Chinese College Admissions and School Choice Reforms: Theory and Evidence*

Yan Chen Onur Kesten

January 6, 2015

Abstract

Within the last decade, many Chinese provinces have transitioned from the 'sequential' to the 'parallel' college admissions mechanisms. We show that all of the provinces that have abandoned the sequential mechanism have moved towards less manipulable and more stable mechanisms. Furthermore, Tibet implements the least manipulable parallel mechanism, whereas Beijing, Gansu and Shangdong have adopted the most manipulable versions. In the laboratory, participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the parallel and then the sequential mechanisms. While stability comparisons follow the same order, efficiency comparisons vary across environments.

Keywords: college admissions, school choice, sequential mechanism, Chinese parallel mechanism, deferred acceptance, experiment

JEL Classification Numbers: C78, C92, D47, D82

*We thank Susan Athey, Dirk Bergemann, Caterina Calsamiglia, Yeon-Koo Che, Isa Hafalir, Rustam Hakimov, Fuhito Kojima, Scott Kominers, Erin Krupka, Morimitsu Kurino, John Ledyard, Antonio Miralles, Herve Moulin, Parag Pathak, Jim Peck, Paul Resnick, Al Roth, Rahul Sami, Tayfun Sönmez, Guofu Tan, Utku Unver, Xiaohan Zhong and seminar participants at Arizona, Autonoma de Barcelona, Bilkent, Carnegie Mellon, Columbia, Florida State, Kadir Has, Johns Hopkins, Michigan, Microsoft Research, Rice, Rochester, Sabanci, Shanghai Jiao Tong, Tsinghua, UCLA, UC-Santa Barbara, USC, UT-Dallas, UECE Lisbon Meetings (2010), the 2011 AMMA, Decentralization, EBES, Stony Brook, WZB, and NBER Market Design Working Group Meeting for helpful discussions and comments, Ming Jiang, Malvika Deshmukh, Tyler Fisher, Robert Ketcham, Tracy Liu, Kai Ou and Ben Spulber for excellent research assistance. Financial support from the National Science Foundation through grants no. SES-0720943 and 0962492 is gratefully acknowledged. Chen: School of Information, University of Michigan, 105 South State Street, Ann Arbor, MI 48109-2112. Email: yanchen@umich.edu. Kesten: Tepper School of Business, Carnegie Mellon University, PA 15213. Email: okesten@andrew.cmu.edu.

> Confucius said, "Emperor Shun was a man of profound wisdom. [...] Shun considered the two extremes, but only implemented the moderate [policies] among the people." - *Moderation*, Chapter 6[1]

1 Introduction

School choice and college admissions have been among the most important and widely-debated education policies in various countries around the world. The past two decades have witnessed major innovations in this domain. In the United States, shortly after Abdulkadiroğlu and Sönmez (2003) was published, New York City high schools decided to replace its allocation mechanism with a capped version of the student-proposing deferred acceptance (DA) mechanism (Gale and Shapley 1962, Abdulkadiroğlu, Pathak and Roth 2005b). Concurrently, presented with theoretical analysis (Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006) and experimental evidence (Chen and Sönmez 2006) that one of the most popular school choice mechanisms, the Boston mechanism, is vulnerable to strategic manipulation, the Boston Public School Committee voted to replace the existing Boston school choice mechanism with the DA in 2005 (Abdulkadiroğlu, Pathak, Roth and Sönmez 2005a).

Like school choice in the United States, college admissions are among the most intensively debated public policies in the past thirty-five years in China. After the establishment of the People's Republic of China in 1949, Chinese universities continued to admit students via decentralized mechanisms. Historians identified two major problems with decentralized admissions during this time period. From the perspectives of the universities, as each student could be admitted into multiple universities, the enrollment to admissions ratio was low, ranging from 20% for ordinary universities to 75% among the best universities in 1949 (Yang 2006, p. 6). From the students' perspectives, however, after being rejected by the best universities, some qualified students missed the application and examination deadlines of ordinary universities and ended up not admitted by any university. To address these coordination problems, in 1950, 73 universities formed three regional alliances, with centralized admissions within each alliance. Based on the success of the alliances,[2] the Ministry of

[1] *Moderation* (*zhōng yōng*) is one of the four most influential classics in ancient Chinese philosophy. Emperor Shun, who ruled China from 2255 BC to 2195 BC, was considered one of the wisest emperors in Chinese history.

[2] This experiment achieved an improved average enrollment to admissions ratio of 50% for an ordinary

Education decided to transition to centralized matching by implementing the first National College Entrance Examination, also known as *gaokao*, in August 1952.

In recent years, each year roughly 10 million high school seniors compete for 6 million seats at various universities in China. The matching of students to universities has profound implications for the education and labor market outcomes of these students. For matching theorists and experimentalists, the regional variations of matching mechanisms and their evolution over time provide a wealth of field observations which can enrich our understanding of matching mechanisms (see Appendix A for a historical account of Chinese college admissions). This paper provides a systematic theoretical characterization and experimental investigation of the major Chinese college admissions (CCA) mechanisms.

The CCA mechanisms are centralized matching processes via standardized tests, with each province implementing an independent matching process. These matching mechanisms fall into three classes: sequential, parallel, and asymmetric parallel. The *sequential mechanism,* which until recently used to be the only mechanism used in Chinese student assignments both at the high school and college level, is what is often referred as the Boston mechanism in the school choice literature (Nie 2007b).[3] A common complaint about the sequential mechanism, one we are familiar with from school choice in the U.S., is that "a good score in the college entrance exam is worth less than a good strategy in the ranking of colleges" (Nie 2007a). In response to the college admissions reform survey conducted by the Beijing branch of the National Statistics Bureau in 2006, a parent complained (Nie 2007b):

> My child has been among the best students in his school and school district. He achieved a score of 632 in the college entrance exam last year. Unfortunately, he was not accepted by his first choice. After his first choice rejected him, his second and third choices were already full. My child had no choice but to repeat his senior year.

To alleviate the problem of high-scoring students not accepted by any universities and

university (Yang 2006, p. 7). The enrollment to admissions ratio for an ordinary university in 1952 was above 95%, a metric used by the Ministry of Education to justify the advantages of the centralized exam and admissions process (Yang 2006, p. 14).

[3] In China this mechanism is executed sequentially across tiers in decreasing prestige. In other words, each college belongs to a tier, and within each tier, the Boston mechanism is used. When assignments in the first tier are finalized, the assignment process in the second tier starts, and so on. In this paper, we do not explicitly model tiers or other restrictions students face in practice.

the general dissatisfaction with the sequential mechanism, the *parallel mechanism* was proposed by Zhenyi Wu, Director of Undergraduate Admissions at Tsinghua University from 1999 to 2002. Wu discussed the problems with the sequential mechanisms and outlined the parallel mechanism in interviews published in *Beijing Daily* (June 13, 2001), and *Guangming Daily* (July 26, 2001), respectively. In the parallel mechanism, students can place several "parallel" colleges for each choice-band. For example, a student's first choice-band can contain a set of three colleges, A, B, and C and the second choice-band can contain another set of three colleges, D, E, and F (both bands in decreasing desirability). Colleges then process student applications, where students gain priority for colleges they have listed in the first band over those who have listed the same college in the second band while assignments for the parallel colleges listed in the same band are temporary until all choices of that band have been considered. Thus, this mechanism lies between the sequential mechanism, where every choice is final, and the DA, where every choice is temporary until all seats are filled.

In 2001, Hunan became the first province to transition to the parallel mechanism in its tier 0 admissions, i.e., the admissions to military academies, which precedes the admissions to other four-year colleges. The results were viewed favorably by students and parents. In 2002, Hunan further allowed parallel choice-bands among tiers 2, 3 and 4 colleges. In 2003, Hunan implemented a full version of the mechanism, allowing 3 parallel colleges in the first choice-band, 5 in the second choice-band, 5 in the third choice-band, 5 in the fourth choice-band, and so on.[4] By 2012, the parallel mechanisms have been adopted by 28 out of 31 provinces.

In China, the *parallel mechanism* is widely perceived to improve allocation outcomes. For example, using survey and interview data from Shanghai in 2008, the first year when Shanghai adopted the parallel mechanism for college admissions, Hou, Zhang and Li (2009) find a 40.6% decrease in the number of students who refused to go to the universities they were matched with, compared to the year before when the sequential mechanism was in place.

[4]Information regarding the Hunan reform was obtained from two documents, *Constructing College Applicants' Highway towards Their Ideal Universities: Five years of Practice and Exploration of the Parallel Mechanism Implementation in Gaokao in Hunan* (2006), and *Summary of the Parallel Mechanism Implementation During the 2008 Gaokao in Hunan* (2008). The latter was circulated among the 2008 Ten-Province Collaborative Meeting of the Provincial Examination Institute Directors. We thank Tracy Liu and Wei Chi for sharing these documents and their interview notes with Guoqing Liu, Director of the Hunan Provincial Admissions Office in the early 2000s.

An interview with a parent in Beijing also underscores the incentives to manipulate the first choice under the sequential versus the parallel mechanisms:[5]

> My child really wanted to go to Tsinghua University. However, [...], in order not to take any risks, we unwillingly listed a less prestigious university as her first choice. Had Beijing allowed parallel colleges in the first choice[-band], we could at least give [Tsinghua] a try.

While variants of the parallel mechanisms, each of which differs in the number of parallel colleges for each choice-band, have been implemented in different provinces, to our knowledge, they have not been systematically studied theoretically or tested in the laboratory. In this paper, we ask two related questions. First, is there any validity to the widespread belief that the parallel mechanisms may better serve the interests of the students than the sequential mechanism? Second, when the number of parallel choices within a choice-band varies, how do manipulation incentives and stability properties change? We investigate these questions both theoretically and experimentally.

In our investigation, we use a more general priority structure than that used in the context of college admissions, as the transition from sequential to parallel mechanisms has happened not only in college admissions, but also in school choice in China. In the latter context, elementary school students applying for middle schools are prioritized based on their residence, whereas middle school students applying for high schools are prioritized based on their municipal-wide exam scores. In the context of school choice, similar manipulations under the sequential mechanism are documented and analyzed in He (2012) using school choice data from Beijing. To our knowledge, Shanghai was the first city to adopt the parallel mechanism for its high school admissions.[6]

To study the performance of the different mechanisms more formally, we first provide a theoretical analysis and present a parametric family of *application-rejection* mechanisms where each member is characterized by some positive number $e \in \{1, 2, \ldots, \infty\}$ of parallel and periodic choices through which the application and rejection process continues before assignments are finalized.

[5] Li Li. "Ten More Provinces Switch to Parallel College Admissions Mechanism This Year." *Beijing Evening News*, June 8, 2009.

[6] `http://edu.sina.com.cn/l/2003-05-15/42912.html`, retrieved on December 12, 2013.

As parameter e varies, we go from the sequential mechanism ($e = 1$) to the Chinese parallel mechanisms ($e \in [2, \infty)$), and from those to the DA ($e = \infty$). In this framework, we find that, as one moves from one extreme member of this family to the other, the experienced trade-offs are in terms of strategic immunity and stability.[7] We provide property-based rankings of the members of this family using some techniques recently developed by Pathak and Sönmez (2013). We show that whenever any given member can be manipulated by a student, any member with a smaller e number can also be manipulated but not vice versa (Theorems 1 & 3). In this sense, for example, the parallel mechanism used in Tibet ($e = 10$) is <u>less manipulable</u> than any other parallel or sequential mechanism currently in use. In fact, we find that all but three (Beijing, Gansu and Shangdong) of the provinces that adopted a parallel mechanism have transitioned to a less manipulable assignment system than the previously used sequential mechanism.

We also show that when $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$, any stable equilibrium of the application-rejection mechanism (e) is also a stable equlibrium of the application-rejection mechanism (e') but not vice versa (Theorems 2 & 4). In this sense, for example, the parallel mechanism used in Hainan ($e = 6$) is <u>more stable</u> than the version used in Jiangsu ($e = 3$).[8] Most remarkably, we find that every newly adopted parallel mechanism is more stable than the sequential mechanism it replaced.

Although it is well-known that the dominant strategy equilibrium outcome of the DA Pareto dominates any equilibrium outcome of the Boston mechanism (Ergin and Sönmez, 2006) which we refer as the sequential mechanism in this paper, we show that there is no clear dominance of the DA over a Chinese parallel mechanism (Proposition 4). Moreover, a parallel mechanism provides the students with a certain sense of "insurance" by allowing them to list their equilibrium assignments under the sequential mechanism as a safety option while listing more desirable options higher up in their preferences, which in turn leads to an outcome at least as good as that of the sequential mechanism for everyone (Proposition

[7] A mechanism is stable if the resulting matching is non-wasteful and there is no unmatched student-school pair (i, s) such that i would rather be assigned to school s where he has higher priority than at least one student currently assigned to it.

[8] Nie and Zhang (2009) investigate the theoretical properties of a variant of the parallel mechanism where each applicant has three parallel colleges, i.e., $e = 3$ in our notation, and characterize the equilibrium when applicant beliefs are i.i.d draws from a uniform distribution. Wei (2009) considers the parallel mechanism where each college has an exogenous minimum score threshold drawn from a uniform distribution. Under this scenario, she demonstrates that increasing the number of parallel options cannot make an applicant worse off.

5). Notably, such insurance does not come at any ex ante welfare cost in a stylized setting (Proposition 6).

The rest of this paper is organized as follows. Section 2 formally introduces the school choice problem and the family of mechanisms. Section 3 presents the theoretical results. Section 4 concludes.

2 School choice problem and the three mechanisms

A school choice problem (Abdulkadiroğlu and Sönmez 2003) is comprised of a number of students each of whom is to be assigned a seat at one of a number of schools. Further, each school has a maximum capacity, and the total number of seats in the schools is no less than the number of students. We denote the set of students by $I = \{i_1, i_2, \ldots, i_n\}$, where $n \geq 2$. A generic element in I is denoted by i. Likewise, we denote the set of schools by $S = \{s_1, s_2, \ldots, s_m\} \cup \{\emptyset\}$, where $m \geq 2$ and $\emptyset$ denotes a student's outside option, or the so-called null school. A generic element in S is denoted by s. Each school has a number of available seats. Let q_s be the number of available seats at school s, or the **quota** of s. Let $q_\emptyset = \infty$. For each school, there is a strict priority order of all students, and each student has strict preferences over all schools. The priority orders are determined according to state or local laws as well as certain criteria of school districts. We denote the priority order for school s by $\succ_s$, and the preferences of student i by P_i. Let R_i denote the at-least-as-good-as relation associated with P_i. Formally, we assume that R_i is a linear order, i.e., a complete, transitive, and anti-symmetric binary relation on S. That is, for any $s, s' \in S$, $s \; R_i \; s'$ if and only if $s = s'$ or $s \; P_i \; s'$. For convenience, we sometimes write $P_i : s_1, s_2, s_3, \ldots$ to denote that, for student i, school s_1 is his first choice, school s_2 his second choice, school s_3 his third choice, etc.

A **school choice problem**, or simply a problem, is a pair $(\succ = (\succ_s)_{s \in S}, P = (P_i)_{i \in I})$ consisting of a collection of priority orders and a preference profile. Let $\mathcal{R}$ be the set of all problems. A **matching** μ is a list of assignments such that each student is assigned to one school and the number of students assigned to a particular school does not exceed the quota of that school. Formally, it is a function $\mu : I \rightarrow S$ such that for each $s \in S$, $|\mu^{-1}(s)| \leq q_s$. Given $i \in I$, $\mu(i)$ denotes the assignment of student i at μ and given $s \in S$, $\mu^{-1}(s)$ denotes the set of students assigned to school s at μ. Let $\mathcal{M}$ be the set of all matchings. A matching μ is **non-wasteful** if no student prefers a school with unfilled quota to his assignment. Formally, for all $i \in I$, $s \; P_i \; \mu(i)$ implies $|\mu^{-1}(s)| = q_s$. A matching μ is **Pareto efficient** if there is no other matching which makes all students at least as well off and at least one student better off. Formally, there is no $\alpha \in \mathcal{M}$ such that $\alpha(i) \; R_i \; \mu(i)$ for all $i \in I$ and $\alpha(j) \; P_j \; \mu(j)$ for some $j \in I$.

A closely related problem to the school choice problem is the *college admissions problem* (Gale and Shapley 1962). In the college admissions problem, schools have preferences over students whereas in a school choice problem, schools are merely objects to be consumed. A key concept in college admissions is "stability," i.e., there is no unmatched student-school pair (i, s) such that student i prefers school s to his assignment, and school s either has not filled its quota or prefers student i to at least one student who is assigned to it. The natural counterpart of stability in our context is defined by Balinski and Sönmez (1999). The **priority of student** i **for school** s **is violated** at a given matching μ (or alternatively, student i justifiably envies student j for school s) if i would rather be assigned to s to which some student j who has lower $s-$priority than i, is assigned, i.e., $s \; P_i \; \mu(i)$ and $i \succ_s j$ for some $j \in I$. A matching is **stable** if it is non-wasteful and no student's priority for any school is violated.

A **school choice mechanism**, or simply a mechanism φ, is a systematic procedure that chooses a matching for each problem. Formally, it is a function $\varphi : \mathcal{R} \to \mathcal{M}$. Let $\varphi(\succ, P)$ denote the matching chosen by φ for problem $(\succ, P)$ and let $\varphi_i(\succ, P)$ denote the assignment of student i at this matching. A mechanism is Pareto efficient (stable) if it always selects Pareto efficient (stable) matchings. A mechanism φ is **strategy-proof** if it is a dominant strategy for each student to truthfully report his preferences. Formally, for every problem $(\succ, P)$, every student $i \in I$, and every report P'_i, $\varphi_i(\succ, P) \; R_i \; \varphi_i(\succ, P'_i, P_{-i})$.

Following Pathak and Sönmez (2013), a mechanism ϕ is *manipulable by student* j at problem $(\succ, P)$ if there exists P'_j such that $\phi_j(\succ, P'_j, P_{-j}) \; P_j \; \phi_j(\succ, P)$. Thus, mechanism ϕ is said to be *manipulable* at a problem $(\succ, P)$ if there exists some student j such that ϕ is manipulable by student j at $(\succ, P)$. Mechanism φ is **more manipulable** than mechanism ϕ if (i) at any problem ϕ is manipulable, φ is also manipulable; and (ii) the converse is not always true, i.e., there is at least one problem at which φ is manipulable but ϕ is not. Mechanism φ is **more stable** than mechanism ϕ if (i) at any problem ϕ is stable, φ is also stable; and (ii) the converse is not always true, i.e., there is at least one problem at which φ is stable but ϕ is not.[9]

We now describe the three mechanisms that are central to our study. The first two are the sequential and the DA mechanisms, while the third one is a stylized version of the simplest parallel mechanism.

2.1 The Sequential Mechanism

The sequential mechanism was the prevalent college admissions mechanism in China in the 1980s and 1990s. It is commonly referred as the Boston mechanism in the context of school choice. The outcome of the *sequential mechanism* can be calculated via the following algorithm for a given problem:

Step 1: For each school s, consider only those students who have listed it as their first choice. Up to q_s students among them with the highest $s-$priority are assigned to school s.

Step k, $k \geq 2$: Consider the remaining students. For each school s with q_s^k available seats, consider only those students who have listed it as their k-th choice. Those q_s^k students among them with the highest $s-$priority are assigned to school s.

The algorithm terminates when there are no students left. Importantly, note that the assignments in each step are final. Based on this feature, an important critique of the sequential mechanism highlighted in the literature is that it gives students strong incentives to misrepresent their preferences. Because a student who has high priority for a school may lose her priority advantage for that school if she does not list it as her first choice, the sequential mechanism forces students to make hard and risky strategic choices (see e.g., Abdulkadiroğlu and Sönmez 2003, Ergin and Sönmez 2006, Chen and Sönmez 2006, and He 2012).

2.2 Deferred Acceptance Mechanism (DA)

A second matching mechanism is the student-optimal stable mechanism (Gale and Shapley 1962), which finds the stable matching that is most favorable to each student. Its outcome

[9]See Kesten (2006 and 2011) for similar problem-wise property comparisons across and within mechanisms for matching problems.

can be calculated via the following *deferred acceptance (DA) algorithm* for a given problem:

Step 1: Each student applies to her favorite school. For each school s, up to q_s applicants who have the highest $s-$priority are tentatively assigned to school s. The remaining applicants are rejected.

Step k, $k \geq 2$: Each student rejected from a school at step $k-1$ applies to her next favorite school. For each school s, up to q_s students who have the highest $s-$priority among the new applicants and those tentatively on hold from an earlier step, are tentatively assigned to school s. The remaining applicants are rejected.

The algorithm terminates when each student is tentatively placed to a school. Note that, in the DA, assignments in each step are temporary until the last step. The DA has several desirable theoretical properties, most notably in terms of incentives and stability. Under the DA, it is a dominant strategy for students to state their true preferences (Roth 1982, Dubins and Freedman 1981). Furthermore, it is stable. Although it is not Pareto efficient, it is the most efficient among the stable school choice mechanisms.

In practice, the DA has been the leading mechanism for school choice reforms. For example, the DA has been adopted by New York City and Boston public school systems, which had suffered from congestion and incentive problems from their previous assignment systems, respectively (Abdulkadiroğlu et al. 2005a, Abdulkadiroğlu et al. 2005b).

2.3 The Chinese Parallel Mechanisms

As mentioned in the introduction, a Chinese parallel mechanism was first implemented in Hunan tier 0 college admissions in 2001. Later, it was adopted as a high school admissions mechanism in Shanghai in 2002. From 2001 to 2012, variants of the mechanism have been adopted by 28 provinces as the parallel college admissions mechanisms to replace the sequential mechanisms (Wu and Zhong 2012).

While there are many regional variations in CCA, from a game theoretic perspective, however, they differ in two main dimensions which impact the students' strategic decisions during the application process. The first dimension is the timing of preference submission, including before the exam (2 provinces), after the exam but *before* knowing the exam scores

(3 provinces), and *after* knowing the exam scores (26 provinces).[10] The second dimension is the actual matching mechanisms used in each province. The sequential mechanism used to be the only college admissions mechanism used in China. In 2012, while the sequential mechanism was still used in 2 provinces, variants of the parallel mechanisms have been adopted by 28 provinces, while the remaining province, Inner Mongolia, uses an admissions process which resembles a dynamic implementation of the parallel mechanism. A brief description of the evolution of Chinese college admissions mechanisms from 1949 to 2012 is contained in Appendix A.

In this study, we investigate the properties of the family of mechanisms used for Chinese school choice and college admissions. We now describe a stylized version of the Chinese parallel mechanisms in its simplest version, with two parallel choices per choice-band, adapted for the school choice context. A more general description is contained in Section 3.

- An application to the first ranked school is sent for each student.

- Throughout the allocation process, a school can hold no more applications than its quota.

  If a school receives more applications than its quota, it retains the students with the highest priority up to its quota and rejects the remaining students.

- Whenever a student is rejected from her first-ranked school, his application is sent to her second-ranked school. Whenever a student is rejected from her second-ranked school, he can no longer make an application in this round.

- Throughout each round, whenever a school receives new applications, these applications are considered together with the retained applications for that school. Among the retained and new applications, the ones with the highest priority up to the quota are retained.

[10] Zhong, Cheng and He (2004) demonstrate that, while there does not exist a Pareto ranking of the three variants in the preference submission timing, the first two mechanisms can sometimes achieve Pareto efficient outcomes. Furthermore, experimental studies confirm the *ex ante* efficiency advantage of the sequential mechanism with pre-exam preference ranking submissions in both small (Lien, Zheng and Zhong 2012) and large markets (Wang and Zhong 2012). Lastly, using a data set from Tsinghua University, Wu and Zhong (2012) find that, while students admitted under the sequential mechanism with pre-exam preference ranking submissions have on average lower entrance exam scores than those admitted under other mechanisms, they perform as well or even better in college than their counterparts admitted under other timing mechanisms.

- The allocation is finalized every two choices. That is, if a student is rejected by her first two two choices in the initial round, then he participates in a new round of applications together with other students who have also been rejected from their first two choices, and so on. At the end of each round the assigned students and the slots assigned to them are removed from the system.

The assignment process ends when no more applications can be rejected. We refer to this mechanism as the *Shanghai mechanism*.[11]

In the next section, we offer a formal definition of the parallel mechanisms and characterize the theoretical properties of this family of matching mechanisms.

3 Theoretical Analysis: A parametric family of mechanisms

In this section, we investigate the theoretical properties of a symmetric family of application-rejection mechanisms. Given student preferences, school priorities, and school quotas, consider the following parametric *application-rejection algorithm* that indexes each member of the family by a *permanency-execution period* e:

Round t $=0$**:**

- Each student applies to his first choice. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his e-th choice school, applies to his next choice. If a student has been rejected from all his first e choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

[11]In Appendix A, we provide a translation of an online Q&A about the Shanghai parallel mechanism used for middle school admissions to illustrate how the parallel choices work.

- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round, i.e., he has been rejected by all his first e choice schools. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

In general,

Round t ≥ 1**:**

- Each unassigned student from the previous round applies to his $te+1$-st choice school. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his $te+e$-th choice school, applies to his next choice. If a student has been rejected from all his first $te+e$ choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.
- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round, i.e., he has been rejected by all his first $te+e$ choice schools. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

The algorithm terminates when each student has been assigned to a school. At this point all the tentative assignments are final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* (e) and denoted by φ^e. This family of mechanisms nests the sequential and the DA mechanisms as extreme cases, the Chinese parallel mechanisms as intermediate cases, and the Chinese asymmetric parallel mechanisms as an extension (see Section 3.3).[12]

Remark 1 *The application-rejection mechanism* (e) *is equivalent to*

(i) the sequential mechanism when $e = 1$,

(ii) the Shanghai mechanism when $e = 2$,

(iii) the Chinese parallel mechanism when $2 \leq e < \infty$, *and*

(iv) the DA mechanism when $e = \infty$.

Remark 2 *It is easy to verify that all members of the family of application-rejection mechanisms, i.e.,* $e \in \{1, 2, \ldots, \infty\}$, *are non-wasteful. Hence, the outcome of an application-rejection mechanism is stable for a given problem if and only if it does not result in a priority violation.*

Next is our first observation about the properties of this family of mechanisms.

Proposition 1 *Within the family of application-rejection mechanisms, i.e.,* $e \in \{1, 2, \ldots, \infty\}$,

(i) there is exactly one member that is Pareto efficient. This is the sequential mechanism;

(ii) there is exactly one member that is strategy-proof. This is the DA mechanism; and

(iii) there is exactly one member that is stable. This is the DA mechanism.

All proofs and examples are relegated to Appendix B.

3.1 Property-specific comparisons of application-rejection mechanisms

As Proposition 1 shows, an application-rejection (e) mechanism is manipulable if $e < \infty$. Hence, when faced with a mechanism other than the DA, students should make careful judgments to determine their optimal strategies, and in particular, when deciding which e schools to list on top of their preference lists. More specifically, since priorities matter for determining the assignments only *within* a round and have no effect on the assignments of past rounds, getting assigned to one of the first e choices is extremely crucial for a student.

[12]From a modeling vantage point, our main analysis could have alternatively been based on the more general setting of Section 3.3. However, as will be seen subsequently, the main findings about both families of mechanisms are essentially driven by the number of choices considered within the initial round (rather than any other round); therefore we have adopted the simpler modeling approach to facilitate the exposition and illustration of ideas.

When $e < \infty$, a successful strategy for a student is one that ensures that he is assigned to his "target school" at the end of the initial round, i.e., round 0. In this sense, missing out on the first choice in the sequential mechanism could be more costly to a student than in a Chinese parallel mechanism such as the Shanghai, which offers a "second chance" to the student before he loses his priority advantage. On the other hand, at the other extreme of this class lies the DA, which completely eliminates any possible loss of priority advantage for a student. The three-way tension among incentives, stability, and welfare that emerges under this class is rooted in this observation.

We next provide an incentive-based ranking of the family of application-rejection mechanisms.

Theorem 1 (Manipulability) *For any e, φ^e is more manipulable than $\varphi^{e'}$ where $e' > e$.*

In Appendix B, we offer two examples before the proof of Theorem 1. Example 1a shows that the sequential mechanism is manipulable when the Shanghai mechanism is, whereas Example 1b shows that the Shanghai mechanism is not manipulable when the sequential mechanism is.

Corollary 1 *Among application-rejection mechanisms, the sequential is the most manipulable and the DA is the least manipulable member.*

Corollary 2 *Any Nash equilibrium of the preference revelation game associated with φ^e is also a Nash equilibrium of that of $\varphi^{e'}$ where $e' > e$.*

Remark 3 *Notwithstanding the manipulability of all application-rejection mechanisms except the DA, it is still in the best interest of each student to report his within-round choices in their true order. More precisely, for a student facing φ^e, any strategy that does not list the first e choices, that are considered in the initial round, in their true order, is dominated by the otherwise identical strategy that lists them in their true order. Similarly, not listing a set of e choices considered in a subsequent round is also dominated by an otherwise identical strategy that lists them in their true order.*

Corollary 2 says that the set of Nash equilibrium strategies corresponding to the preference revelation games associated with members of the application-rejection family has a nested structure.[13] A useful interpretation is that when making problemwise comparisons across the members of the application-rejection family (e.g., see Proposition 2), such comparisons might as well be made across equilibria of two different members.

We now turn to investigate a possible ranking of the members of the family based on stability. An immediate observation is that under an application-rejection (e) mechanism, no student's priority for one of his first e choices is ever violated. This is simply because all previous assignments are tentative in the application-rejection algorithm until the student is rejected from all his first e choices. This observation hints that one might expect mechanisms to become more stable as parameter e grows. The next result shows that this may not always be the case.

Proposition 2 (Stability) *Let $e' > e$.*

(i) If $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$, then $\varphi^{e'}$ is more stable than φ^e.

(ii) *If $e' \neq ke$ for any $k \in \mathbb{N} \cup \{\infty\}$, then $\varphi^{e'}$ is not more stable than φ^e.*

Corollary 3 *The DA is more stable than the Shanghai mechanism, which is more stable than the sequential mechanism.*

Corollary 4 *Any other (symmetric) application-rejection mechanism is more stable than the sequential mechanism.*

Proposition 2 indicates that while it is possible to rank all three special members of the family of application-rejection mechanisms, i.e., $e \in \{1, 2, \infty\}$, according to the stability of their outcomes, within the Chinese parallel mechanisms, however, there may not be a problemwise systematic ranking in general. Nevertheless, if the number of choices considered in each round by one mechanism is a multiple of that of the other mechanism, in this case the mechanism that allows for more choices is the more stable one. Proposition 2 coupled with Theorem 1 allows us to compare stability properties of certain members across equilibria.

[13]A similar observation is made by Haeringer and Klijn (2008) for the revelation games under the Boston mechanism when the number of school choices a student can make (in her preference list) is limited by a quota.

Theorem 2 (Stable Equilibria) *Let $e' = ke$ for some $k \in \mathbb{N} \cup \{\infty\}$. Any equilibrium of φ^e that leads to a stable matching is also an equilibrium of $\varphi^{e'}$ and leads to the same stable matching. However, the converse is not true, i.e., there are stable equilibria of $\varphi^{e'}$ that may not be equilibrium nor stable under φ^e.*

Theorem 2 shows that the set of stable equilibrium profiles (i.e., the equilibrium profiles that lead to a stable matching under students' true preferences) for an application-rejection mechanism φ^e is (strictly) smaller than that of $\varphi^{e'}$ whenever e' is a multiple of e. This implies, for example, that the Shanghai mechanism admits a larger set of stable equilibrium profiles than the sequential mechanism.

A common, albeit questionable, metric often used by practitioners as a measure of students satisfaction is based on considering the number of students assigned to their first choices.[14] As it turns out, the sequential is the most generous in terms of first choice accommodation, whereas the DA is the least.

Proposition 3 (Choice accommodation) *Within the class of application-rejection mechanisms,*

(i) *φ^e assigns a higher number of students to their first choices than $\varphi^{e'}$ where $e < e'$.*

(ii) *φ^e assigns a higher number of students to their first e choices than $\varphi^{e'}$ where $e \neq e'$.*

Corollary 5 *Within the class of application-rejection mechanisms, the sequential mechanism maximizes the number of students receiving their first choices.*

Corollary 6 *Within the class of application-rejection mechanisms, the Shanghai mechanism maximizes the number of students receiving their first or second choices.*

Nonetheless, one needs to be cautious when interpreting Proposition 3. Since all members of the family with the exception of the DA violate strategy-proofness, student preference submission strategies may also vary across mechanisms and the reported preferences

[14]For example, in evaluating the outcome of the Boston mechanism, Cookson Jr. (1994) reports that 75% of all students entering the Cambridge public school system at the K-8 levels gained admission to the school of their first choice. Similarly, the analysis of the Boston and NYC school district data by Abdulkadiroğlu, Pathak, Roth and Sönmez (2006) and Abdulkadiroğlu, Pathak and Roth (2009) also report the number of first choices of students.

may not represent students' true choices. To address this issue, in the next section we turn to investigate the properties of Nash equilibrium outcomes of the family of application-rejection mechanisms.

3.2 Equilibria of the Induced Preference Revelation Games: Ex post equilibria

Ergin and Sönmez (2006) show that every Nash equilibrium outcome of the preference revelation game induced by the sequential mechanism leads to a stable matching under students' true preferences, and that any given stable matching can be supported as a Nash equilibrium of this game. This result has a clear implication. Since the DA is strategy-proof and chooses the most favorable stable matching for students, the sequential mechanism can *at best* be as good as the DA in terms of the resulting welfare. Put differently, there is a clear welfare loss associated with the sequential mechanism relative to the DA.

To analyze the properties of the equilibrium outcomes of the application-rejection mechanisms, we next study the Nash equilibrium outcomes induced by the preference revelation games under this family of mechanisms. It turns out that the DA does not generate a clear welfare gain relative to the Chinese parallel mechanisms.

Proposition 4 (Ex post equilibria) *Consider the preference revelation game induced by φ^e under complete information.*

(i) If $e = 1$, then for every problem every Nash equilibrium outcome of this game is stable and thus it is Pareto dominated by the DA under the true preferences.

(ii) If $e \notin \{1, \infty\}$, there exist problems where the Nash equilibrium outcomes, in undominated strategies, of this game are unstable and Pareto dominate the DA under the true preferences.[15]

Proposition 4 shows that the welfare comparison between the equilibria of the DA and the Chinese parallel mechanisms is ambiguous. On the other hand, the fact that both the sequential and parallel mechanisms admit multiple equilibria, precludes a direct equilibrium-wise comparison between the two mechanisms. Nevertheless, a curious question at this

[15]Note that the DA also admits Nash equilibria that lead to unstable matchings that Pareto dominate the DA outcome under the true preferences. However, any such equilibria necessarily involves a dominated strategy.

point is then whether there could be any validity to the widespread belief (also expressed in a quote in the introduction) that the parallel mechanisms may better serve the interests of students than the sequential mechanism. The next result provides a formal sense in which a parallel mechanism may indeed be more favorable for each student relative to the sequential mechanism.

Proposition 5 (Insurance under the Parallel Mechanisms) *Let μ be any equilibrium outcome under the sequential mechanism. Under φ^e if each student i lists $\mu(i)$ as one of his first e choices and any schools he truly likes better than $\mu(i)$ as higher-ranked choices, then each student's assignment is at least as good as that under the sequential mechanism.*[16]

Remark 4 *It is worth emphasizing that Proposition 5 does not generalize to any two application-rejection mechanisms as this result crucially hinges on part (i) of Proposition 4. For example, let μ be an equilibrium outcome of the Shanghai. If each student lists his assignment at μ as one of his first e choices similarly to the above, then the resulting outcome of φ^e with any $e > 2$ need not be weakly preferred to that of Shanghai by each student.*[17]

From a practical point of view, Proposition 5 says that whatever school a student is "targeting" under the sequential mechanism, he would be at least as well off under a parallel mechanism by simply including it among his first e choices while ranking better options higher up in his preferences, provided that other students are doing the same. In other words, the Chinese parallel mechanisms may allow students to retain their would-be assignments under the sequential mechanism as "insurance" options while keeping more desirable options within reach. Practitioners seem to understand this aspect of the parallel mechanism. For example, the official Tibetan gaokao website starts with the following introduction to its admissions mechanism:

> To reduce the risks applicants bear when submitting their college rank order lists, and to reduce the applicants' psychological pressure, the Tibet Au-

[16] We stipulate that the e-th choice is the last choice when $e = \infty$. For expositional simplicity, we also assume that student i has $e - 1$ truly better choices than $\mu(i)$.

[17] To illustrate this point for the Shanghai vs. the DA, for example, let μ correspond to an unstable equilibrium outcome that Pareto dominates the DA matching under truthtelling.

tonomous Region [will] implement the parallel mechanism among ordinary colleges in 2012."[18]

An alternative interpretation of Proposition 5 concerns the level of coordination among students. Let μ^{DA} be the DA outcome under a given profile of students' true preferences. This is indeed also an equilibrium outcome of the sequential mechanism for a profile of reports where each student lists his DA assignment as his first choice. Nevertheless, as our experimental analysis also confirms, in general it is unlikely to expect to observe students coordinating on one such equilibrium. In practice, the use of such strategies may as well entail potentially large costs for students in cases of miscoordination. Proposition 5 suggests that if each student includes his DA assignment among his first e choices under φ^e and is otherwise truthful about the choices he declares to be more desirable, he will be guaranteed an assignment no worse than that he would be getting under the DA. Notably, this conclusion does not depend on whether or not the profile of student reports constitutes an equilibrium of φ^e : the outcome of φ^e always Pareto dominates that of the DA. Interestingly, if such a profile is a disequilibrium under φ^e, then the outcome of φ^e strictly Pareto dominates that of the DA, making at least one student strictly better off under φ^e in comparison to the DA. In this sense, one can argue that the Chinese parallel mechanisms may do a better job relative to the sequential mechanism by facilitating coordination on desirable outcomes and may help reduce the high costs of miscoordination under the sequential mechanism. In particular, the higher the e parameter, the easier it becomes for students to include their DA assignments among the first e choices. In the extreme case of $e = \infty$, the DA assignment is necessarily one of the e choice of each student and the resulting outcome is that of the DA itself, which is also an equilibrium.

Abdulkadiroğlu, Che and Yasuda (2011) [henceforth, ACY] study an incomplete information model of school choice that captures two salient features from practice: correlated ordinal preferences and coarse school priorities. More specifically, they consider a highly special setting where students share the same ordinal preferences but different and unknown cardinal preferences and schools have no priorities, i.e., priorities are determined via a random lottery draw after students submit preference rankings. Nonetheless, the DA outcome

[18] `http://gaokao.chsi.com.cn/gkxx/ss/201201/20120130/278496141-3.html`, accessed on January 9, 2014.

coincides with a purely random allocation in this stylized setting.[19] ACY focus on the symmetric Bayesian Nash equilibria under the sequential mechanism and show that every student is at least weakly better off in any such equilibrium than under the DA. This result suggests that there may be a welfare loss to every student under the DA relative to the sequential mechanism in such circumstances.[20]

We next investigate whether or not the *ex ante* dominance of the sequential mechanism in this restricted setting prevails when compared with a Chinese parallel mechanism.[21] It turns out the answer is negative.

Proposition 6 (Ex ante equilibrium) *In the Bayesian setting of ACY (see the Appendix B for a formal treatment),*

(i) each student is weakly better off in any symmetric equilibrium of the Shanghai than she is in the DA, and

(ii) no ex ante Pareto ranking can be made between the sequential mechanism and the Shanghai, i.e., there exists problems where some student types are weakly better off at the equilibrium under the Shanghai than they are under the sequential mechanism and vice versa.

Part (i) says that just like the sequential mechanism, the Shanghai mechanism also leads to a clear welfare gain over the DA in the same setting. This shows that in special settings, just like the sequential mechanism, the Shanghai mechanism may also allow students to communicate their preference intensities. Part (ii) shows the non-dominance of the sequential mechanism over the Shanghai in the same Bayesian setting.[22]

[19]Since this model assumes no priorities, any stable mechanism always induces an equal weighted lottery over all feasible allocations. In this restricted setting, the DA and the well-known top trading cycles mechanisms (Abdulkadiroğlu and Sönmez (2003)) both coincide with a random serial dictatorship mechanism.

[20]However, this finding is not robust to changes in the priority structure. Indeed, Troyan (2012) shows that when school priorities are introduced into the same setting, Boston no longer dominates DA in terms of ex ante welfare.

[21]As noted earlier, out of the 31 provinces in China, two of them, Beijing and Shanghai, require students to submit preference rankings before taking the college entrance exam.

[22]The reason why some students may prefer the Shanghai to the Boston, unlike the case against the DA, as in this example, can be intuitively explained as follows. Under the Boston mechanism, students' first choices are crucial and thus students target a single school at equilibrium. Under the Shanghai mechanism, the first two choices are crucial and students target a pair of schools. This difference, however, may enable a student to guarantee a seat at an unpopular school under the Shanghai by ranking it as his second choice and still give him some chance to obtain a more preferred school by ranking it as his first choice. See the Appendix B for a more thorough illustration through an example.

3.3 The Asymmetric Class of Chinese Parallel Mechanisms

Thus far our benchmark analysis has focused on the symmetric Chinese parallel mechanisms where the same number of student choices are considered periodically, i.e., the parameter e has been constant across rounds. In fact, in 15 Chinese provinces, the college admission mechanisms allow for variations in the number of choices that are considered within a round. For example, in Hebei province, the number of parallel choices are set to $e = 5, 1, 6, 1, 6$ in 2012. Table 1 in Appendix A provides the complete list of choice sequences used in various provinces across China in 2012.

We first slightly augment the application-rejection family to accommodate for the asymmetric class. Given a problem, let φ^S denote the *application-rejection mechanism* that is associated with a choice sequence $S = (e_0, e_1, e_2, \ldots)$, where the terms in the sequence respectively denote the number of choices to be tentatively considered in each round. (See Appendix B for a more precise description.)

We next investigate the incentive and stability properties within the asymmetric class of Chinese parallel mechanisms.

Theorem 3 (Manipulability of the Asymmetric Class) *An application-rejection mechanism associated with a choice sequence $S = (e_0, e_1, e_2, \ldots)$ is more manipulable than any application-rejection mechanism associated with a choice sequence $S' = (e'_0, e'_1, e'_2, \ldots)$ where $e_0 < e'_0$.*

Theorem 3 says that a mechanism using a choice sequence of fewer number of parallel colleges in the initial round is more manipulable than a corresponding asymmetric parallel mechanism with a greater number of such parallel colleges. This result in turn underscores the importance of the initial round relative to all other rounds, a point much emphasized in the previous literature in the context of the sequential mechanism.[23] Using Theorem 3 we obtain the following complete manipulability ranking among the CCA mechanisms.

Corollary 7 *The following is the manipulability order of mechanisms in various provinces of China, starting with those that are most manipulable: $\{$Heilongjiang, Qinghai, Shandong, Gansu, Beijing$\}$ $>$ $\{$Guangdong, Jiangsu, Liaoning$\}$ $>$ $\{$Anhui, Shǎnxi, Guangxi,*

[23] Intuitively, the reason why the ranking depends only on the number of parallel choices of the initial round is because manipulations that happen in subsequent rounds can always be "translated" to the initial round by including the target school among the parallel choices of the initial round. Consequently, the number of choices in subsequent rounds do not matter for manipulability.

Jiangxi, Fujian, Ningxia, Shanghai, Xinjiang$\}$ $>$ $\{$*Sichuan, Hebei, Hubei, Shānxi, Hunan, Zhejiang, Guizhou, Yunan, Jilin, Tianjin*$\}$ $>$ $\{$*Hainan, Henan, Chongqing*$\}$ $>$ *Tibet.*

In the above ranking, Tibet stands out as the home to the least manipulable parallel mechanism, whereas Heilongjiang, Qinghai, Shandong, Gansu, and Beijing lie at the other end of the spectrum although the latter three have partially moved away from the sequential mechanism that is still in use in the former two. Before we turn to investigate the stability properties of the asymmetric class, a useful definition is in order.

Definition 4 *A choice sequence $S = (e_0, e_1, e_2, \ldots)$ is an* <u>*additive decomposition*</u> *of another choice sequence $S' = (e'_0, e'_1, e'_2, \ldots)$ if and only if there exist indexes $t_0 < t_1 < \cdots < t_k < \cdots$ such that*

$$e'_0 = \sum_{i=0}^{t_0} e_i;\ e'_1 = \sum_{i=t_0+1}^{t_1} e_i; \cdots ; e'_k = \sum_{i=t_{k-1}+1}^{t_k} e_i, \cdots, \textit{etc.}$$

In words, if the sequence S is an additive decomposition of the sequence S', then it possible to write each term in S' as a sum of distinct but consecutive terms in S starting with the first term and following the order of the indexes. For example, observe that the sequence corresponding to the sequential mechanism, represented by $S^{SEQ} = (1, 1, 1, \ldots)$, is an additive decomposition of the Shanghai sequence, represented by $S^{SH} = (2, 2, 2, \ldots)$. In fact, any sequence can be obtained from the sequential sequence.

Remark 5 *The sequence corresponding to the sequential mechanism is an additive decomposition of any sequence corresponding to any symmetric or asymmetric member of the application-rejection family.*

The next result, which is an analogue of Proposition 2, shows that any two members of the application-rejection family represented by sequences that are comparable according to additive decomposition are also comparable according to their stability properties.

Proposition 7 (Stability of the Asymmetric Class) *Let φ^S and $\varphi^{S'}$ be two application-rejection mechanisms, represented by the choice sequences S and S', respectively.*

(i) If S is an additive decomposition of S', then $\varphi^{S'}$ is more stable than φ^S.

(ii) If S is not an additive decomposition of S', then $\varphi^{S'}$ is not more stable than φ^{S}.

Proposition 7 has a remarkable implication. In all the provinces where the sequential mechanism was abandoned, all the successors are more stable mechanisms.

Corollary 8 *All CCA mechanisms that replaced the sequential mechanism are more stable than the sequential mechanism.*

Proposition 7 also enables us to obtain cross-province stability comparisons among some of the parallel mechanisms currently in use.

Corollary 9 *The following are the stability rankings among some of the parallel mechanisms that are being used in various provinces.*

- *Sichuan and Shānxi are more stable than Shandong.*
- *Anhui, Shǎnxi, Guangxi, Jiangxi and Ningxia are more stable than Gansu and Beijing.*
- *Tibet is more stable than Hebei, Hunan, Zhejang, Tianjin, Yunan and Guizhou.*
- *Hainan is more stable than Jiangsu.*

The following analogue of Theorem 2 obtained from Proposition 7 coupled with Theorem 3 allows us to compare stability properties across equilibria and is applicable to all the comparisons given in Corollary 9.

Theorem 4 (Stable Equilibria of the Asymmetric Class) *Let φ^{S} and $\varphi^{S'}$ be two application-rejection mechanisms, respectively represented by choice sequences S and S', where S is an additive decomposition of S' and $e_0 < e'_0$. Any equilibrium of φ^{S} that leads to a stable matching is also an equilibrium of $\varphi^{S'}$ and leads to the same stable matching. However, the converse is not true, i.e., there are stable equilibria of $\varphi^{S'}$ that may not be equilibrium nor stable under φ^{S}.*

4 Conclusions

School choice and college admissions have profound implications for the education and labor market outcomes of the students involved in these processes worldwide. Whereas much of the debate on school choice in the literature exclusively focused on the Boston vs. DA comparisons, in this paper we synthesize these well-known mechanisms with those used for school choice and college admissions in China, and characterize them as members of a family of application-rejection mechanisms, with the Boston and the DA being special cases. A key insight is that the Chinese parallel mechanism used for school choice, and for college admissions in 28 provinces in China bridges the well studied Boston and the DA mechanisms.

Our theoretical analysis indicates a systematic change in the incentive and stability properties of this family of mechanisms as one goes from one extreme member to the other. We also see that the Nash equilibrium strategies corresponding to the induced preference revelation games associated with members of the application-rejection family are nested. Although the DA has been shown to dominate the equilibria of the sequential mechanism under complete information, no such conclusion holds relative to the parallel mechanism.

We show that all of the Chinese provinces that have moved away from the sequential mechanism have moved towards less manipulable and more stable mechanisms. Furthermore, any student can ensure that she does at least as well under the parallel mechanism as under the sequential mechanism. Unlike with the DA, such insurance does not entail any ex ante welfare cost since the parallel mechanism also allows students to communicate their preference intensities more efficiently relative to the DA.

To test our theoretical predictions and to search for behavioral regularities where theory is silent, we conduct laboratory experiments in two environments differentiated by their complexity. In the laboratory, participants are most likely to reveal their preferences truthfully under the DA mechanism, followed by the parallel and then the sequential mechanisms. Furthermore, while the DA is significantly more stable than the parallel mechanism, which is more stable than the sequential mechanism, efficiency comparisons vary across environments. Whereas theory is silent about equilibrium selection, we find that stable Nash equilibrium outcomes are more likely to arise than unstable ones.

Our study represents the first systematic theoretical and experimental investigation of the Chinese parallel mechanisms. The analysis yields valuable insights which enable us to treat this class of mechanisms as a family, and systematically study their properties and performance. More importantly, our results have policy implications for school choice and college admissions. As the parallel mechanism is less manipulable than the sequential mechanism, and its achieved efficiency is robustly sandwiched between the two extremes whose efficiency varies with the environment, it might be a less radical replacement for the Boston mechanism compared to the DA.

While variants of the parallel mechanism have been implemented in different provinces to replace the sequential mechanism since 2001, the choice of the number of parallel colleges (e) is likely to be set for reasons other than game-theoretic or welfare reasons. In Hunan, for example, Guoqing Liu, Director of the Hunan Provincial Admissions Office during the early 2000s, explained that the reason they set the number of parallel colleges for the first choice-band to three was because they found three "1" looking symbols, i.e., the Arabic number "1," the Roman numeral "I," and the English letter "l" (*elle*). He conjectured that this listing would make each of the three parallel colleges perceive that they were ranked number one, despite the decreased desirability from 1 to l.[24] Our study provides the first theoretical analysis on the effects of the number of parallel colleges on the incentives and stability of these mechanisms. Of the variants of the parallel mechanisms adopted since 2001, our analysis indicates that the parallel mechanism implemented in Tibet ($e = 10$) is the least manipulable one, whereas the partial reforms adopted in Beijing, Gansu and Shangdong are the most manipulable ones.

Appendix A: Evolution of the Chinese College Admissions Mechanisms (For Online Publication)

In this Appendix, we present the evolution of the Chinese College Admissions mechanisms from 1949 to 2012. In summarizing its main variations, we rely primarily on several books written by educators, policy-makers and historians. In particular, Yang (2006) provides the historical and political contexts of Chinese college admissions from 1949 to 1999. Liu (2009) reports the policy debates surrounding college admissions reforms up to 2009, including survey data around some major policy reforms. In comparison, Qiu and Zhao (2011) offer practical advice for high school seniors and their parents on recent admission statistics of each university, the admissions mechanisms, and application strategies. While Chinese

[24]We are grateful to Tracy Xiao Liu and Wei Chi for sharing their interview notes with Guoqing Liu (August 14, 2013).

college admissions have been traditionally studied by educators, Chinese economists recently started to analyze their game-theoretic properties. We reference most of the latter in the main text of this paper. As matching mechanisms in historical documents are not described in game-theoretic terms, we provide the translation of the relevant paragraphs and our own interpretation in game-theoretic terms.

For more up to date information on college admissions rules and policies in various provinces, we refer the reader to the official Ministry of Education website on college admissions, `http://gaokao.chsi.com.cn/`.[25]

4.1 From Decentralized to Centralized Examinations and Admissions (1952 - 1957)

After the establishment of the People's Republic of China in 1949, Chinese universities continued to admit students via decentralized mechanisms, i.e., each university administered its own entrance exams and admissions processes. In 1950, there were 227 universities and colleges, with 134,000 students (Yang 2006, p. 5).[26] Historians identified two major problems with decentralized admissions during this time period. From the perspectives of the universities, as each student could be admitted into multiple universities, the enrollment to admissions ratio was low, ranging from 20% for some ordinary universities to 75% among the best universities in 1949 (Yang 2006, p. 6). Therefore, many ordinary universities could not fill their first-year classes. From the students' perspectives, however, after being rejected by the best universities, some qualified students missed the application and examination deadlines of ordinary universities and ended up not admitted by any university. To address these coordination problems, in 1950, 73 universities formed three regional alliances, with centralized admissions within each alliance. This experiment achieved an improved average enrollment to admissions ratio of 50% for an ordinary university (Yang 2006, p. 7).

Based on the success of the alliances, the Ministry of Education decided to transition to centralized matching in 1952 by implementing the first National College Entrance Examination, also known as *gaokao*, in August 1952.[27] The exam consisted of eight subjects (math, physics, chemistry, biology, foreign language, history and geography, politics, and

[25] This website has remained stable at least since 2006. We last accessed it on December 12, 2013.

[26] In reporting statistics, we exclude universities in Taiwan, Hong Kong and Macau. Also note that Chinese sources prior to 1977 typically report statistics in units of ten thousand (*wàn*).

[27] Using a national examination to select talent for various government positions had been a long tradition in China, dating back to 605 A.D. (Liu 2009, p. 2).

Chinese), and lasted for three consecutive days, a format that more or less persisted to 2012, with various adjustments on the content of the exam. The enrollment to admissions ratio for an ordinary university in 1952 was above 95%, a metric used by the Ministry of Education to justify the advantages of the centralized exam and admissions process (Yang 2006, p. 14).

Between 1952 and 1957, the Ministry of Education made several adjustments to the centralized admissions process. First, minority-serving institutions, fine arts and music institutions were allowed to include institution-specific admissions processes in addition to gaokao, such as interviews, auditions and portfolio presentations. Second, the single-track gaokao evolved into two tracks in 1954, and three tracks in 1955. The three tracks included the science and engineering track, the medicine, biology and agriculture track, and the humanities and social sciences track. The first two tracks were recombined into a single track in 1964, forming the present-day two-track exam system. Lastly, *key universities*, such as Peking (Beijing), Tsinghua, and Jiaotong, were allowed to recruit nationwide, while *ordinary universities* were restricted to recruit within their respective province, which created the *tier system* among universities.[28]

From a game-theoretic perspective, the centralized admissions mechanism used during this time period, "Exam-Score Based Admissions" (*fēn jí lù qǔ*), resembled a serial dictatorship mechanism. "[Admissions] should proceed in decreasing exam scores, starting with the highest score, and proceeding to the next score after [the admission of the student with] the highest score is finished. For each student, proceed based on the student's preference ranking. That is, send the student's application to his first choice. If that university decides to admit the student, it keeps his application file and marks 'Admitted' in the Admission Results column. If the university decides not to admit the student or if its quota is full, it should mark 'Not Admitted' on the student's application, and pass his file to his second-choice university (with the same process as described above). And so on." (Yang 2006, p. 76-77)

The transition from decentralized to centralized matching was designed to alleviate co-

[28]According to Weidong Liu (interviewed by Yan Chen on August 9, 2013), historically, tiers were created as a result of the manual admissions process. During the admissions process, each university sent 4-5 admissions officers to each province. Typically all admissions officers stayed at the same hotel to finish the admissions process. A province could not accommodate all university admissions officers concurrently in the same hotel because of limited hotel capacities, therefore, they dealt with one tier at a time. From the students' perspective, however, the tier system reduces the risks associated with the sequential mechanism.

ordination failure and excess demand. In 1956, for example, universities had a target of admitting 165,500 students, whereas 156,000 students graduated from high school that year (Yang 2006, p. 40). By encouraging cadres from workplaces to apply for colleges,[29] the situation changed in 1957, with a target of admitting 120,000 students and 199,000 applicants (Yang 2006, p. 45). After a nation-wide debate of whether to go back to the decentralized admissions processes, used in the Soviet Union at the time, the Ministry of Education decided to continue the centralized admissions processes, mainly based on its advantages of better coordination and lower transaction costs, i.e., students did not have to participate in multiple exams administered by different universities. It appears that, after a national exam, separate admissions processes within each province was established after the 1957 debate.

4.2 The Leftists' Attacks on College Admissions (1958 - 1965)

Since 1958, gaokao had been scrutinized and attacked by the leftists in the Communist Party, on its intellectual focus and its lack of communist ideology. In response, the Ministry of Education stepped up the screening of student political backgrounds in the admissions process, and implemented the Guaranteed Admissions of cadres from proletariat families who went through the Crash Training Schools for Workers and Farmers. Prior to 1958, the cadres were required to take gaokao and go through the same admissions process after bonus points were added to their scores. In contrast, they were exempt from gaokao since 1958 (Yang 2006, p. 91). The admissions rate was a staggering 97% in 1958 (Yang 2006, p. 139).

To our knowledge, the first documented tiered admissions appeared in 1959. "Admissions of new students should proceed in tiers. National key universities admit students first." The second tier included provincial and ministry-level key universities, whereas the third tier included all other universities and colleges (Yang 2006, p. 104).

After the Great Leap Forward (1958 - 1961) ended in a disastrous famine, in 1962, college admissions rate reached its lowest point prior to the Cultural Revolution, 24%, with 107,000 students admitted among 440,000 applicants.

In 1963, it appeared that the college admissions mechanism transitioned from a serial dictatorship into a hybrid of serial dictatorship and priority matching mechanism, "Exam-Score Interval Based Admissions" (*fēn duàn lù qǔ*). Average exam scores were chunked

[29] Affirmative action, in the form of adding 10-15 points per subject (out of a 100-point scale), was implemented in 1954 to increase the number of cadres in universities (Yang 2006, p. 55).

into (typically) five-point intervals (*duàn*), e.g., [80, 100], [75, 79], [70, 74], [65, 69], etc. Admissions proceeded sequentially from the highest interval downward, clearing one interval before starting the next (*duàn duàn qīng*). Within an interval, admissions proceeded in the order of student preference ranking of universities and exam scores (Yang 2006, p. 135-136). Under this mechanism, each student could apply for five national key universities. Within each university, he could apply for three different departments. Admission decisions were made by each university. This mechanism was designed to reduce the disparity of student qualities between different departments within a university (Yang 2006, p. 150).

Meanwhile, because of the increased competitiveness, some students considered that "gaokao is a battle that determines your fate: one point [difference] in gaokao can determine whether you go to heaven [i.e., universities] or hell [i.e., becoming a farmer]" (Yang 2006, p. 171), which underscores the importance of gaokao in labor market outcomes. Until recently, labor market mobility had been constrained by the Household Registration (*hù kǒu*) system. For millions of youths from rural areas, gaokao offered the only way of breaking away from a life time on the farms.

4.3 Demise of Gaokao During the Cultural Revolution (1966-1976)

The year 1966 marked the start of the ten-year Cultural Revolution, and the abolition of gaokao. In its place, farmers, workers and soldiers who had the equivalence of a high school education could be recommended to go to universities. The political turmoil dictated that none of the universities recruited new students for the subsequent six years. From 1972 to 1976, university education resumed based on a recommendation system. Students had to have completed at least two years of real-life work experience, i.e., having worked on farms, in factories or served in the armed forces, to be eligible. The recommendation system opened the door for rampant corruption in college admissions during this time period.

4.4 College Admissions Reform (1977 - 2012)

With the end of the Cultural Revolution in October 1976, gaokao resumed in 1977. As a result, 5.7 million applicants participated in gaokao, including many from the ten-year backlog of high school graduates together with the class of 1977, with 4.8% of all applicants admitted into universities. In 1977, each province wrote its own exams and administered its own admissions process. Starting 1978, gaokao again became a national exam, written by the Ministry of Education. A record 6.1 million students participated in the 1978 gaokao, with admissions rate again at 4.8%. To further curb corruption, every applicant's score was publicly posted.[30] Compared with gaokao before the Cultural Revolution, where the average admissions rate was 55.92%, the average admissions rate between 1977 and 1982 was 6.05% (Yang 2006, p. 278), indicating a much more competitive process.

While the hybrid serial dictatorship and priority matching mechanism, "Exam-Score Interval Based Admissions," continued to be used till 1984, to grant more autonomy to individual universities, starting from 1985, it was gradually replaced by a priority matching mechanism, which resembled the Boston mechanism with tiers (Yang 2006, p 314-315; Liu 2009, p. 41). Using this mechanism, based on the distribution of gaokao exam scores, the number of applicants who list it as their first choice, and its quota, each university determines a minimum threshold. It then receives applications that list it as the applicants' first choice. After admitting first-choice applicants in the order of high to low exam scores up to its quota, the first round allocations are finalized and the first round is closed. After the first round, universities which have not fulfilled their quotas each review applicants who list it as their second choice; etc. This mechanism is called the *sequential mechanism* (*shùn xù zhì yuàn*), which prioritizes students' preference orderings over their score rankings (*zhì yuàn yōu xiān*).

The sequential mechanism places huge strategic importance on an applicant's first choice. Among those admitted into a key university in 2010, more than 95% of them list it as his or her first choice, whereas 80% of those admitted into an ordinary university list it as his or her first choice (Qiu and Zhao 2011, p. 243). Therefore, Qiu and Zhao (2011) warn the applicants that if their first- and second-choice universities are too close in quality, they might not get into any university in the first tier (p. 243). An obvious problem is that some students with very high scores do not get into any university in the first tier simply because they miss their first choice, leading to the popular saying that "a good score in the college entrance exam is worth less than a good strategy in the preference ranking of universities" (Nie 2007b).

To remedy the strategic manipulation inherent in the sequential mechanism, the *paral-*

[30]In comparison, individual gaokao scores were kept secret before the Cultural Revolution (Yang 2006, p. 269-270).

lel mechanism (*píng xíng zhì yuàn*) was first introduced into college admissions in Hunan Province in 2001. Jiangsu and Zhejiang adopted the mechanism in 2005 and 2007, respectively (Liu 2009, p. 382). The main innovation of the parallel mechanism is that students can put several "parallel" universities for each choice-band. For example, a student's first choice-band can contain four universities, A, B, C and D, in decreasing desirability. Applicants are ranked by exam scores. Starting from the applicant with the highest score to the one with the lowest score, each applicant applies for the parallel universities in the order of her preference ranking, from A to D. She gets into the first university with unfulfilled quota. After every applicant has applied to his first choice-band universities, the first round is closed. Those who are not admitted in the first round start the same process in the second round, and so on. Our interpretation is that it is a modified deferred acceptance mechanism as we formalize in our paper. This interpretation has a broad set of applications as it can also be applied to the school choice context where priorities are not unique.

In addition to the matching mechanisms, many other important components of the college admissions process underwent changes in the 1990s and the early 21 century. While these components are not the focus of our paper, we include five of them below to illustrate the scope of the reform. First, the content of the exam, i.e., subjects that should be covered and the number of tracks, changed several times. For example, in 1999, "$3+X$" system was implemented, where 3 refers to the three exams required for every applicant, math, Chinese, and foreign language, and X refers to any number of exams taken from physics, chemistry, biology, geography, history, politics. Second, a controversial institutionalized feature started in the 1990s is the guaranteed admissions for up to 5% of the high school graduates, each recommended by his high school. Third, standardized test techniques, such as an increase in multiple choice problems and machine grading, were gradually implemented in the late 80s and 90s. Fourth, as of 1985, Shanghai has been implementing its own exams. By 2006, 16 provinces each implemented its own exams. Lastly, computerized admissions process was first implemented in Guangxi and Tianjin in 1998. By 2001, nation-wide computerized admissions through the Internet was completed (Liu 2009, p. 41).

Compared to the historical accounts and qualitative analysis of Chinese college admissions, game-theoretic analysis of Chinese college admissions mechanisms has been relatively new. The latter mainly focuses on two issues, the timing of preference ranking submissions (Zhong et al. 2004, Wu and Zhong 2012, Lien et al. 2012, Wang and Zhong 2012) and the matching mechanisms themselves (Nie 2006, Nie 2007a, Nie 2007b, Nie and Zhang 2009, Wei 2009). We discuss both aspects in the main text of our paper. Lastly, Chiu and Weng (2009) present a theoretical model that investigates the incentives for schools to pre-commit to admitting qualified applicants who rank them as their top choices over more qualified applicants who do not. Such incentives are relevant in CCA as well.

4.5 Shanghai Mechanism: Online Q&A

We translate the following question and answer from an online Q&A forum about the parallel choices in Shanghai high school admissions, posted in May 2003.

> Question: If a student lists a school as his first choice or second choice, what difference does it make in the admission process?[31]
>
> Answer: Middle school admission principles are: based on the student exam scores and school preference ranking, place the applications accordingly, while also considering their moral, intellectual and physical aspects, choose the best from high to low scores. For each individual student, the Middle School Admissions Office will submit his application in the order of his preference ranking. Only when he cannot get into his first choice, will his second choice be considered. In the admissions process of the entire district, each school has only one threshold. If a student's score is above the school threshold, whether he lists it as his first or second choice, he should be admitted.
>
> For example, if student A's first choice is Luwan Middle School, and student B's second choice is Luwan. If A and B's scores are both above the Luwan minimum threshold, then both should be admitted into Luwan. However, if student B is already admitted by his first choice, it is impossible for him to get into Luwan. On the other hand, if the two students have different scores, e.g., A's score is low and below the Luwan threshold, while B (whose second choice is Luwan) has a high score, which is above the Luwan threshold, then A (whose first choice is Luwan) cannot be admitted into Luwan because his score is below

[31] Translated from `http://edu.sina.com.cn/l/2003-05-15/42912.html`, accessed on December 12, 2013.

Table 1: Chinese College Admissions Mechanisms by Province in 2012

Province	**Mechanism Type**	**Sequence**	**No. of Applicants in 2012**
Heilongjiang	sequential	(1, 1, 1, ...)	208,000
Qinghai	sequential	(1, 1, 1, ...)	38,000
Jiangsu	symmetric parallel	(3, 3, 3, ...)	500,000
Anhui	symmetric parallel	(4, 4, 4, ...)	506,000
Guangxi	symmetric parallel	(4, 4, 4, ...)	292,000
Jiangxi	symmetric parallel	(4, 4, 4, ...)	289,000
Ningxia	symmetric parallel	(4, 4, 4, ...)	60,000
Shǎnxi	symmetric parallel	(4, 4, 4, ...)	384,000
Hebei	symmetric parallel	(5, 5, 5, ...)	459,000
Hunan	symmetric parallel	(5, 5, 5, ...)	352,000
Yunan	symmetric parallel	(5, 5, 5, ...)	230,000
Zhejiang	symmetric parallel	(5, 5, 5, ...)	300,000
Tianjin	symmetric parallel	(5, 5, 5, ...)	65,000
Hainan	symmetric parallel	(6, 6, 6, ...)	54,000
Tibet	symmetric parallel	(10, 10, 10, ...)	18,000
Beijing	asymmetric parallel	(1, 3, 1, 3, 1, 3, ...)	73,000
Gansu	asymmetric parallel	(1, 3, 1, 3, 1, 3, ...)	296,000
Shandong	asymmetric parallel	(1, 4, 1, 4, 1, 4, ...)	587,000
Liaoning	asymmetric parallel	(3, 1, 8, 1, 8, ...)	245,000
Guangdong	asymmetric parallel	(3, 3, 1, ...)	692,000
Fujian	asymmetric parallel	(4, 4, 6, ...)	267,000
Shanghai	asymmetric parallel	(4, 6, 8, ...)	61,000
Xinjiang	asymmetric parallel	(4, 6, 1, 8, 1, 5, ...)	155,000
Guizhou	asymmetric parallel	(5, 5, 1, 3, 1, ...)	248,000
Jilin	asymmetric parallel	(5, 1, 7, 1, 6, ...)	165,000
Hubei	asymmetric parallel	(5, 1, 6, 1, 6, ...)	457,000
Shānxi	asymmetric parallel	(5, 1, 4, 1, 4, ...)	339,000
Sichuan	asymmetric parallel	(5, 1, 4, 1, 4, ...)	514,000
Chongqing	asymmetric parallel	(6, 1, 5, 1, 5, ...)	230,000
Henan	asymmetric parallel	(6, 6, 1, 4, ...)	855,000
Inner Mongolia	dynamic adjustment	-	206,000

Note: The sequences do not include tier 0, which is primarily for military academies.

the threshold; whereas B (whose second choice is Luwan), if not admitted by his first choice, should be admitted by Luwan, even though he listed Luwan as his second choice.

In the next section, we explain how the minimum threshold score is determined in the admissions process.

4.6 The Computer Software for CCA and the Minimum Threshold

In this section, we describe the history of computerization in CCA, and the determination of the minimum threshold for each college. This section is based on the first author's interview with Professor Weidong Liu at the Department of Computer Science and Technology, Tsinghua University, on August 9, 2013. Professor Liu is the Principal Investigator for the Chinese College Admissions Software Development Project, which was commissioned by the Ministry of Education in 1998. The main reasons for the Ministry of Education to push for computerization is to decrease the time period for admissions and to reduce the error rate.

The Ministry mandates that the official version of the software should be able to accommodate regional variations, thus the software is modularized, with major modules provided by the Tsinghua group. To adapt the software to the mechanism used in each province, the provincial officials just need to set the parameters. One key parameter is the number of parallel colleges for each choice-band, i.e., the parameter e in our notation. Later on, each province can also add modules to accommodate province-specific policies, such as affirmative action policies.

The software has several modules: planning, student application submission, student application review, and final allocation. The final allocation is simultaneously announced by each college and the Provincial Admissions Office (PAO), which serves as an advocate for the students. The beta version of the software was implemented in 1998 in Tianjin and Guangxi. However, key elements of the manual admissions process were still present. For example, each college still sent admissions officers to the provinces using the beta version of the software. The major official versions are described as follows:

1. Version 1.0 was released in 2001 nationwide. The parallel mechanism could be accommodated in this version by setting the choice parameter greater than one (in our

notation, $e > 1$).

2. Version 2.0 was released in 2003 with major security improvements.

3. Version 3.0 was released in 2006, with the USB key identification of the college identity, based on security considerations.

4. Version 4.0 was implemented in 2008, which supports IPv6.

5. In 2010, another version was released which enables each province to add small modules to the major modules sanctioned by the Ministry. Source code for the major modules is provided to the provinces upon request. Provincial modules can be added to reflect province-specific policies, such as affirmative actions.

We next describe the minimum threshold under each mechanism. Under the sequential mechanism, three thresholds are relevant in the final allocation:

1. Tier threshold (*pī cì kòng zhì xiàn*): each PAO determines the threshold for each tier, which is the minimum score for a student to be eligible for that tier. This threshold is determined by the quota of each college and the distribution of scores.

2. Student application submission threshold (*tóu dàng xiàn*): each college has its own threshold, i.e., its minimum score for a student's application to be reviewed. The Ministry allows each college to review no more than 120% of its quota, which gives the college considerable flexibility in the allocation of students among its various departments.

3. Minimum admissions threshold (*lù qǔ xiàn* $\geq$ *tóu dàng xiàn*): This is the minimum score of those finally admitted to a college.

In comparison, under the parallel mechanism, the admissions threshold for each college is determined through a simulation process. The simulations can be viewed as a negotiation process between the PAO and the colleges. The colleges would not agree to review the same number of applications as its quota. The question is how many more applications can a college review without jeopardizing the students' welfare. This is determined by several rounds of simulations. The simulation results for each round are not released to the public.

Round 1: each college receives student applications up to 110% of its quota. The software goes through the entire allocation process and let each college know the maximum and minimum scores in the college as well as in each of its departments. Note the college does not see who are in each round of simulation, but only the summary statistics.

Round 2: based on the round 1 statistics, each college then proposes to adjust its percentage of applications, e.g., to 106% of its quota. The software takes the new percentages and performs another round of simulated allocations. Summary statistics are again given to each college.

⋮

The simulation process continues until every college is satisfied with its summary statistics or until time has run out. In some provinces, the PAO determines the number of rounds of simulations. The minimum admissions threshold for each college is determined at the end of the simulation phase.

When the simulation phase is over, the student application review phase starts. Each college can view individual student applications of those who apply for it and whose scores are above its minimum threshold, and make admission decisions. If an application is reviewed and the corresponding student is rejected during this phase, the student is likely to have violated some written guidelines.

4.7 College admissions in Hong Kong: JUPAS

College admissions in Hong Kong use a centralized system called the Joint University Programmes Admissions System (JUPAS). Under JUPAS, each student submits preferences for up to 25 programmes. These 25 choices are further divided into 5 bands, A, B, C, D, and E. For each student, the first three choices are band A choices, next three are band B, etc. Each programme ranks its applicants with an objective formula (based on academic performance and other considerations) to form a base priority ranking. Each programme is informed of the band it is placed by a student, but not the precise ranking. Most programmes use this information to adjust the base priority ranking. Finally given student preferences and adjusted priority rankings, the outcome is obtained via the student-proposing deferred acceptance algorithm (Liu and Chiu 2011).

The flexibility for colleges to modify priorities under JUPAS has led some colleges to strategically choose their priority construction formula in response to the formulas chosen by more popular colleges. "The rating criterion is independently determined by each programme: although some would adopt Boston-like criterion which assigns band A student highest priority, some may also rate students only by their eligibility [based on their academic performances, interview outcomes and extracurricular activities]. Some unpopular programmes tend to employ the latter strategy if they find most excellent students listed it as band B or C choices rather than band A." (p. 4 and 5, Liu and Chiu 2011)

While the parallel mechanisms in CCA and JUPAS have some similarities, they also have important differences. Specifically, to determine the priority order under JUPAS, each college uses a combination of the student academic performances together with the band the student places the college in his preferences. It is important to note that each college has its own formula for doing this, which is captured by the α_c parameter in Liu and Chiu (2011). This means that, depending on the formula of the college, a student who places a school in band B may still have higher priority than another student who places it in band A if the former has a sufficiently higher exam score. This situation can never happen under the parallel mechanism in CCA. Under CCA, the priority construction is lexicographic, first based on the band, and second based on the exam score (for those in the same band). Moreover, this is the same for each college, i.e. there is no college-specific formula.

Appendix B: Proofs and Examples (For Online Publication)

Proof of Proposition 1: (Part i). It is easy to see that the sequential mechanism is Pareto efficient. Now consider the following problem with four students and four schools each with one seat. Priority orders and student preferences are as follows.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_4	i_2	$\vdots$	$\vdots$
i_2	i_3		
i_1	i_4		
i_3	i_1		

P_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}
s_1	s_1	s_2	s_2
s_4	s_2	s_3	s_1
$\vdots$	$\vdots$	$\vdots$	$\vdots$

The outcome of the application-rejection mechanism (e) for all $e \geq 2$ is the following Pareto inefficient matching

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 & i_4 \\ s_4 & s_2 & s_3 & s_1 \end{pmatrix}.$$

(Parts ii & iii). Fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \ldots, i_{e+2}\}$ and $S = \{s_1, s_2, \ldots, s_{e+2}\}$, where each school has a quota of one. Each $i_k \in I$ with $k \in \{1, 2, \ldots, e\}$ ranks school s_k first and each $i_k \in I$ with $k \in \{1, 2, \ldots, e+1\}$ has the highest priority for school s_k. The preferences of student i_{e+1} are as follows: $s_1 \; P_{e+1} \; s_2 \; P_{e+1} \ldots s_{e+1} \; P_{e+1} \; s_{e+2}$. And student i_{e+2} ranks school s_{e+1} first. Let us apply the application-rejection (e) mechanism to this problem. Consider student i_{e+1}. It is easy to see that he applies to school s_{e+1} in step $e+1$ of the algorithm when a lower priority student is already permanently assigned to it in round 0. Hence he is rejected from school s_{e+1} and his final assignment is necessarily worse than s_{e+1}. Then the outcome of the application-rejection (e) mechanism for this problem is clearly unstable. Moreover, student i_{e+1} can secure a seat at school s_{e+1} when he submits an alternative preference list in which he ranks school s_{e+1} first. ∎

Example 1a. (The Sequential mechanism is manipulable whenever the Shanghai mechanism is) Consider the following example with five students and four schools. Schools s_1, s_2, and s_4 each have a quota of one, while school s_3 has a quota of two.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$	$\succ_{s_4}$
i_4	i_1	$\vdots$	i_5
i_1	i_3		i_1
i_2	i_4		$\vdots$
$\vdots$	$\vdots$		

P_{i_1}	P'_{i_1}	P_{i_2}	P_{i_3}	P_{i_4}	P'_{i_4}	P_{i_5}
s_1	s_2	s_1	s_2	s_2	s_1	s_4
s_4	$\vdots$	s_3	s_3	s_1	$\vdots$	$\vdots$
s_2	$\vdots$	s_2	s_1	s_3		
s_3		s_4	s_4	s_4		

The following two tables illustrate the steps of the Shanghai mechanism applied to the problem $(\succ, P)$. A student tentatively placed at a school at a particular step is outlined in a box.

Round 0	$s_1\ (q^{r=0}_{s_1}=1)$	$s_2\ (q^{r=0}_{s_2}=1)$	$s_3\ (q^{r=0}_{s_3}=2)$	$s_4\ (q^{r=0}_{s_4}=1)$
Step 1	$\boxed{i_1}$, i_2	$\boxed{i_3}$, i_4		$\boxed{i_5}$
Step 2	$\boxed{i_4}$, i_1	$\boxed{i_3}$	$\boxed{i_2}$	
Step 3				$\boxed{i_5}$, i_1

Round 1	$s_1\ (q^{r=1}_{s_1}=0)$	$s_2\ (q^{r=1}_{s_2}=0)$	$s_3\ (q^{r=1}_{s_3}=1)$	$s_4\ (q^{r=1}_{s_4}=0)$
Step 4		i_1		
Step 5	⋮		$\boxed{i_1}$	

In the above tables, observe that student i_1 ends up at his last choice at problem $(\succ, P)$. Now consider the following two tables that illustrate the steps of the Shanghai mechanism when student i_1 reports P'_{i_1}, as opposed to P_{i_1}.

Round 0	$s_1\ (q^{r=0}_{s_1}=1)$	$s_2\ (q^{r=0}_{s_2}=1)$	$s_3\ (q^{r=0}_{s_3}=2)$	$s_4\ (q^{r=0}_{s_4}=1)$
Step 1	$\boxed{i_2}$	$\boxed{i_1}$, i_3, i_4		$\boxed{i_5}$
Step 2	$\boxed{i_4}$, i_2	$\boxed{i_1}$	$\boxed{i_3}$	
Step 3			$\boxed{i_2, i_3}$	

In this case, student i_1 is assigned to school s_2. Thus, the Shanghai mechanism is manipulable by student i_1 at problem $(\succ, P)$. Next, let us apply the Boston mechanism to problem $(\succ, P)$. The specifications are illustrated in the following tables.

Round 0	$s_1\ (q^{r=0}_{s_1}=1)$	$s_2\ (q^{r=0}_{s_2}=1)$	$s_3\ (q^{r=0}_{s_3}=2)$	$s_4\ (q^{r=0}_{s_4}=1)$
Step 1	$\boxed{i_1}$, i_2	$\boxed{i_3}$, i_4		$\boxed{i_5}$

Round 1	$s_1\ (q^{r=1}_{s_1}=0)$	$s_2\ (q^{r=1}_{s_2}=0)$	$s_3\ (q^{r=1}_{s_3}=1)$	$s_4\ (q^{r=1}_{s_4}=0)$
Step 2	i_4		$\boxed{i_2}$	
Step 3	⋮		$\boxed{i_2, i_4}$	

Observe that student i_1 ends up at s_1 (his first choice), and thus cannot gain by a misreport, but student i_4 ends up at s_3 (his third choice) at problem $(\succ, P)$. Next consider the following tables that illustrate the steps of the Boston mechanism when student i_4 reports P'_{i_4}, as opposed to P_{i_4}.

Round 0	$s_1\ (q_{s_1}^{r=0}=1)$	$s_2\ (q_{s_2}^{r=0}=1)$	$s_3\ (q_{s_3}^{r=0}=2)$	$s_4\ (q_{s_4}^{r=0}=1)$
Step 1	$\boxed{i_4}, i_1, i_2$	$\boxed{i_3}$		$\boxed{i_5}$
Round 1	$s_1\ (q_{s_1}^{r=1}=0)$	$s_2\ (q_{s_2}^{r=1}=0)$	$s_3\ (q_{s_3}^{r=1}=2)$	$s_4\ (q_{s_4}^{r=1}=0)$
Step 2			$\boxed{i_2}$	i_1
Round 2	$s_1\ (q_{s_1}^{r=2}=0)$	$s_2\ (q_{s_2}^{r=2}=0)$	$s_3\ (q_{s_3}^{r=2}=1)$	$s_4\ (q_{s_4}^{r=2}=0)$
Step 3			$\boxed{i_1}$	

Now student i_4 ends up at school s_1. Thus, the Boston mechanism is also manipulable at problem $(\succ, P)$.

Example 1b. (Shanghai mechanism is not manipulable when the sequential mechanism is) Consider the following example with the given priority structure and the profile of preferences. Each school, s_1, s_2, and s_3, has a quota of one.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_1	i_2	$\vdots$
i_2	i_3	
$\vdots$	$\vdots$	

P_{i_1}	P_{i_2}	P'_{i_2}	P_{i_3}
s_1	s_1	s_2	s_2
$\vdots$	s_2	$\vdots$	s_3
	s_3		$\vdots$

Clearly, at problem $(\succ, P)$ under the Boston mechanism, student i_2 can obtain a seat at s_2 by submitting P'_{i_2} as opposed to P_{i_2} which places him at s_3. Note, however, that under the Shanghai mechanism no student can ever gain by a misreport at problem $(\succ, P)$.

Proof of Theorem 1:

We start with a useful definition. Given a preference relation P_i of a student i, let $rank_i(a)$ denote the rank of school a in student i's preferences.

Definition: Given a preference profile P, student i ranks school a at a higher e-class than student j iff

$$\left\lceil \frac{rank_i(a)}{e} \right\rceil < \left\lceil \frac{rank_j(a)}{e} \right\rceil .$$

Intuitively, a student who lists a school among his first e choices ranks that school at a higher e-class than those who do not list it as one of their first e choices; a student who lists a school among his first $e+1$ through $2e$ choices ranks that school at a higher e-class than those who do not list it as one of their first $2e$ choices; etc. The following construction will be instrumental in the proof of Theorem 1 as well as some of the subsequent proofs.

For a given problem $(\succ, P)$, the corresponding *e-augmented priority profile* $\widetilde{\succ}$ is constructed as follows. For each $a \in S$, and all $i, j \in I$, we have $i \widetilde{\succ}_a j$ if and only if either

(1) i ranks school a at a higher e-class than j, or

(2) i and j both rank school a in the same e-class and $i \succ_a j$.

Lemma 1: Given a problem $(\succ, P)$ and the corresponding e-augmented priority profile $\widetilde{\succ}$, $\varphi^e(\succ, P) = \varphi^\infty(\widetilde{\succ}, P)$.

Proof of Lemma 1: Let J^r denote the set of students who are permanently assigned to some school at the end of round r of φ^e at problem $(\succ, P)$. We first argue that the students in J^0 receive the same assignments under the DA at problem $(\widetilde{\succ}, P)$. First observe that by the construction of the e-augmented priority profile $\widetilde{\succ}$, a student who ranks a school in a higher e-class than some other student can never be rejected by that school under the DA at $(\widetilde{\succ}, P)$ because of the application of that other student. Then since round 0 of φ^e is equivalent to applying the DA algorithm to the first e choices of all students and the assignments are made permanent at the end of round 0 of φ^e, the assignments of students in J^0 under φ^e at problem $(\succ, P)$ has to coincide with their assignments under the DA at problem $(\widetilde{\succ}, P)$. Decreasing each school's quota under φ^e before round 1 and applying the same reasoning to this round the students in J^1 must receive the same assignments under the DA at problem $(\widetilde{\succ}, P)$. Iterating this reasoning for the next rounds in turn we conclude that $\varphi^e(\succ, P) = \varphi^\infty(\widetilde{\succ}, P)$. ∎

Given $i \in I$ and $x \in S$, let P_i^x denote a preference relation where student i ranks school x as his first choice.

Lemma 2: Given a problem $(\succ, P)$, let $\varphi_i^E(\succ, P) = x$. Then $\varphi_i^E(\succ, P) = \varphi_i^e(\succ, P_i^x, P_{-i}) = x$ where $e < E$.

Proof of Lemma 2: By Lemma 1, $\varphi_i^E(\succ, P) = \varphi_i^\infty(\widetilde{\succ}, P) = x$ where $\widetilde{\succ}$ is the E-augmented priority profile corresponding to $(\succ, P)$. By the strategy-proofness of the DA, $\varphi_i^\infty(\widetilde{\succ}, P) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i})$. Hence, we have

$$\varphi_i^E(\succ, P) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i}). \tag{1}$$

On the other hand, by Lemma 1,

$$\varphi_i^E(\succ, P_i^x, P_{-i}) = \varphi_i^\infty(\widehat{\succ}, P_i^x, P_{-i}) \tag{2}$$

where $\widehat{\succ}$ is the E-augmented priority profile corresponding to $(\succ, P_i^x, P_{-i})$. Note that $\widehat{\succ}_{-x}$ and $\widetilde{\succ}_{-x}$ agree on all students' relative priority orderings but i and $\widehat{\succ}_x$ (weakly) improves the priority of student i for school x in comparison to $\widetilde{\succ}_x$. Then it follows from the working of the DA algorithm that

$$\varphi_i^\infty(\widehat{\succ}, P_i^x, P_{-i}) = \varphi_i^\infty(\widetilde{\succ}, P_i^x, P_{-i}). \tag{3}$$

Last we claim that

$$\varphi_i^E(\succ, P_i^x, P_{-i}) = \varphi_i^e(\succ, P_i^x, P_{-i}). \tag{4}$$

To see this note that when applied to $(\succ, P_i^x, P_{-i})$, the set of students who apply to school x in round 0 of φ^E is weakly larger than that in round 0 of φ^e and since student i is not rejected from school x after applying to it in the first step under φ^E, he cannot be rejected from it under φ^e either.

Combining (1), (2), (3), and (4), we obtain $\varphi_i^E(\succ, P) = \varphi_i^e(\succ, P_i^x, P_{-i}) = x$. ∎

Now we are ready to prove Theorem 1. Let $(\succ, P)$ be a problem such that there exists $i \in I$ and preferences P_i' where $\varphi_i^{e'}(\succ, P_i', P_{-i})\ P_i\ \varphi_i^{e'}(\succ, P)$. We show that there exists $j \in I$ and preferences P_j' such that $\varphi_j^e(\succ, P_j', P_{-j})\ P_j\ \varphi_j^e(\succ, P)$ where $e < e'$. Let $\varphi_i^{e'}(\succ, P_i', P_{-i}) = x$. We consider two cases.

Case 1. $x\ P_i\ \varphi_i^e(\succ, P)$: Since $\varphi_i^{e'}(\succ, P_i', P_{-i}) = x$, by Lemma 2 $\varphi_i^e(\succ, P_i^x, P_{-i}) = x$. Thus, i manipulates φ^e at $(\succ, P)$.

Case 2. $\varphi_i^e(\succ, P)\ R_i\ x$: We claim that for all $k \in I$, $\varphi_k^e(\succ, P)\ R_k\ \varphi_k^{e'}(\succ, P)$. Suppose not. Then, there exists $j \in I$ such that $\varphi_j^{e'}(\succ, P)\ P_j\ \varphi_j^e(\succ, P)$. By Lemma 2, $\varphi_j^e(\succ, P_j^{\varphi_j^{e'}(\succ, P)}, P_{-i}) = \varphi_j^{e'}(\succ, P)$ and thus j manipulates φ^e at $(\succ, P)$. Hence the claim is true.

Moreover, since $\varphi_i^e(\succ, P)\ R_i\ x$ and $x\ P_i\ \varphi_i^{e'}(\succ, P)$, by transitivity we have $\varphi_i^e(\succ, P)\ P_i\ \varphi_i^{e'}(\succ, P)$. This together with the preceding claim implies that $\varphi^e(\succ, P)$ Pareto dominates $\varphi^{e'}(\succ, P)$.

Next, consider the rounds of $\varphi^{e'}$ when applied to problem $(\succ, P)$. Let $y = \varphi_i^{e'}(\succ, P)$. Let also r be the round at the end of which student i is (permanently) assigned to school y. We claim that $r \geq 1$. Suppose for a contradiction that $r = 0$. Then since $\varphi_i^{e'}(\succ, P_i', P_{-i}) = x\ P_i\ y = \varphi_i^{e'}(\succ, P)$, student i ranks school x at the same (and the highest) e'-class at both (P_i', P_{-i}) and P. Let $\widetilde{\succ}$ and $\widehat{\succ}$ be the e'-augmented priority profiles corresponding to $(\succ, P_i', P_{-i})$ and $(\succ, P)$ respectively. Thus, by Lemma 1, $\varphi_i^\infty(\widetilde{\succ}, P_i', P_{-i}) = x$ and $\varphi_i^\infty(\widehat{\succ}, P) = y$. Let P_i^{xy} be a relation where i ranks x first and y second. By the strategy-proofness of the DA, $\varphi_i^\infty(\widetilde{\succ}, P_i^{xy}, P_{-i}) = x$. Note that since student i ranks school x at the same (and the highest) e'-class at both (P_i', P_{-i}) and P, $\widehat{\succ}_x = \widetilde{\succ}_x$. Thus, $\varphi_i^\infty(\widehat{\succ}_x, \widetilde{\succ}_{-x}, P_i^{xy}, P_{-i}) = x$. Recall that $\varphi_i^\infty(\widehat{\succ}, P) = y$. By the strategy-proofness of the DA, $\varphi_i^\infty(\widehat{\succ}, P_i^{xy}, P_{-i}) = y$. But then, at both $(\widehat{\succ}_x, \widetilde{\succ}_{-x}, P_i^{xy}, P_{-i})$ and $(\widehat{\succ}, P_i^{xy}, P_{-i})$ the preference profiles are the same and student i lists school x as first choice. Since the priority order for x is also identical at both problems, the DA should give i the same assignment for both problems. A contradiction. Thus, $r \geq 1$ as claimed.

Let $z_0 = \varphi_i^e(\succ, P)$. Since $\varphi_i^e(\succ, P)\ P_i\ \varphi_i^{e'}(\succ, P)$ and $\varphi_i^{e'}$ is nonwasteful, there exists $j_1 \in \varphi^{e'}(\succ, P)(z_0) \backslash \varphi^e(\succ, P)(z_0)$. Since $\varphi^e(\succ, P)$ Pareto dominates $\varphi^{e'}(\succ, P)$, we must have $\varphi_{j_1}^e(\succ, P)\ P_{j_1}\ \varphi_{j_1}^{e'}(\succ, P)$. Letting $z_1 = \varphi_{j_1}^e(\succ, P) \neq z_0$, there exists $j_2 \in \varphi^{e'}(\succ, P)(z_1) \backslash \varphi^e(\succ, P)(z_0)$. Since I is finite, iterating this reasoning we obtain a set $J = \{i, j_1, \ldots, j_k\}$ of students with $k \geq 1$ each of whom is assigned to a distinct school from the set $A = \{z_0, z_1, \ldots, z_k = y\}$ at $\varphi^e(\succ, P)$. Reconsidering the $\varphi^{e'}$ algorithm when applied to problem $(\succ, P)$, each student in J must then be assigned to the corresponding school in A in the same round. For otherwise, the school from the set A that admits a student at a later round will still have a vacant position in all previous rounds which contradicts the fact that the student from the set J assigned to it at $\varphi^e(\succ, P)$ is better off compared to $\varphi_i^{e'}(\succ, P)$. In other words, all Pareto improving assignment exchanges from $\varphi^{e'}(\succ, P)$ to $\varphi^e(\succ, P)$ must involve students who receive their (permanent) assignments in the same round. Hence, each student in J are (permanently) assigned to the corresponding school in A in round $r \geq 1$.

Consider round r of the $\varphi^{e'}$ algorithm when applied to problem $(\succ, P)$. Let $J^r \supset J$

be the set of students such that (1) they each receive their (permanent) assignments at the end of round r, and (2) they each are better off at $\varphi^e(\succ, P)$ compared to $\varphi^{e'}(\succ, P)$.[32] Let $j^* \in J^r$ be the last student in J^* to apply to his assignment at $\varphi^e(\succ, P)$ in that round and let $z^* = \varphi_j^* e'(\succ, P)$. Let k^* be the student who is kicked out from z^* at that step. Note that k^* necessarily exists since a student from J^r has already been kicked out from z^* at a previous step in that round. Thus, $z^* \; P_k^* \; \varphi_k^* e'(\succ, P)$. Moreover, by the choice of j^*, $k^* \notin J^r$. If student k^* receives his (permanent) assignment at the end of round r, then $\varphi_k^* e(\succ, P) = \varphi_k^* e'(\succ, P)$. Otherwise, student k^* receives his (permanent) assignment at a later round than r and by the argument in the preceding paragraph pertaining to students who are better off at $\varphi^e(\succ, P)$, $z^* \; P_k^* \; \varphi_k^* e(\succ, P)$.

Finally, since school z^* has a vacancy before round $r \geq 1$, it follows that $\varphi_{k^*}^{e'}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = z^*$. Then by Lemma 2, $\varphi_{k^*}^{e'}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = \varphi_{k^*}^{e}(\succ, P_{k^*}^{z^*}, P_{-k^*}) = z^* \; P_{k^*} \; \varphi_{k^*}^{e}(\succ, P)$. Hence, student k^* manipulates φ^e at $(\succ, P)$.

We next prove that $\varphi^{e'}$ may not be manipulable when φ^e is. Fix $e < \infty$. Consider the following problem. Let $I = \{i_1, i_2, \ldots, i_{e+2}\}$ and $S = \{s_1, s_2, \ldots, s_{e+1}\}$ where each school has a quota of one. Each student $i \in I$ has the following preferences: $s_1 \; P_i \; s_2 \; P_i \ldots s_e \; P_i \; s_{e+1} \; P_i \; \emptyset$. There is a single priority order for each school given as follows: for each $s \in S$, suppose $i_k \succ_s i_{k'}$ whenever $k < k'$, i.e., i_1 has the highest priority, i_2 has the second highest priority and so on. Let us apply the application-rejection (e) mechanism to this problem. Consider student i_{e+2}. It is easy to see that he is unassigned in round 0 and is assigned to his last choice (i.e., the null school) at step $e + 2$ of round 1 after being rejected from school s_{e+1}. If student i_{e+2} were to report school s_{e+1} as his first choice, he would clearly be assigned to it in round 0. Hence, φ^e is manipulable by student i_{e+1} at this problem. It is easy to see that no student can manipulate $\varphi^{e'}$ via a preference misreport at this problem. ∎

Proof of Proposition 2:

(Part i). Let $e' = ke$. If $k = \infty$, Proposition 1 implies that the DA is more stable than φ^e for any $e < \infty$. So let $k \in \mathbb{N}$. We show that if $\varphi^{e'}$ is unstable at a problem, then so is φ^e. We prove the contrapositive of this statement. Let $(\succ, P)$ be a problem at which $\varphi^e(\succ, P)$ is stable. We show that $\varphi^e(\succ, P) = \varphi^{e'}(\succ, P)$.

Consider mechanism φ^e when applied to problem $(\succ, P)$. Since $\varphi^e(\succ, P)$ is stable, any

[32] Note that the set J^r is well-defined by the argument made in the previous paragraph.

unassigned student of round 0 (who was rejected from all his first $e-$choices) must have lower priority at his first $e-$choice schools than every student who obtained a seat at any such school in round 0. Similarly, since $\varphi^e(\succ, P)$ is stable, any unassigned student of round 1 (who was rejected from all his first $2e-$choices) must have lower priority at his first $2e-$choice schools than every student who obtained a seat at any such school in round 0 or round 1. In general, any unassigned student of round $k-1$ must have lower priority at his first $ke-$choice schools than every student who obtained a seat at any such school in round $k-1$ or any previous round. But this implies that any student who is unassigned at the end of round $k-1$ of φ^e is also unassigned at the end of round 0 of $\varphi^{e'}$ as he applies to and gets rejected from the same set of schools in the same order under both mechanisms. Similarly, any student who is assigned to some school s in some round $t \leq k-1$ of φ^e is also assigned to school s in round 0 of $\varphi^{e'}$ as he cannot be rejected by a student who does not list school s among his first $(t+1)e-$choices. Then the students who participate in rounds k through $2k-1$ of φ^e are the same as those who participate in round 1 of $\varphi^{e'}$ and by the same argument they apply to and get rejected from the same set of schools in the same order under both mechanisms. Iterating this reasoning, we conclude that $\varphi^{e'}(\succ, P) = \varphi^e(\succ, P)$.

The problem given at the end of the proof of Theorem 1 shows a situation where $\varphi^{e'}$ is stable while φ^e is not.

(Part ii). Since $e' \neq ke$ for any $k \in \mathbb{N} \cup \{\infty\}$, there exists $t \in \mathbb{N}$ such that $te < e' < (t+1)e$. Consider the following problem $(\succ, P)$. Let $I = \{i_1, i_2, \ldots, i_{te+e'+2}\}$ and $S = \{s_1, s_2, \ldots, s_{te+e'+1}\}$ where $q_s = 1$ for all $s \in S$. Each $i_j \in I \backslash \{i_{te+1}, i_{te+e'+2}\}$ top-ranks school s_j and has the highest priority for it. The remaining two students' preferences are as follows. $P_{i_{te+1}} : s_1, s_2, \ldots, s_{te+1}, \emptyset$ and $P_{i_{te+e'+3}} : s_{te+2}, s_{te+3}, \ldots, s_{te+e'+1}, s_{te+1}, \emptyset$. Let $i_{te+e'+2} \succ_{s_{te+1}} i_{te+1}$.

It is not difficult to calculate that for each $i_j \in I \backslash \{i_{te+1}, i_{te+e'+3}\}$, $\varphi^e_{i_j}(\succ, P) = \varphi^{e'}_{i_j}(\succ, P) = s_j$, $\varphi^e_{i_{te+1}}(\succ, P) = \varphi^{e'}_{i_{te+e'+3}}(\succ, P) = \emptyset$, and $\varphi^{e'}_{i_{te+1}}(\succ, P) = \varphi^e_{i_{te+e'+3}}(\succ, P) = s_{te+1}$. Clearly, $\varphi^e(\succ, P)$ is stable whereas $\varphi^{e'}(\succ, P)$ is not. The problem given at the end of the proof of Theorem 1 shows a situation where $\varphi^{e'}$ is stable while φ^e is not. ∎

Proof of Theorem 2: The first statement follows from the proof of Theorem 1, Corollary 2, and Proposition 2. For the second statement we construct a problem under which $\varphi^{e'}$ has a stable equilibrium which neither is equilibrium nor leads to a stable matching under

φ^e.Consider the following problem $(\succ, P)$. Let $I = \{i_1, i_2, \ldots, i_{e'}\}$ and $S = \{s_1, s_2, \ldots, s_{e'}\}$ where $q_s = 1$ for all $s \in S$. Each $i \in I$ ranks school s_1 first, s_2 second, ..., and $s_{e'}$ last. For each school $s \in S$, i_1 has the highest priority, i_2 has the second-highest priority, ..., and $i_{e'}$ has the lowest priority. At the unique stable matching i_1 is assigned to s_1, i_2 is assigned to s_2, ..., and $i_{e'}$ is assigned to $s_{e'}$. Let us denote it by μ. Consider the following profile of reports. Each student but student $i_{e'}$ reports truthfully, while student $i_{e'}$ only switches the positions of s_e and s_{e+1} and is truthful otherwise. These reports constitute an equilibrium under $\varphi^{e'}$ and lead to μ. However, the same profile is not an equilibrium under φ^e since $i_{e'}$ is now assigned to s_{e+1} and, any $i \in \{i_{e+1}, \ldots, i_{e'-1}\}$ can profitably deviate by replacing any one of his first e choices by s_{e+1}. Nor does this profile lead to a stable matching since any $i \in \{i_{e+1}, \ldots, i_{e'-1}\}$ can form a blocking pair with school s_{e+1}.

∎

Proof of Proposition 3:

Part (i). Fix a problem $(\succ, P)$. Take any two mechanisms φ^e and $\varphi^{e'}$ with $e' > e$. We contrast round 0 of φ^e with that of $\varphi^{e'}$. For any school $s \in S$, the set of students who apply to s in round 0 of $\varphi^{e'}$ is weakly larger than the set of students who apply to s in round 0 of φ^e. This implies that any student who is assigned to his first choice at the end of round 0 of $\varphi^{e'}$ is also assigned to his first choice at the end of round 0 of φ^e but not vice versa. In other words, a student who is assigned to his first choice under φ^e, may be rejected from that school under $\varphi^{e'}$ due to the application a higher priority student who ranks it as one of his $e+1$ through e' choices.

Part (ii). Fix a problem. Suppose $e' < e$. Consider any student-say i- who is assigned to one of his first e choices-say s- under $\varphi^{e'}$ but not under φ^e. Since assignments under φ^e are final after the first e choices have been considered (or alternatively, since the equivalent the DA algorithm constructed in Lemma 1 prioritizes the first e choices), student i's slot at s is filled by another student who also ranks s as one of his first e choices. Thus, the number of students who receive one of their first e choices cannot decrease under φ^e.

Suppose $e' > e$. Take any student-say j- who is assigned to one of his first e choices under $\varphi^{e'}$. Note that the corresponding $e-$augmented priority profile for this problem gives (weakly) higher priority to student j for all his first e choices than the corresponding $e'-$augmented priority profile. Then by Lemma 1 and the stability of the DA, student j must be assigned to one of his first e choices under φ^e as well. ∎

Proof of Proposition 4: Part (i) is established in Theorem 1 of Ergin and Sönmez (2006). We prove part (ii). Let $I = \{i_1, i_2, i_3\}$ and $S = \{s_1, s_2, s_3\}$, where each school has a quota of one. Consider the following priority profile $\succ$ and true preferences $P = (P_1, P_2, P_3)$ of students.

$\succ_{s_1}$	$\succ_{s_2}$	$\succ_{s_3}$
i_3	i_2	i_2
i_2	$\vdots$	i_1
i_1		i_3

P_{i_1}	P_{i_2}	P_{i_3}
s_1	s_1	s_3
s_3	s_2	s_1
s_2	s_3	s_2

the DA outcome for problem $(\succ, P)$ is the following matching

$$\mu = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_3 & s_2 & s_1 \end{pmatrix}.$$

Consider a strategy profile $Q = (Q_1, Q_2, Q_3)$ where $Q_1 = P_{i_1}$, $Q_3 = P_{i_3}$, and $s_2\ Q_2\ s_3\ Q_2\ s_1$. For problem $(\succ, Q)$ the outcome of the application-rejection mechanism (e), for any $e \geq 2$, is the unstable matching

$$\mu' = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix},$$

where μ' Pareto dominates μ. To see that Q is indeed an equilibrium profile (in undominated strategies), it suffices to consider possible deviations by student i_2. For any preferences in which he ranks s_1 first, he gets rejected from s_1 at the third step. If he ranks s_2 first, clearly his assignment does not change. If she ranks s_3 first, he is assigned to s_3. ∎

Proof of Proposition 5: Let $(\succ, P)$ be the problem where P is the list of true student preferences. By Proposition 2, μ is stable under $(\succ, P)$. Let P' be a preference profile where each $i \in I$ lists $\mu(i)$ as his e-th choice and such that for any $s \in S$, $s\ P_i'\ \mu(i)$ implies $s\ P_i\ \mu(i)$. We show that for each $i \in I$, $\varphi_i^e(\succ, P)\ R_i\ \mu(i)$ for any e. Suppose to the contrary that student i remains unassigned at the end of round 0. This means that school $\mu(i)$ is full at the end of round 0, and in particular, there is $j \neq i$ such that $\varphi_j^e(\succ, P) = \mu(i) \neq \mu(j)$ and $j \succ_{\mu(i)} i$. Then, since $\mu(i)\ P_j\ \mu(j)$ and $j \succ_{\mu(i)} i$, μ is not stable under $(\succ, P)$. ∎

Ex ante Equilibria: Incomplete information view

To gain a clear insight into the *ex ante* welfare issues we focus on the Boston (sequential) and the DA together with Shanghai, the simplest member of the Chinese parallel mechanisms. We show that, in the same setting as ACY, there may be students who are better off in a Bayesian equilibrium of Shanghai than in one of Boston. The following example illustrates the intuition.

Let there be four students of three types, with values $\{\mathbf{v}_L, \mathbf{v}_M, \mathbf{v}_H\}$, two from the low type and one each from the medium and high types, and four schools $\{s_0, s_1, s_2, s_3\}$, each with one seat. There are no priorities *a priori*, students have common ordinal preferences, and each student type has the von Neumann Morgenstern (vNM) utility values given in the following table.

	$\mathbf{v}_L$	$\mathbf{v}_M$	$\mathbf{v}_H$
s_0	.9	.53	.36
s_1	.09	.36	.35
s_2	.01	.11	.29
s_3	0	0	0

First, consider Boston with random tie-breaking. Type $\mathbf{v}_L$ students have a dominant strategy of ranking schools truthfully. Given that, type $\mathbf{v}_M$ student has a best response of ranking s_1 as his first choice (regardless of what type $\mathbf{v}_H$ does). And, given all these strategies, type $\mathbf{v}_H$ student has a best response of ranking s_2 as his first choice. This constitutes the unique equilibrium under the Boston mechanism, where type $\mathbf{v}_H$ student obtains an expected utility of .29.

Now let us consider the Shanghai mechanism with random tie-breaking. Type $\mathbf{v}_L$ students again have a dominant strategy of ranking schools truthfully. Given that, type $\mathbf{v}_M$ student has a best response of ranking schools truthfully (regardless of what type $\mathbf{v}_H$ does). And, given all these strategies, type $\mathbf{v}_H$ student has a best response of respectively ranking s_1 and s_2 as his first and second choices (see the proof of Proposition 4 part (ii) for details). This constitutes the unique equilibrium under the Shanghai mechanism, where type $\mathbf{v}_H$ student now obtains an expected utility of .32.

The reason why some students may prefer the Shanghai to the Boston, unlike the case against the DA, as in this example, can be intuitively explained as follows. Under the Boston mechanism, students' first choices are crucial and thus students target a single school at equilibrium. Under the Shanghai mechanism, the first two choices are crucial and students target a pair of schools. This difference, however, may enable a student to guarantee a seat at an unpopular school under the Shanghai by ranking it as his second choice and still give him some chance to obtain a more preferred school by ranking it as his first choice. For example, in the above scenario, type $\mathbf{v}_H$ student "gains priority" at school s_2, her sure outcome in Boston, when others do not include it in their first two choices and enjoys as well a positive chance of ending up at s_1.[33]

Although we have assumed in the above example that students have complete information about their cardinal preferences, it is possible to use the same insight to show the non-dominance of Boston over Shanghai in a Bayesian setting.

Proof of Proposition 6:

Part (i). We start by adopting the ACY model. Let $S = \{s_0, s_1, \ldots, s_m\}$ with $m \geq 1$ be the set of schools (without the outside option). Each student privately draws vNM utility values $v = (v_0, \ldots, v_m)$ from a finite set $\mathcal{V} = \{(v_0, \ldots, v_m) \in [0,1]^m | v_0 > v_1 \ldots > v_m\}$ with probability $f(\mathbf{v})$, which is common knowledge. Without loss of generality, we assume that $\sum_{s \in S} q_s = n = |I|$. Let Π be the set of all ordinal preferences over S, and $\Delta(\Pi)$ the set of probability distributions over Π. A symmetric Bayesian strategy is a mapping $\sigma : \mathcal{V} \to \Delta(\Pi)$.

In showing the dominance of Shanghai over the DA, we use exactly the same proof strategy as ACY. Following ACY, the probability that any student is assigned to school $s \in S$ is given by

$$P_s^{DA} = \frac{q_s}{n}.$$

[33]Loosely speaking, the Boston lottery (i.e., Boston with random tie-breaking) when compared with the Shanghai lottery (i.e., Shanghai with random tie-breaking) can be seen as a weighted average over more extreme choices (when the lotteries are non-degenerate). In the above example, for instance, a low type student faces a lottery between his first and last choices under Boston. This is because if he misses his first choice, his second and third choices will already be taken. On the other hand, the Shanghai lottery always puts positive weight on the first *and* the second choices. At the other extreme, the DA lottery is an equal weighted average over all choices.

For any equilibrium strategy $\sigma \in \{\sigma^*(v)\}_{\mathbf{v}\in\mathcal{V}}$, let $P_s^{SHA}(\sigma)$ be the probability that a student is assigned to school s if he plays σ when all other students play σ^*. Then, in equilibrium, for each $s \in S$,

$$\sum_{\mathbf{v}\in\mathcal{V}} n P_s^{SHA}(\sigma^*(\mathbf{v})) f(\mathbf{v}) = q_s.$$

Suppose a type $\widetilde{\mathbf{v}} \in \mathcal{V}$ student chooses to play $\sigma^*(\mathbf{v})$ with probability $f(\mathbf{v})$. Denote that strategy by $\widetilde{\sigma}$. Then he is assigned to $s \in S$ with probability

$$P_s^{SHA}(\widetilde{\sigma}) = \sum_{\mathbf{v}\in\mathcal{V}} P_s^{SHA}(\sigma^*(\mathbf{v})) f(\mathbf{v}) = \frac{q_s}{n} = P_s^{DA}.$$

That is, by playing $\widetilde{\sigma}$, which is not necessarily an equilibrium strategy, a student can guarantee himself the same random assignment as that he would get under the DA.

Part (ii). We start by showing that the specified strategies for the complete information example given in the text indeed constitute the unique equilibrium of Shanghai. Let $u_i(s)$ denote the vNM utility of student i for school s and σ_i denote a (pure) strategy of student i. Suppose students 1 and 2 are of the low type, student 3 and 4 are respectively of the medium and high types. Let $EU_i^{SHA}(\sigma^*)$ be the expected utility of student i at the specified strategy profile, i.e., when $\sigma_i^* = s_0s_1s_2s_3$ for $i = 1, 2, 3$ and $\sigma_4^* = s_1s_2s_0s_3$. Then we have $EU_i^{SHA} = \frac{1}{3}u_i(s_0) + \frac{1}{6}u_i(s_1) + \frac{1}{6}u_i(s_2) + \frac{1}{3}u_i(s_3)$ for $i = 1, 2, 3$ and $EU_4^{SHA} = \frac{1}{2}u_4(s_1) + \frac{1}{2}u_4(s_2) = .32$.

Clearly, for any student, ranking s_3 at any position but the bottom is dominated. Moreover, σ_1^* and σ_2^* are dominant strategies. We first claim that σ_3^* is a best response to σ_1^* and σ_2^* regardless of what 4 does. To show this, we fix σ_1^* and σ_2^*, and consider three possibilities for σ_4^*.

1. $\sigma_4^* = s_0s_1s_2s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_1s_2s_0s_3) = .24 > EU_3^{SHA}(\sigma_3 = s_0s_2s_1s_3) = .23.$[34]

2. $\sigma_4^* = s_1s_2s_0s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_0s_2s_1s_3) = .22 > EU_3^{SHA}(\sigma_3 = s_1s_2s_0s_3) = .21$.

3. $\sigma_4^* = s_0s_2s_1s_3$. Then, $EU_3^{SHA}(\sigma_3^*) = .25 > EU_3^{SHA}(\sigma_3 = s_1s_2s_0s_3) = .23 >$

[34]Upon fixing σ_1^* and σ_2^*, we calculate that $EU_i^{SHA}(\sigma_3 = s_1s_2s_0s_3, \sigma_4^* = s_0s_1s_2s_3) = \frac{1}{4}u_i(s_0) + \frac{1}{3}u_i(s_1) + \frac{1}{12}u_i(s_2) + \frac{1}{3}u_i(s_3)$ for $i = 1, 2, 3$ and $EU_4^{SHA}(\sigma_3 = s_1s_2s_0s_3, \sigma_4^* = s_0s_1s_2s_3) = \frac{1}{4}u_4(s_1) + \frac{3}{4}u_4(s_2)$.

$EU_3^{SHA}(\sigma_3 = s_0s_2s_1s_3) = .19$.

Last, we claim that σ_4^* is a best response to σ_1^*, σ_2^*, and σ_3^*. Indeed, $EU_4^{SHA}(\sigma_4^*) = .32 > EU_4^{SHA}(\sigma_4 = s_0s_2s_1s_3) = .31 > EU_4^{SHA}(\sigma_4 = s_0s_1s_2s_3) = .25$. Thus, we have confirmed that profile σ^* constitutes the unique equilibrium of Shanghai.

We next prove part (ii) of Proposition 3 building on the example given in the main text. Let $I = \{1, 2, 3, 4\}$, $S = \{s_0, s_1, s_2, s_3\}$, and $\mathcal{V} = \{\mathbf{v}_L, \mathbf{v}_M, \mathbf{v}_H\}$ (as in the example) with probabilities $p_L = \frac{3}{4} - \frac{\varepsilon}{2}$, $p_M = \frac{1}{4} - \frac{\varepsilon}{2}$, and $p_H = \varepsilon$, where $\varepsilon > 0$ can be chosen arbitrarily close to zero. Consider the following strategies under Boston: $\sigma^{BOS}(\mathbf{v}_L) = s_0s_1s_2s_3$, $\sigma^{BOS}(\mathbf{v}_M) = s_1s_0s_2s_3$, and $\sigma^{BOS}(\mathbf{v}_H) = s_2s_0s_1s_3$. We claim that these strategies constitute a symmetric Bayesian Nash equilibrium for a sufficiently small ε.

Since an exact analysis would be unnecessarily lengthy and cumbersome, we provide only rough arguments. For a low type student it is still a dominant strategy to rank truthfully. Consider a high type student. Fixing the strategies of the other students as above, the following table provides possible realizations of the types of the remaining three students and a corresponding best response of a high type student to the particular realization in each case. With an abuse of notation, let $|\mathbf{v}_x|$ denote the number of students of type $\mathbf{v}_x$. Note that we do not display those realizations involving a high type student as they will have no affect on equilibrium verification when ε is chosen to be sufficiently close to zero.

Realization	Probability	Best response	Payoff loss from $\sigma^{BOS}(\mathbf{v}_H)$	Minimum gain from $\sigma^{BOS}(\mathbf{v}_H)$
$\lvert\mathbf{v}_L\rvert = 3$	.42	$\sigma = s_1$	$-.06$	-
$\lvert\mathbf{v}_L\rvert = 2, \lvert\mathbf{v}_M\rvert = 1$	.42	$\sigma^{BOS}(\mathbf{v}_H)$	-	.11
$\lvert\mathbf{v}_L\rvert = 1, \lvert\mathbf{v}_M\rvert = 2$	.14	$\sigma^{BOS}(\mathbf{v}_H)$	-	.11
$\lvert\mathbf{v}_M\rvert = 3$	.02	$\sigma^{BOS} = s_0$	$-.07$	-

For example, the first row of the table represents the case when all three students are of low type, which occurs with probability $p_L^3 \cong .42$. In this case, a high type maximizes his payoff by ranking s_1 first, by which he receives a payoff of .35. But since $\sigma^{BOS}(\mathbf{v}_H)$ is not a best response to this realization, a high type receives only .29 by playing $\sigma^{BOS}(\mathbf{v}_H)$. The second row represents the case when two students are of low type and one of medium type, which occurs with probability $3p_L^2p_M \cong .42$. In this case, $\sigma^{BOS}(\mathbf{v}_H)$ is a best response of a high type to this realization, by which he receives a payoff of .29. The next-best action of a high type to this realization is playing $\sigma = s_1$, by which he receives $\frac{.35}{2} \cong .18$. Hence playing $\sigma^{BOS}(\mathbf{v}_H)$ gives him an extra payoff of at least .11 over any other strategy. The rest of the table is filled in similarly. It follows from the table that expected utility loss of a high type due to playing $\sigma^{BOS}(\mathbf{v}_H)$ when it is not a best response, is more than offset by his gain from playing $\sigma^{BOS}(\mathbf{v}_H)$ when it is a best response.

Consider a medium type student. Fixing the strategies of the other students as above, the following table provides possible realizations for the types of the remaining three students and the corresponding best responses of a medium type student to the particular realization in each case. Once again, we do not display those realizations involving a high type student.

Realization	Probability	Best response	Payoff loss from $\sigma^{BOS}(\mathbf{v}_M)$	Minimum gain from $\sigma^{BOS}(\mathbf{v}_M)$
$\lvert\mathbf{v}_L\rvert = 3$	.42	$\sigma^{BOS}(\mathbf{v}_M)$	-	.12
$\lvert\mathbf{v}_L\rvert = 2, \lvert\mathbf{v}_M\rvert = 1$	.42	$\sigma^{BOS}(\mathbf{v}_M)$	-	.04
$\lvert\mathbf{v}_L\rvert = 1, \lvert\mathbf{v}_M\rvert = 2$	.14	$\sigma = s_0$	$-.15$	-
$\lvert\mathbf{v}_M\rvert = 3$	.02	$\sigma = s_0$	$-.44$	-

It follows from the table that expected utility loss of a medium type due to playing $\sigma^{BOS}(\mathbf{v}_M)$ when it is not a best response, is more than offset by his gain from playing $\sigma^{BOS}(\mathbf{v}_M)$ when it is a best response. Thus, $(\sigma^{BOS}(\mathbf{v}_L), \sigma^{BOS}(\mathbf{v}_M), \sigma^{BOS}(\mathbf{v}_H))$ is a Bayesian equilibrium under Boston. In particular, $EU^{BOS}_{\mathbf{v}_H} \cong .29$.

Next consider the following strategies under Shanghai: $\sigma^{SHA}(\mathbf{v}_L) = \sigma^{SHA}(\mathbf{v}_M) = s_0s_1s_2s_3$ and $\sigma^{SHA}(\mathbf{v}_H) = s_1s_2s_0s_3$. We claim that these strategies constitute a symmetric Bayesian Nash equilibrim for a sufficiently small ε. For a low type student, it is a dominant strategy to rank truthfully. Consider a high type student. Fixing the strategies of the other students as above, for any particular realization (that does not involve a high type), a high type student faces three students that are playing $\sigma^{SHA}(\mathbf{v}_L)$, and as calculated above for the example with complete information, it is then a best response for him to play $\sigma^{SHA}(\mathbf{v}_H)$. Similarly, for a medium type student, it is also a best response for him to play $\sigma^{SHA}(\mathbf{v}_H)$ for any particular realization (that does not involve a high type). Thus, $(\sigma^{SHA}(\mathbf{v}_L), \sigma^{SHA}(\mathbf{v}_M), \sigma^{SHA}(\mathbf{v}_H))$ is a Bayesian equilibrium under Shanghai. In particular, $EU^{SHA}_{\mathbf{v}_H} \cong .32 > EU^{BOS}_{\mathbf{v}_H}$. ∎

Description of the Algorithm for the Asymmetric Class of Parallel Mechanisms:

Let $S = (e_0, e_1, e_2, \ldots)$ be a given choice sequence.

Round t $=0$**:**

- Each student applies to his first choice. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his e_0-th choice school, applies to his next choice. If a student has been rejected from all his first e_0-choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.
- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

In general,

Round t ≥ 1**:**

- Each unassigned student from the previous round applies to his $\sum_{i=0}^{t-1} e_i + 1$-st choice school. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

In general,

- Each rejected student, who is yet to apply to his $\sum_{i=1}^{t} e_i$-th choice school, applies to his next choice. If a student has been rejected from all his first $\sum_{i=1}^{t} e_i$-choices, then he remains unassigned in this round and does not make any applications until the next round. Each school x considers its applicants. Those students with highest $x-$priority are tentatively assigned to school x up to its quota. The rest are rejected.

- The round terminates whenever each student is either assigned to some school or has remained unassigned in this round. At this point all tentative assignments are final and the quota of each school is reduced by the number students permanently assigned to it.

The algorithm terminates when each student has been assigned to a school. At this point all the tentative assignments are final. The mechanism that chooses the outcome of the above algorithm for a given problem is called the *application-rejection mechanism* S and denoted by φ^S.

Proof of Theorem 3: Clearly, Theorem 1 shows this result for the special case when all the terms in a choice sequence are identical. It is fairly straightforward to check that the proof of Theorem 1 depends only on the number of choices that are considered in round 0 and not on the number of choices considered in any subsequent round of the application-rejection algorithm. Hence, the same proof still applies once Lemmas 1 and 2 are appropriately modified for the extended class. For brevity, we omit these details. ∎

Proof of Proposition 7: Since the proof is analogous to that of Proposition 3, for brevity we only describe the necessary modifications.

Part (i). If S is an additive decomposition of S', it is straightforward to show analogously to the proof of part (i) of Proposition 2 that at any problem the outcome of φ^S is stable, the outcome of $\varphi^{S'}$ is exactly the same stable matching. For the converse, a simple variant of the same example could be used to show that φ^S can choose an unstable matching for a problem $\varphi^{S'}$ chooses a stable matching.

Part (ii). Suppose that S is not an additive decomposition of S'. Let t be the smallest index such that $e_t \neq e'_t$. Similarly to the proof of part (ii) of Theorem 1, one can construct a problem where a priority violation occurs for the $\sum_{i=0}^{t} e_i + 1$-st choice of a student, which leads to an unstable matching under φ^S but not under $\varphi^{S'}$.

We next describe the construction of a problem where the outcome of φ^S is stable while that of $\varphi^{S'}$ is not. Since S is not an additive decomposition of S', there exists an index t such that $\sum_{i=0}^{t} e'_i \neq \sum_{i=0}^{l} e_i$ for any l. Then choose the largest k and the smallest k' such that $\sum_{i=0}^{k} e_i < \sum_{i=0}^{t} e'_i < \sum_{i=0}^{k'} e_i$. Once again using a variant of the problem in the proof of part (ii) of Theorem 1, one can construct a problem where a priority violation occurs for the $\sum_{i=0}^{t} e'_i + 1$-st choice of a student, which leads to an unstable matching under $\varphi^{S'}$ but not under φ^S. ∎

Proof of Theorem 4: The first statement follows from Theorem 3 and Proposition 7. The second statement be shown similarly to the proof of Theorem 2 replacing e and e' respectively by e_0 and e'_0. ∎

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