - \frac{c}{1 - \delta} > 0$$

therefore,

$$R_i^\nu \geq \frac{1 - \delta\, \rho_{ii}^\nu - \delta^2(1 - \rho_{ii}^\nu)}{\delta\, (1 - \delta)} c$$

Since $U_{j,j} \geq 0$ and

$$1 - \delta\, \rho_{jj} - \delta^2(1 - \rho_{jj}) \geq 1 - \delta$$

we have $R_j^\nu \geq c$. By applying the lemma, we have the specified regulatory exchange market.

The transition rate in location i falls from $T_i = \tilde{q}_j^{LEC}$ to $T_i' = \tilde{q}_j^{LEC} - \tilde{q}_i^{LEC} - V_i^{LEC}$ and the transition rate it location j goes from $T_j = \tilde{q}_i^{LEC}$ to 0. The difference in surplus is

$$WTS^{REM} - WTS^{LEC} = \omega c(2\tilde{q}_i^{LEC} - V_i^{LEC}) + \phi_i\left(\tilde{q}_i^{LEC} - V_i^{LEC}\right) + \phi_j \tilde{q}_i^{LEC} > 0$$

□

Proposition 7. *Every metropolitan level regulation equilibrium is payoff equivalent to a regulatory exchange market equilibrium. As a result the regulatory exchange market is preferred to metropolitan level regulation.*

Proof. Let a metropolitan level equilibrium have $\{p_i^{ml}, V_i^{ml}, Q_i^{ml}, T_i^{ml}\}_{i\in\{1,2\}}$.

Let

$$n_{i,a} = \frac{n_i^{ml}\, n_a^{ml}}{n_1^{ml} + n_2^{ml}}$$

then

$$\sum_{i\in\{1,2\}} n_{i,a} = N_a \qquad \sum_{a\in\{1,2\}} n_{i,a} = n_i^{ml}$$

Under metropolitan-level regulation, WLOG, we can assume all transitioning taxis choose to participate in the regulatory exchange. Then choosing

$$\sigma_{1,1}(s_i) = \sigma_{1,1}^{ml}(s_i) \qquad \sigma_{2,1}(s_i^m) = \sigma_{2,1}^{ml}(s_i^m)$$

$$\sigma_{1,2}(s_i^m) = \sigma_{1,1}^{ml}(s_i) \qquad \sigma_{2,2}(s_i) = \sigma_{2,1}^{ml}(s_i^m)$$

Then the number of searching taxicabs is

$$\begin{aligned}
n_i^{\nu_{tot}} &= n_{i,i}(\sigma_{i,i}(\nu) + \sigma_{i,i}(\tilde{\nu})) + n_{i,j}(\sigma_{i,j}(\nu^m) + \sigma_{i,j}(\tilde{\nu}^m)) \\
&= (n_{i,i} + n_{i,j})(\sigma_{i,i}(\nu) + \sigma_{i,i}(\tilde{\nu})) \\
&= n_i^{ml}(\sigma_{i,i}(\nu) + \sigma_{i,i}(\tilde{\nu})) = n_i^{\nu_{tot},ml}
\end{aligned}$$

therefore, the quantity and vacancy rates are the same.

Since the strategies are the same for each location, the probability of becoming occupied and driving to the other location, the probability of remaining unoccupied and relocating, and the probability of transitioning will be the same as under metropolitan level regulation; therefore,

$$\tilde{q}_{i,a} = \frac{n_a^{ml}}{n_1^{ml} + n_2^{ml}} \tilde{q}_i^{ml} \qquad \tilde{v}_{i,a} = \frac{n_a^{ml}}{n_1^{ml} + n_2^{ml}} \tilde{v}_i^{ml}$$

$$T_{i,a} = \frac{n_a^{ml}}{n_1^{ml} + n_2^{ml}} T_i^{ml}$$

This implies

$$\begin{aligned}
\tilde{q}_{1,a} + \tilde{v}_{1,a} + T_{1,a} &= \frac{n_a}{n_1 + n_2}(\tilde{q}_{1,1}^{ml} + \tilde{v}_{1,1}^{ml} + T_{1,1}^{ml}) \\
&= \frac{n_a}{n_1 + n_2}(\tilde{q}_{2,1}^{ml} + \tilde{v}_{2,1}^{ml} + T_{2,1}^{ml}) \\
&= \tilde{q}_{2,a} + \tilde{v}_{2,a} + T_{2,a}
\end{aligned}$$

Choose a regulatory exchange price $\tau_i = 0$ for $i \in \{1, 2\}$. Since the pice and vacancy rate are the same

$$\pi_{i,i}^{\nu} = \pi_{i,j}^{\nu^m} = R_i^{\nu} - c \qquad \pi_{i,i}^{\tilde{\nu}} = \pi_{i,j}^{\tilde{\nu}^m} = R_i^{\tilde{\nu}} - c$$

$$\pi_{i,i}^{T} = \pi_{i,j}^{T^m} = -c$$

Therefore

$$U_{i,a} = U_{i,1}^{ml}$$

and individual rationality and incentive compatibility are satisfied since the conditions are satisfied under metropolitan level regulation.

□

Lemma 1. *Assume, for a given* $\{p_i, V_i\}_{i\in\{1,2\}}$ *that*

1. *for* $i \in \{1, 2\}$

$$V_i \geq \tilde{q}_j(p_j, V_j) - \tilde{q}_i(p_i, V_i)$$

2. *The no subsidy condition is satisfied at* $\{p_i, V_i\}_{i\in\{1,2\}}$

Then there exists a steady-state regulatory exchange market with $N_i = Q_i + V_i$ *and* $\tau_i = R_i^\nu - R_j^\nu$ *such that the prices and vacancy rates are* $(p_i, V_i)_{i\in\{1,2\}}$*,*

$$q_i = q_i(p_i, V_i) \qquad \tilde{q}_i = \tilde{q}_i(p_i, V_i) \qquad T_i = 0$$

and

$$\pi_{1,a}^\nu = \pi_{2,a}^\nu$$

for all $a \in \{1, 2\}$*.*

Conversely, any market equilibrium such such that $\pi_{1,a}^\nu = \pi_{2,a}^\nu$ *satisfies*

$$V_i \geq \tilde{q}_j(p_j, V_j) - \tilde{q}_i(p_i, V_i)$$

with

$$q_i = q_i(p_i, V_i) \qquad \tilde{q}_i = \tilde{q}_i(p_i, V_i) \qquad T_i = 0$$

Proof. After relabeling as necessary, assume that $\tilde{q}_2 > \tilde{q}_1$. Then the steady-state flow equality is satisfied with $\tilde{v}_1 = \tilde{q}_2 - \tilde{q}_1$ and $\tilde{v}_2 = 0$. The mixed strategies

$$\sigma_{1,i}(\tilde{\nu}) = \frac{\tilde{v}_1}{V_1} \qquad \sigma_{1,i}(\nu) = 1 - \sigma_{1,i}(\tilde{\nu}) \qquad \sigma_{2,1}(\tilde{\nu}) = 1$$

induce these vacancy rates when $n_i = Q_i + V_i$

When we set $\tau_i = R_i^\nu - R_j^\nu$ then

$$\begin{aligned} \pi_{i,i}^{\nu} &= R_i^\nu - c & \pi_{i,j}^{\nu} &= R_j^\nu - \tau_i - c = R_i^\nu - c \\ \pi_{i,i}^{\tilde{\nu}} &= R_i^\nu - c & \pi_{i,j}^{\tilde{\nu}} &= R_j^\nu - \tau_i - c = R_i^\nu - c \\ \pi_{i,i}^{T} &= -c & \pi_{i,j}^{T} &= -c \end{aligned}$$

IC: Since the maximal single period payoffs are the same, any set of strategies that achieves the maximal payoff is incentive compatible; therefore, the choice of strategies is incentive compatible.

IR: The expected discounted profit is

$$\begin{aligned} U_{i,i} &= R_i^\nu - c + \delta(\rho_{ii} U_{i.i} + (1 - \rho_{ii}) U_{j,i}) \\ U_{j,i} &= R_i^\nu - c + \delta(\rho_{jj} U_{j.i} + (1 - \rho_{jj}) U_{i,i}) \end{aligned}$$

or

$$U_{i,i} = U_{j,i} = \frac{R_i^\nu - c}{1 - \delta}$$

Since the no subsidy condition is satisfied $R_i^\nu - c > 0$, and the allocation is individually rational.

SS: Since $T_{i,a} = 0$, the outcome is stationary since

$$\begin{aligned}
\tilde{q}_{1,a} + \tilde{v}_{1,a} &= \tilde{q}_1 \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k} + \tilde{v}_1 \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k} \\
&= (\tilde{q}_1 + \tilde{v}_1) \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k} \\
&= (\tilde{q}_2 + \tilde{v}_2) \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k} \\
&= \tilde{q}_2 \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k} + \tilde{v}_2 \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k} \\
&= \tilde{q}_{2,a} + \tilde{v}_{2,a}
\end{aligned}$$

REM: Since

$$\begin{aligned}
\tau_1 &= R_1^\nu - R_2^\nu \\
&= -(R_2^\nu - R_1^\nu) = -\tau_2
\end{aligned}$$

the exchange fees represent a transfer.

When

$$n_{i,a} = (Q_i + V_i) \frac{Q_a + V_a}{\sum_{k \in \{1,2\}} Q_k + V_k}$$

Then,

$$n_i = \sum_{k \in \{1,2\}} n_{i,k} = Q_i + V_i \qquad N = \sum_{k \in \{1,2\}} n_{k,i} = Q_i + V_i$$

and

$$n_{1,2} = (Q_1 + V_1)\frac{Q_2 + V_2}{\sum_{k\in\{1,2\}} Q_k + V_k}$$
$$= (Q_2 + V_2)\frac{Q_1 + V_1}{\sum_{k\in\{1,2\}} Q_k + V_k} = n_{2,1}$$

So the outcome is a regulatory exchange market.

To prove the converse, we note that if $T_i = 0$ then $\sigma_{i,k}(T) > 0$ and $n_{i,a} > 0$ for some $a \in \{1, 2\}$. When this occurs, taxis affiliated with location a receive $\pi^T_{i,a} = -c$ with positive probability; therefore they are not maximizing.

When $T_{i,a} = 0$ for all $i \in \{1, 2\}$, $a \in \{1, 2\}$, the flow inequality can be written as

$$\tilde{q}_{i,a} + \tilde{v}_{i,a} = \tilde{q}_{j,a} + \tilde{v}_{j,a}$$

summing across a gives

$$\tilde{q}_i + \tilde{v}_i = \tilde{q}_j + \tilde{v}_j$$

Noting that $V_i \geq \tilde{v}_i$ and $\tilde{v}_j \geq 0$ gives the inequality. The quantities follow from the demand equation. □

Lemma 2. *Assume that for a given* $\{p_i, V_i\}_{i\in\{1,2\}}$ *such that*

$$V_i < \tilde{q}_j(p_j, V_j) - \tilde{q}_i(p_i, V_i)$$

for some $i \in \{1, 2\}$*, then if*

$$R^\nu_i \geq \frac{1 - \delta\,\rho^\nu_{ii} - \delta^2(1 - \rho^\nu_{ii})}{\delta\,(1 - \delta)}c \qquad R^\nu_j \geq c$$

then there exists a regulatory exchange market with the same price and vacancy rates *such that for,*

$$q_k = q_k(p_k, V_k) \qquad \tilde{q}_k = \tilde{q}_k(p_i, V_i)$$

$$T_i = \tilde{q}_j(p_j, V_j) - \tilde{q}_i(p_i, V_i) - V_i \qquad T_j = 0$$

for $k \in \{1, 2\}$. Under this regulatory exchange market

$$\pi^{\nu}_{i,a} < \pi^{\nu}_{j,a}$$

for $a \in \{1, 2\}$.

Conversely, any regulatory exchange market with $\tau_i > R^{\nu}_i - R^{\nu}_j$, satisfies $\pi^{\nu}_{i,a} < \pi^{\nu}_{j,a}$ for all $i \in \{1, 2\}$. For such equilibrium

$$V_i \leq \tilde{q}_j(p_j, V_j) - \tilde{q}_i(p_i, V_i)$$

and

$$q_k = q_k(p_k, V_k) \qquad \tilde{q}_k = \tilde{q}_k(p_i, V_i)$$

$$T_i = \tilde{q}_j(p_j, V_j) - \tilde{q}_i(p_i, V_i) - V_i \qquad T_j = 0$$

for $k \in \{1, 2\}$

Proof. Assume that $V^{LEC}_i < \tilde{q}^{LEC}_i - \tilde{q}^{LEC}_j$ for some $i \in \{1, 2\}$, then after relabeling we can assume

$$V^{LEC}_1 < \tilde{q}^{LEC}_2 - \tilde{q}^{LEC}_1$$

We implement a regulatory exchange market with $T_1 = \tilde{q}_2 - \tilde{q}_1 - V_1$ and $T_2 = 0$. We choose a price $\tau_1 > R^{\nu}_1 - R^{\nu}_2$. For such prices, we show that the set of possible maximizing strategies are $S^{BR}_{2,a} = \nu$ and $S^{BR}_{1,a} \in \{\tilde{\nu}, T\}$ for $a \in \{1, 2\}$ and for taxis of each regulatory affiliation, there exists a critical value τ^{cr}_a such that:

$$
\begin{aligned}
\tau_1 < \tau^{cr}_a \quad &\Rightarrow \quad S^{BR}_{1,a} \in \{\tilde{\nu}\} \\
\tau_1 = \tau^{cr}_a \quad &\Rightarrow \quad S^{BR}_{1,a} = \{\tilde{\nu}, T\} \\
\tau_1 > \tau^{cr}_a \quad &\Rightarrow \quad S^{BR}_{1,a} \in \{T\}
\end{aligned}
$$

Since $\tau_1 > R^{\nu}_1 - R^{\nu}_2$, the expected single period profit in each location by being affiliated with regulator 1 and 2 is

$$
\begin{aligned}
\pi^{\nu}_{1,1} &= R^{\nu}_1 - c & \pi^{\nu}_{2,1} &= R^{\nu}_2 + \tau_1 - c > R^{\nu}_1 - c \\
\pi^{\nu}_{1,2} &= R^{\nu}_1 - \tau_1 - c < R^{\nu}_2 - c & \pi^{\nu}_{2,2} &= R^{\nu}_2 - c
\end{aligned}
$$

respectively. For taxis with each affiliation, the single-period profit in location 2 is greater than that of location 1.

We show that the $s_{1,k} \notin \nu$ and $s_{2,k} = \nu$. The continuation value of choosing strategies s_i, s_j along the equilibrium path is

$$U^{s_i,s_j}_{i,a} = R^{s_i}_{i,a} - c + \delta \left(\rho^{s_i}_{ii} U^{s_i,s_j}_{i,a} + \rho^{s_i}_{ij} U^{s_j,s_i}_{j,a}\right)$$

Setting $\rho^{s_i}_{ii} = 1 - \rho^{s_i}_{ij}$ and simplifying the previous equation gives

$$(1 - \delta)U^{s_i,s_j}_{i,a} = R^{s_i}_{i,a} - c - \delta\, \rho^{s_i}_{ij}\, (U^{s_i,s_j}_{i,a} - U^{s_j,s_i}_{j,a}) \tag{17}$$

Finding $(1 - \delta)(U^{s_i,s_j}_{i,a} - U^{s_j,s_i}_{j,a})$ and rearranging gives

$$U^{s_i,s_j}_{i,a} - U^{s_j,s_i}_{j,a} = \frac{R^{s_i}_{i,a} - R^{s_i}_{j,a}}{1 - \delta + \delta\rho^{s_i}_{ij} + \delta\rho^{s_j}_{ji}}$$

Therefore,

$$(1-\delta)U_{i,a}^{s_i,s_j} = R_{1,a}^{s_i} - c - \delta\ \rho_{ij}^{s_i}\frac{R_{i,a}^{s_i} - R_{j,a}^{s_i}}{1-\delta+\delta\rho_{ij}^{s_i}+\delta\rho_{ji}^{s_j}}$$

Therefore, since

$$\frac{\partial U_{i,a}}{\partial \rho_{ij}^{s_i}} < 0 \qquad\qquad \frac{\partial U_{1,a}}{\partial R_{i,a}^{s_i}} > 0$$

and

$$\rho_{ij}^{\nu} < \rho_{ij}^{\tilde{\nu}} < \rho_{ij}^{T} \qquad\qquad R_{i,a}^{\nu} = R_{i,a}^{\tilde{\nu}} > R_{i,a}^{T}$$

the value for $U_{2,a}$ is maximized when $s_2 = \nu$.

Since

$$\frac{\partial U_{1,a}}{\partial \rho_{ij}^{s_i}} > 0 \qquad\qquad \frac{\partial U_{1,a}}{\partial R_{i,a}^{s_i}} > 0$$

the strategy $s_1 = \tilde{\nu}$ provides a higher payoff than $s_1 = \nu$.

Now, we show that there is a critical value. Let $U_{i,a}^{s_i,s_j}$ be the payoff from choosing a strategies s_i and s_j along the equilibrium path. For each of the strategies we find out what happens as we change τ_1. So

$$\begin{aligned}
\frac{\partial}{\partial \tau_1}(1-\delta)U_{2,2}^{\nu,T} &= \frac{\partial}{\partial \tau_1}\left(R_2^{\nu} - c - \delta\ \rho_{21}^{\nu}\frac{R_2^{\nu}}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{T}}\right) = 0 \\
\frac{\partial}{\partial \tau_1}(1-\delta)U_{2,2}^{\nu,\tilde{\nu}} &= \frac{\partial}{\partial \tau_1}\left(R_2^{\nu} - c - \delta\ \rho_{21}^{\nu}\frac{R_2^{\nu} - (R_1^{\nu} - \tau_1)}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{\tilde{\nu}}}\right) \\
&= \frac{-\delta\ \rho_{21}^{\nu}}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{\tilde{\nu}}}
\end{aligned}$$

so

$$\frac{\partial}{\partial \tau_1}(1-\delta)(U_{2,2}^{\nu,T} - U_{2,2}^{\nu,\tilde{\nu}}) = \frac{\delta\, \rho_{21}^{\nu}}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{\tilde{\nu}}}$$

Since

$$\begin{aligned}
\frac{\partial}{\partial \tau_1}(1-\delta)U_{2,1}^{\nu,T} &= \frac{\partial}{\partial \tau_1}\left(R_2^{\nu}+\tau_1-c-\delta\, \rho_{21}^{\nu}\frac{R_2^{\nu}+\tau_1}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{T}}\right) \\
&= 1-\frac{\delta\, \rho_{21}^{\nu}}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{T}} \\
\frac{\partial}{\partial \tau_1}(1-\delta)U_{2,1}^{\nu,\tilde{\nu}} &= \frac{\partial}{\partial \tau_1}\left(R_2^{\nu}+\tau_1-c-\delta\, \rho_{21}^{\nu}\frac{R_2^{\nu}+\tau_1-R_1^{\nu}}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{\tilde{\nu}}}\right) \\
&= 1-\frac{\delta\, \rho_{21}^{\nu}}{1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{\tilde{\nu}}}
\end{aligned}$$

so

$$\frac{\partial}{\partial \tau_1}(1-\delta)(U_{2,1}^{\nu,T} - U_{2,1}^{\nu,\tilde{\nu}}) = \frac{(\rho_{12}^{T}-\rho_{12}^{\tilde{\nu}})\delta\, \rho_{21}^{\nu}}{(1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{\tilde{\nu}})(1-\delta+\delta\rho_{21}^{\nu}+\delta\rho_{12}^{T})}$$

Since, when $\tau_1 = R_1^{\nu} - R_2^{\nu}$, $U_{2,a}^{\nu,\tilde{\nu}} > U_{2,a}^{\nu,T}$, there exists a τ_r^{cr} for each $a \in \{1,2\}$, such that $U_{2,a}^{\nu,\tilde{\nu}} = U_{2,a}^{\nu,T}$.

There are three possibilities

1. The critical value for taxis affiliated with regulator 2 is lower

2. The critical value for taxis affiliated with regulator 1 is lower and there are a sufficiently large number of taxis affiliated with location 1 to handle transitioning.

3. The critical value for taxis affiliated with regulator 1 is lower and there are a not enough taxis affiliated with location 1 to handle transitioning.

Case 1: We let $n^1 = Q_1 + V_1$ and $n^2 = T_1 + Q_2 + V_2$. Then let $\tilde{q}_2^T = T_1$ be the number of taxis necessary to make up for the transition. Then, partition the taxis affiliated with location 2 into 2 groups, the transitioning taxis of size

$$\frac{T_1}{\tilde{q}_2}(Q_2 + V_2) \qquad\qquad \left(1 - \frac{T_1}{\tilde{q}_2}\right)(Q_2 + V_2)$$

Then,

$$\tilde{q}_1 = \left(1 - \frac{T_1}{\tilde{q}_2}\right)(\tilde{q}_2)$$

So, by the previous proposition, there exists $n^{\tilde{v}}_{i,a}$ such that the outcome forms a market equilibrium. Further, let

$$n^T_{1,2} = T_1 \qquad\qquad n^T_{2,2} = \frac{T_1}{\tilde{q}_2}(Q_2 + V_2)$$

This is feasible since $\frac{T_1}{\tilde{q}_2} < 1$.

The outcome is an equilibrium with

$$n_{i,1} = n^{\tilde{v}}_{i,1} \qquad\qquad n_{i,2} = n^{\tilde{v}}_{i,2} + n^T_{1,2}$$

With strategies

$$\sigma_{1,1}(\tilde{\nu}) = 1 \qquad\qquad \sigma_{2,1}(\nu) = 1$$

$$\sigma_{1,2}(\tilde{\nu}) = \frac{n^{\tilde{v}}_{i,a}}{n_{i,1}} \qquad\qquad \sigma_{2,2}(\nu) = 1$$

$$\sigma_{1,2}(T) = 1 - \sigma_{1,2}(\tilde{\nu})$$

We show that it is a steady-state and a market equilibrium. Best Responses.

Case 2 We look at the case where

$$T_1 \leq \rho_{21}^{\nu}(Q_1 + V_1)$$

We let $n^1 = Q_1 + V_1 + T_1$ and $n^2 = Q_2 + V_2$. Choose $\tau_1 = \tau_1^{cr}$, and maintain the same prices. Then let

$$n_1^{T,\nu} = T_1 + \frac{T_1}{\rho_{21}^{\nu}} \qquad n_1^{\tilde{\nu},\nu} = Q_1 + V_1 + T_1 - n_1^{T,\nu}$$

Then

$$n_{1,1}^{T,\nu} = \frac{\rho_{21}^{\nu}}{1 + \rho_{21}^{\nu}} n_1^{T,\nu} \qquad n_{2,1}^{T,\nu} = \frac{1}{1 + \rho_{21}^{\nu}} n_1^{T,\nu}$$

$$n_{1,1}^{\tilde{\nu},\nu} = \frac{\rho_{21}^{\nu}}{\rho_{12}^{\tilde{\nu}} + \rho_{21}^{\nu}} n_1^{\tilde{\nu},\nu} \qquad n_{2,1}^{\tilde{\nu},\nu} = \frac{\rho_{12}^{\tilde{\nu}}}{\rho_{12}^{\tilde{\nu}} + \rho_{21}^{\nu}} n_1^{\tilde{\nu},\nu}$$

Setting

$$n_{1,1} = n_{1,1}^{T,\nu} + n_{1,1}^{\tilde{\nu}} \qquad n_{2,1} = n_{2,1}^{T,\nu} + n_{2,1}^{\tilde{\nu}}$$

$$n_{1,2} = n_1 - n_{1,1} \qquad n_{2,2} = n_2 - n_{2,1}$$

So the mixed strategies

$$\sigma_{1,1}(T) = \frac{n_{1,1}^{T,\nu}}{n_{1,1} = n_{1,1}^{T,\nu}} \qquad \sigma_{1,1}(\tilde{\nu}) = 1 - \sigma_{1,1}(T) \qquad \sigma_{1,2}(\nu) = 1$$

$$\sigma_{1,2}(\tilde{\nu}) = 1 \qquad \sigma_{2,2}(\nu) = 1$$

Has T_1 transitioning taxis at location 1, the remaining choose the other strategies; therefore, since the number of taxis affiliated with location 1 is in steady-state, so is the number of taxis affiliated with location 2.

Further, since

$$n_1 = n_{1,1} + n_{1,2} \qquad\qquad n_1 = n_{1,1} + n_{1,2}$$

then

$$n_{1,2} = n_{2,1}$$

Best responses

Case 3 We look at the case where

$$T_1 > \rho_{21}^{\nu}(Q_1 + V_1)$$

Then set $T_{1,1} = \rho_{21}^{\nu}(Q_1 + V_1)$ and $T_{1,2} = T_1 - T_{1,1}$. Choose $\tau_1 = \tau_2^{cr}$, and maintain the same prices.

We let $n^1 = Q_1 + V_1 + T_{1,1}$ and $n^2 = Q_2 + V_2 + T_{1,2}$ with

$$\begin{aligned} n_{1,1} &= T_{1,1} & n_{2,1} &= Q_1 + V_1 \\ n_{1,2} &= T_{1,2} + Q_1 + V_1 & n_{2,2} &= Q_2 + V_2 - Q_1 - V_1 \end{aligned}$$

Then

$$n_{1,1} + n_{1,2} = T_1 + Q_1 + V_1 \qquad\qquad n_{2,1} + n_{2,2} = Q_2 + V_2$$

So the mixed strategies

$$\begin{aligned} &\sigma_{1,1}(T) = 1 & & & &\sigma_{1,2}(\nu) = 1 \\ &\sigma_{1,2}(\tilde{\nu}) = \frac{T_{1,2}}{T_{1,2} + Q_1 + V_1} & &\sigma_{1,2}(T) = 1 - \sigma_{1,2}(\tilde{\nu}) & &\sigma_{2,2}(\nu) = 1 \end{aligned}$$

These are best responses.

The number of searching taxicabs is $n_1^{\nu_{tot}} = Q_1 + V_1$ and $n_2^{\nu_{tot}} = Q_2 + V_2$. So the equilibrium vacancy levels are maintained. The number of taxis participating in the regulatory exchange market is

$$Q_{1,2} + V_{1,2} = Q_1 + Q_2 = Q_{2,1} + V_{2,1}$$

We need to show that the outcome is a steady-state equilibrium.

$$T_{1,1} = \tilde{q}_{2,1} = \rho_{21}(Q_1 + V_1)$$

so since

$$\tilde{q}_1 + \tilde{v}_1 + T_1 = \tilde{q}_2 + T_2$$

we have

$$\tilde{q}_{1,2} + \tilde{v}_{1,2} + T_{1,2} = \tilde{q}_{2,2} + T_{2,2}$$

□

Lemma 3. *Under local exclusive cruising, The payoff maximizing strategies are* $s_{i,i} = \nu$ *and* $s_{i,j} = T$*. The discounted payoff is*

$$\begin{aligned} U_{i,i}^{LEC} &= \frac{R_i^\nu}{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})} - \frac{c}{1-\delta} \\ U_{j,i}^{LEC} &= \frac{\delta\,R_i^\nu}{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})} - \frac{c}{1-\delta} \end{aligned}$$

Proof. Under local exclusive cruising drivers choose pure strategies with $s_{i,i} = \nu$ and

$s_{i,j} = T$. The payoff from choosing $s_{i,i} = T$ and $s_{i,j} = T$ is

$$U_{i,i} = U_{i,j} = \frac{-c}{1-\delta}$$

The payoff of choosing $s_{i,i} = \nu$ and $s_{i,j} = T$, and $s_{i,i} = \tilde{\nu}$ and $s_{i,j} = T$ is given by solving

$$\begin{aligned} U_{i,i}^{LEC} &= R_i^\nu - c + \delta\left(\rho_{ii} U_{i,i}^{LEC} + (1-\rho_{ii}) U_{j,i}^{LEC}\right) \\ U_{j,i}^{LEC} &= -c + \delta U_{i,i}^{LEC} \end{aligned}$$

Therefore the payoffs are given by

$$\begin{aligned} U_{i,i}^{LEC} &= \frac{R_i^\nu}{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})} - \frac{c}{1-\delta} \\ U_{j,i}^{LEC} &= \frac{\delta\, R_i^\nu}{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})} - \frac{c}{1-\delta} \end{aligned}$$

which is maximized when ρ_{ii} is as high as possible, or under $s_{i,i} = \nu$ and $s_{i,j} = T$.

So since

$$\frac{R_i^\nu}{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})} - \frac{c}{1-\delta} = U_{i,i} \geq 0$$

$$\begin{aligned} R_i^\nu &\geq c\frac{1 - \delta\,\rho_{ii} - \delta^2(1-\rho_{ii})}{1-\delta} \\ &= c\frac{1 - \delta + (\delta - \delta^2)(1-\rho_{ii})}{1-\delta} \\ &\geq c \end{aligned}$$

□