Default and Prepayment Modelling in Participating Mortgages

Yusuf Varli and Yildiray Yildirim*

Current version: July 11, 2014

Abstract

Since the 2007 financial crisis, the mortgage market has been renovating its tools and instruments in order to avoid a new crisis. One such innovative instrument is the participating mortgage, in which the lender gains part of the net operating income and/or future appreciation. In this paper, we establish a financing model for participating mortgages, incorporating early termination options such as default and two prepayment clauses, defeasance and prepayment penalty. Later, we illustrate a detailed sensitivity analysis and get practical results. The values of early termination options depend on the choice of parameters in the model, as well as the term structure of short term rates. Finally, we show that a participation rate of 11.24% results in zero mortgage interest rate using the parameters in our simulation.

Keywords: Participating mortgages; Credit risk; Prepayment risk
JEL Classification: G21; G32; R30

*Varli, yusuf.varli@borsaistanbul.com, Research Department, Borsa İstanbul; Yildirim (Corresponding Author), yildiray@syr.edu, Whitman School of Management, Syracuse University, 721 University Ave Suite 500, Syracuse, NY 13244. We thank Brent Ambrose, Anthony Sanders, Shahid Ebrahim, and the seminar participants at the BIFEC conference in Istanbul for their helpful comments and suggestions.

1 Introduction

Over the past two decades, mortgage products have become more prominent in the fixed-income market. The need for such products varies in accordance with the demand of the borrower and specific characteristics of the market. One of the most interesting types of loans is called a participation mortgage (i.e. participating mortgages). Participating mortgages are based on the risk sharing between the lender and the borrower. They allow the borrower to have the ownership in the property while sharing the risk of changes in the market with the lender. In return the lender is compensated with excess payoffs from the mortgaged property. However, until relatively recently, little has been written on these mortgages, and even now, literature has not addressed the effects of default and prepayment risks in pricing such mortgages.

A participation mortgage (i.e. PM) allows borrowers to obtain below-market interest rates in return for a percentage of the property's future appreciation and/or net operating income. PM's were first introduced mid-1980s, as an alternative to fixed rate mortgages, when interest rates were high. However they were unpopular, because borrowers were reluctant to share in the appreciation of the property and adjustable rate mortgages were also introduced around the same time which had lower initial rates. Pension fund and insurance firms were the primary investors, because they could invest in equity and enjoy the profits from property appreciation. However, due to poorly written loan origination agreements coupled with the capital requirements of the Financial Institutions Reform, Recovery, and Enforcement Act of 1989 (FIRREA)[1] participating mortgages were never popular.

Rather than packaging mortgages and creating mortgage-backed securities to reduce the mortgage rate[2] for affordable housing, the recent financial crisis has proven that risk sharing could reduce the magnitude of the impact in case of the market crash. Caplin et al. (2008) argues that "development of shared appreciation mortgage (i.e. SAM) markets

[1]FIRREA chartered the Resolution Trust Corporation to manage insolvent thrifts formerly insured by the Federal Saving and Loan Insurance Corporation. It adapted a new regulation, making it difficult for saving institutions to hold certain amount of real estate loans. The total regulatory capital amount became 8% thereafter. The banks stopped in real estate loans held by commercial banks, and had a 100% risk-weighted classification. Lastly, it also made banks onerous to liquidate commercial mortgages and curtail originating them (see Hayre (2001).

[2]Separating certain type of illiquid asset from the firm's general risk will allow the company raise funds at a lower cost than if it could have raised the fund by issuing debt or equity (Pennacchi (1988)). Similarly, when mortgages are packaged and mortgage backed securities are created, it reduces the mortgage interest rates further.

in the United States would moderate the impending decline in homeownership and lower the risk of future housing crashes. SAMs can increase the affordability of homeownership by reducing the amount of monthly payments and spreading risk more broadly between borrower and lender…". The PM can serve as a vaccine and prevent the next potential financial crisis. However the risk of default and prepayment under PM's needs assessing in order to prevent problem areas from arising. This paper examines these potential problem areas and creates a path around them.

In conventional banking, the mortgage lender is interested with the refund of a given debt and does not consider the property's appreciation. For a commercial participation mortgage the expected performance and risk of the investment determines credit and debt positions of the lender and the borrower respectively. While the lender can receive a return higher than the market interest rate, borrowers may also have advantageous mortgage rates. Similar condition can be transcribed for the borrower of a residential mortgage. She forgoes a ratio of the property's rent or sale proceeds in order to get lower mortgage payments. Additionally, participation conditions for any kind of property can be adjusted in the contract depending on the negotiation and agreement between the borrower and lender.

Few of the earlier studies emphasize the general framework of participating mortgages. The rest of the literature focuses on a similar but more specific type of mortgage called a shared appreciation mortgage. For example, Alvayay et al. (2005) represents a partial equilibrium model to estimate the extent of the lender's participation and conducts a comparative analysis of the factors affecting it. Ebrahim (1996) demonstrates that participating mortgages improve social welfare which implies that they are pareto superior to conventional mortgages. Ebrahim and Hussain (2011) establishes a basic framework of participating mortgages and describes a facility to the mortgage system. However, they use constant risk free interest rate as a discount rate in their model. The definition of general participating mortgage in the paper is split up to different forms such as shared income, shared equity and shared appreciation mortgages. We extend their structure into more realistic case incorporating default and prepayment options, adopting stochastic interest rate model.

Initial studies on participating mortgages rely on the model as an attempt to reduce the levels of high interest payments in the U.S. (See Dougherty et al. (1982)). Additionally, Page and Sanders (1986) and Dougherty et al. (1990) also focus on the effects of interest rate risk on the SAM's. One of the more comprehensive studies is Sanders and Slawson (2005) which forms the mortgage pricing model for SAM's only in adapting the fixed rate mortgage model of Kau et al. (1992).

The purpose of this paper is to contribute to the theoretical understanding of pricing participating mortgages by incorporating early termination clauses due to default and prepayment. We employ three types of options namely default, and two prepayment clauses, defeasance and prepayment penalty which are widely used in commercial and residential mortgages respectively. The option pricing method which is similar to Hilliard et al. (1998) is embedded into the model and Longstaff and Schwartz (2001) where the simulation method is used to calculate prices of options.

Our numerical analysis documents that an increase in the participation rate for appreciation increases in prepayment and does not have a significant increases in default values. However, an increase in income shares increases both the prepayment and default values. For shared equity mortgages, the lender forgoes interest payments from the borrower by getting a proportion of both net operating income and sale proceeds. In an example, we show that if income and appreciation participation rates are 11.24%, then fixed mortgage interest rate becomes zero.

The remainder of the paper is as follows: the next section introduces participation mortgage model; section three includes prepayment and default risks into model; section four documents the simulation results, finally, section five summarizes the finding and concludes.

2 The model

Following Ebrahim and Hussain (2011), we assume that net operating income from operations by renting the property, namely the profit process P_t, can be defined as

$$dP_t = (r_t - \delta_P)P_t dt + \sigma_P P_t dZ_t^P \tag{1}$$

where r_t is risk free rate of interest and δ_P is constant periodic cash yield. Additionally, σ_P denotes the volatility of profit process and Z_t^P is the Brownian motion under risk neutral. The current profit flow P_0 is assumed as a proportional of present value of expected profit process assuming the property has infinite duration, defined as

$$P_0 = \delta_P \int_{s=0}^{\infty} e^{-r_s(t)} E[P_t] dt, \tag{2}$$

We define the real estate property value, H_t, is generated from the following stochastic process

$$dH_t = (r_t - \delta_H) H_t dt + \sigma_H H_t dZ_t^H, \tag{3}$$

where δ_H is referred as rental rate and represents the stream of rental income that the property provides, which can be viewed as a percentage of the value of the property. Note that borrower and lender shares the maintenance cost for the property, in proportion to their participation in the mortgage. Kau et al. (1992) defines δ_H as a service flow from using the real estate over time. The volatility σ_H in the process indicates how the property value deviates from its mean. Z_t^H is donated as standard risk neutral Brownian motion for the process. For simplicity and without loss of any generality, we do not allow a correlation for Brownian motions of both profit and property values, $E[dZ_t^P dZ_t^H] = \rho^{PH} dt = 0$.

We assume the initial loan balance of Q_0, the loan to value ratio of L, and the maturity of the mortgage as T. The loan includes continuous mortgage payments of a_t for all $t \in [0, T]$ and terminal payment B_T at maturity. The outstanding loan balance (i.e. OLB), Q_s, at any time $s \in [0, T]$ is equal to sum of discounted expected value of future mortgage payments and terminal balance such that

$$Q_s = \int_s^T e^{-r_s(t)(t-s)} E_s[a_t] dt + e^{-r_s(T)(T-s)} E_s[B_T] \tag{4}$$

For simplicity, we assume a non-amortizing mortgage, and outstanding loan balance for each period equals to initial loan payment, implying $Q_t = Q_0 = B_T$ for all $t \in [0, T]$. Continuous mortgage payments are determined with a constant proportion i of OLB and $a_t = iQ_t = iQ_0$ for all $t \in [0, T]$, where i is the mortgage rate representing the cost of using mortgage. If there is no prepayment, default, and any other risk, then the mortgage rate i equals to the risk free interest rate of r.

In comparison to conventional mortgage, participating mortgages offer a participative contract between lender and borrower. In return for reduced mortgage rate, PM's promise lenders to a part of either excess payoff from periodic profit stream or gain of sale proceeds or both. In other words, the borrower compensates the declined mortgage rate in the mortgage contract by giving a share of excess profit flow (i.e. $(P_t - K)^+$) or appreciation of the property value at the maturity time of mortgage (i.e. $(H_T - H_0)^+$) to the lender. K and H_0 denotes the fixed threshold for profit flow and initial value of property respectively.

These threshold points can change depending on negotiation and agreement between the borrower and lender. The share of excess profit flow is binding by the contract, so both the lender and the borrower declare to accept the amount of profit. To determine the amount of the excess profit in each period, an index or a special structure can be utilized. Therefore, continuous mortgage payments, a_t, and remaining balance at maturity B_T in participating mortgages now becomes

(5) $$a_t = iQ_t + \theta_P(P_t - K)^+, and$$

(6) $$B_T = Q_T + \theta_H(H_T - H_0)^+$$

where θ_P and θ_H are participation rates for excess profit flow and appreciation of property value respectively.

We include the term structure of short term rates into the model to construct a discount process for future payments (see Bakshi et al. (1997) and Deng (1997)) . Two types of short term rates are considered. First, is the defaultable short term rates that are used to discount the borrower's future payments, following Vasicek (1977) process as

(7) $$dr_t = \alpha(\theta - r_t)dt + \sigma_r dZ_t^r$$

where α is the speed of mean reversion, θ is the mean value of short term rates in the long-run and σ_r denotes the volatility of the rates. Also, correlation between profit cash flow and property value is assumed to be zero, that is $E[dZ_t^P dZ_t^H] = \rho^{PH} dt = 0$. However, we allow short rates to be correlated with both profit cash flow and property value such like $E[dZ_t^P dZ_t^r] = \rho^{Pr} dt$ and $\rho^{Pr} \neq 0$ and $E[dZ_t^H dZ_t^r] = \rho^{Hr} dt$ and $\rho^{Hr} \neq 0$.

Secondly, the risk-free short term rate process, which is used for discounting the Treasury security payments, is given by

$$d\tilde{r}_t = \alpha(\tilde{\theta} - \tilde{r}_t)dt + \sigma_{\tilde{r}} dZ_t^{\tilde{r}}, \tag{8}$$

which is similar to the process defined in Equation (10). The long term mean rate and the volatility has declining trends such as $\tilde{\theta} < \theta$ and $\sigma_{\tilde{r}} < \sigma_r$. We assume Brownian motions in risky and risk-free short term processes are not correlated.

We can now write the OLB at s as

$$\begin{aligned} Q_s &= \int_s^T e^{-r_s(t)(t-s)} E_s[iQ_t]dt + \theta_P \int_s^T e^{-r_s(t)(t-s)} E_s[(P_t - K)^+]dt + e^{-r_s(T)(T-s)} Q_T \\ &\quad + \theta_H e^{-r_s(T)(T-s)} E_s[(H_T - H_0)^+]. \end{aligned} \tag{9}$$

For simplicity, assuming non-amortizing loans such that $Q_0 = Q_s = Q_T$, the OLB becomes

$$Q_s = \int_s^T e^{-r_s(t)(t-s)} iQ_0 dt + \theta_P \int_s^T c(P_s, K, t, r_s(t))dt + e^{-r_s(T)(T-s)} Q_0 + \theta_H c(H_s, H_0, T, r_s(T)), \tag{10}$$

where $c(.)$ represents the pricing formula for European call option. The value of call option written on the profit cash flow, with strike K and at time s for the any maturity time $t \in (s, T]$ is

$$c(P_s, K, t, r_s(t)) = P_s e^{-\delta_P(t-s)} N(d_1(s,t)) - K B_{s,t} N(d_0(s,t)) \tag{11}$$

where N denotes the standard normal cumulative distribution function. The values as an input for N in Equation (14) are given by

$$d_\lambda(s,t) = \frac{(\ln P_s) - (\ln K) - (\ln B_{s,t}) - \delta_P(t-s) + (\lambda - \frac{1}{2})v_{s,t}^2}{v_{s,t}} \qquad \lambda \in \{1, 0\} \tag{12}$$

where $v_{s,t}^2$ satisfies

$$v_{s,t}^2 = \frac{\sigma_r}{\alpha^2}\Big((t-s) + \frac{2}{\alpha}e^{-\alpha(t-s)} - \frac{1}{2\alpha}e^{-2\alpha(t-s)} - \frac{3}{2\alpha}\Big) + \sigma_P^2(t-s) + \frac{2\rho^{Pr}\sigma_r\sigma_P}{\alpha}\big[(t-s) - (D_{s,t})\big]. \tag{13}$$

Furthermore, $B_{s,t}$ represents the price of zero-coupon bond with maturity t and formulation of that bond in this context is

$$B_{s,t} = e^{\left[D_{s,t}(k - r_s) - (t - s)k - \left(\frac{\sigma_r D_{s,t}}{2\sqrt{\alpha}}\right)^2\right]} \tag{14}$$

where $D_{s,t} = \frac{1 - e^{-\alpha(t-s)}}{\alpha}$ and $k = \theta + \frac{\sigma_r q}{\alpha} - \frac{\sigma_r^2}{2\alpha^2}$. q is market price of risk and we allow that it is zero. As a reminder, value of call option written on the property value $\theta_H c(H_s, H_0, T, r_s(T))$ can also be calculated the same way considering the respective parameters. The description of the term structure $r_s(t)$ of interest rate is illustrated by

$$r_s(t) = -\frac{lnB_{s,t}}{(t - s)} = \frac{\left[-D_{s,t}(k - r_s) + (t - s)k + \left(\frac{\sigma_r D_{s,t}}{2\sqrt{\alpha}}\right)^2\right]}{(t - s)}. \tag{15}$$

Equations from (14) to (18) are compatible with the findings of Merton (1973) and Brigo and Mercurio (2006).

2.1 Mortgage Rate Calculation

The borrower has to decide what percent of the excess profit flow (i.e. θ_P) and appreciation (i.e. θ_H) to share with the lender in order to lower the mortgage rate. At the time of mortgage origination (i.e. $s = 0$), we have

$$i = \frac{1 - e^{-r_0(T)T}}{\int_0^T e^{-r_0(t)t}dt} - \left[\frac{\theta_P \int_0^T c(P_0, K, t, r_0(t))dt + \theta_H c(H_0, H_0, T, r_0(T))}{Q_0 \int_0^T e^{-r_0(t)t}dt}\right], \tag{16}$$

where the part in brackets on the right hand side reflects the reduction in mortgage rate in the case of a participating mortgage in comparison to conventional mortgage. An increase in participation rate lowers the mortgage rate. Other things effecting the mortgage rate are initial loan-to-value (i.e. LTV) ratio L and Q_0, i.e. $Q_0 = L\text{x}H_0$.

Table 1 shows the parameters used in calculating the base case mortgage rate. In Figure 1, we plot the mortgage rate for different state variables. For each of the subfigures, it is clear that mortgage rate decreases with increasing level of participation rates. While mortgage rate in conventional mortgage (i.e. $\theta_P = \theta_H = 0$) for LTV of 80% is calculated as 8.04%, the rate is reduced to 0.89% in participating mortgage case with 10% participation for profit cash and property value (i.e. $\theta_P = \theta_H = 10\%$).

The desired level of mortgage rates can be reached by altering loan-to-value ratio.

The base case scenario indicates that cost of using mortgage can be reduced in participating mortgage setup by changing the values of participation rates and loan-to-value ratio. However the reduction depends on the sensitivity of loan condition with respect to parameters chosen. Furthermore, profit threshold and initial value of the property determine the continuous excess profit and appreciation of the property respectively. Figure 1 also indicates that mortgage rate moves up when each of these threshold points increases in participating mortgages.

2.2 Types of participating mortgage

Depending on the values of participation rates, various kinds of agreements can be arranged between the borrower and the lender. Three common types of participating mortgages are defined at Ebrahim and Hussain (2010) and Ebrahim and Hussain (2011). These are Shared Income, Shared Appreciation and Shared Equity Mortgages. In the first one, the lender participates in the mortgage by receiving only a proportion of the net operating income in exchange for allowing coupon rate below the market interest rate. In shared appreciation mortgages, the lender participates the mortgage by receiving only a proportion of the sale proceeds in exchange for allowing coupon rate below the market interest rate. For shared equity mortgages, the lender waives interest payments of the borrower by receiving a proportion of the net operating income and sale proceeds.

Figure 2 examines the conventional and three types of participation mortgages. Mortgage rate in the base case is 4.46%, if both participation rates are given as 5% ($\theta_P = \theta_H = 5\%$). The mortgage rate increases to 4.69% in shared income mortgages ($\theta_P = 5\%, \theta_H = 0$) and 7.81% in shared appreciation mortgages ($\theta_P = 0, \theta_H = 5\%$). Additionally, the lender fully trades off interest payments in a shared equity mortgage contract whenever the participation brings the same return. When both participation rates are nearly 11.24%, then mortgage rate becomes zero in the base case scenario.

3 Prepayment and Default Modelling

Kalotay et al. (2004), Deng et al. (2000), Ciochetti and Vandell (1999), Riddiough and Thompson (1993) among others use option pricing models in conventional mortgage pricing. In this section, we will introduce the early termination clauses of prepayment and default for the PM using the option pricing model.

3.1 Prepayment

Commercial mortgages differ from residential loans in several ways. Commercial mortgages are backed by income producing properties, like office buildings, retail shops, multifamily apartments, and hotels. Moreover, the commercial mortgages can be either non-amortizing or partially amortizing with a balloon payment. Their prepayment provisions are also different then single-family residential. Mortgage contracts generally include a penalty to restrict borrower's ability to refinance the loan. The most commonly used clauses in commercial mortgage agreement are yield maintenance clauses that enable the lender attain the same yield as if the borrowers continue making the promised payments; lockout periods, allowing the lender to charge a diminishing penalty on the outstanding loan balance; and defeasance where the borrower pledges to the lender U.S Treasury securities whose cash flows equal to the mortgage payment. On the other hand, residential mortgages can only have prepayment penalties. However, our paper is applicable to both residential and commercial mortgages. Therefore, we concentrate on the prepayment penalty and defeasance in quantifying prepayment risk.

Dierker et al. (2005) indicates that prepayment conditions for residential mortgages and commercial mortgages differ. For example, residential mortgages may prepay when interest rates fall, while commercial mortgages may prepay when interest rates rise. For this reason, prepayment penalty and defeasance analyses are generalized for residential mortgages and commercial mortgages respectively.

3.1.1 Prepayment Penalty

In the case of declining short term rates, the borrower may want to adjust the costly loan balance by refinancing, or to cash out the equity built up to buy another property. The lender induces a penalty to avoid the prepayment. Thus, the saving comes from the difference between borrower's new and existing mortgage interest payments and the cost is due to prepayment.

Let us define the mortgage rate as $i(r_0)$ in the contract at the beginning and it is constant for each period until maturity. However, there is a refinancing possibility in each time s after the mortgage originated at time 0 due to declining short term rates. If the borrower chose to refinance, s/he has to pay the existing OLB in the amount $Q_s(i(r_0))$. The new OLB at s after refinancing is $Q_s(i(r_s))$ which is less than the old balance $Q_s(i(r_0))$. We define the prepayment penalty as a proportion p of old OLB $Q_s(i(r_0))$ at the time of prepayment. Therefore, prepayment takes place if the savings is more than the prepayment penalty. The condition of prepayment at time s for participating mortgages is

$$(17) \qquad Q_s(i(r_0)) - Q_s(i(r_s)) = Saving \geq Prepayment\,Penalty = p\text{x}Q_s(i(r_0))$$

where p is the constant penalty rate.

3.1.2 Defeasance

Another clause of prepayment generally used in commercial mortgages is called defeasance. It can be defined as an exchange of mortgage payments with Treasury securities enabling same amount of payments. We follow Dierker et al. (2005) to describe the defeasance condition in participating mortgages. Here, we make a critical assumption that there is a liquid tradable participating mortgage market, and the borrower substitutes the payments of commercial participating mortgage by a treasury security that provides same amount of payments. Namely, at time s the investor pays the amount of

(18)
$$Q_s(\tilde{r}_s(t)) = \int_s^T e^{-\tilde{r}_s(t)(t-s)} iQ_0 dt + \theta_P \int_s^T c(P_s, K, t, \tilde{r}_s(t))dt + e^{-\tilde{r}_s(T)(T-s)}Q_0 + \theta_H c(H_s, H_0, T, \tilde{r}_s(T))$$

in order to refinance the the payments of the loan that amounts of

(19)
$$Q_s(r_s(t)) = \int_s^T e^{-r_s(t)(t-s)} iQ_0 dt + \theta_P \int_s^T c(P_s, K, t, r_s(t))dt + e^{-r_s(T)(T-s)}Q_0 + \theta_H c(H_s, H_0, T, r_s(T))$$

where the risk free rates are less than the defaultable rates, namely $\tilde{r}_s(t) < r_s(t)$ for all $t \in [s, T]$. Also, $Q_s(\tilde{r}_s(t))$ is the loan balance amount of security which pays same amount of OLB of the loan $Q_s(r_s(t))$ but $Q_s(\tilde{r}_s(t)) > Q_s(r_s(t))$. This relation explains the cost of defeasance.

The advantage of defeasance is provided by increased liquidity that is created by equity position after defeasance which is given as $H_s - Q_s(\tilde{r}_s(t))$. Investor funds this liquidity as a down payment for another project and so invests up to $\left[\frac{H_s - Q_s(\tilde{r}_s(t))}{1 - L}\right]$ in the new project. As a result, this investment contributes an extra amount of $\left[\frac{L\text{x}H_s - Q_s(\tilde{r}_s(t))}{1 - L}\right]$ in comparison to property value H_s at the time of defeasance. As proposed in Dierker et al. (2005), investing in the new project brings a benefit equal to the constant k per unit of capital such that $(e^{\mu(T-s)} - 1)$, where μ denotes the project's excess return above its required rate of return. Consequently, benefit of defeasance Π_s by liquidation is given by

$$\Pi_s = \left[\frac{L\text{x}H_s - Q_s(\tilde{r}_s(t))}{1 - L}\right]\text{x}\left(e^{\mu(T-s)} - 1\right). \tag{20}$$

We conclude that defeasance in participating mortgages occurs if the benefit from defeasance exceeds the cost of the defeasance, that is

$$\Pi_s \geq Q_s(\tilde{r}_s(t)) - Q_s(r_s(t)). \tag{21}$$

Plugging Equations (18), (19) and (20) into Equation (21), we write the condition of defeasance given by

$$\begin{aligned} (22)\quad & \left[\frac{L\text{x}H - Q_s(\tilde{r}_s(t))}{1 - L}\right]\text{x}\left(e^{\mu(T-s)} - 1\right) \geq \\ & \left[\int_s^T e^{-\tilde{r}_s(t)(t-s)} iQ_0 dt + \theta_P \int_s^T c(P_s, K, t, \tilde{r}_s(t))dt + e^{-\tilde{r}_s(T)(T-s)}Q_0 + \theta_H c(H_s, H_0, T, \tilde{r}_s(T))\right] \\ & - \left[\int_s^T e^{-r_s(t)(t-s)} iQ_0 dt + \theta_P \int_s^T c(P_s, K, t, r_s(t))dt + e^{-r_s(T)(T-s)}Q_0 + \theta_H c(H_s, H_0, T, r_s(T))\right]. \end{aligned}$$

3.2 Default

A borrower defaults on her mortgage obligation when she stops making the promised monthly payments. The factors influencing the mortgage defaults are generally loan-to-value ratio (LTV) and debt service coverage ratio (i.e. DSCR) (see Vandell et al. (1993), Vandell et al. (1993), Ambrose and Sanders (2003)). Default happens when LTV ratio is higher than one, causing negative equity. Another condition of default depends on the DSCR which is the ratio of income available for debt servicing of mortgage. Whenever DSCR is less than one, the borrower may default on the mortgage. We define the default condition as

$$Q_s \geq H_s, and \tag{23}$$

$$iQ_0 + \theta_P(P_s - K)^+ \geq P_s. \tag{24}$$

The first one represents the condition where the OLB Q_s is higher than the value of the property H_s at time $s \leq T$. The second condition indicates the situation in which the periodic debt payment is not satisfied adequately by the net operating income of the property.

4 Simulation Results

In order to value early termination options for participating mortgages, we employ American options using the Longstaff and Schwartz (2001)'s simulation algorithm based on the use of least-squares estimation for the conditional expected payoff to the option holder. Table 2 documents the parameters used in the simulation. We assume the long term mean of the interest rate process is $\theta = 0.075$. Therefore, we use initial values for short rates of 5%, 7.5%, and 10% to indicate upward sloping, flat, and downward sloping term structure cases respectively.

Table 3 illustrates the default, defeasance and prepayment option values using the base parameters in Table 2 for three term structure cases. Note that default option values decrease as interest rates increase. This happens due to increasing value of underlying asset. As the asset value rises with increasing interest rate, default becomes less profitable. Therefore, simulation values in default options decrease.

On the other hand, value of defeasance increases with an increase in the initial value of short rate, because the treasury securities become cheaper. As an example, taking as $r_0 = 0.05$, it is more advantageous to defease the loan later in time, because the term structure has upward slope in this case. Furthermore, as initial value of short rates increases, the term structure slopes downward. Since the short rates has a diminishing trend, saving due prepayment increases. As a result, a rise in the value of prepayment option is to be explicit.

Table 4 investigates the value of each American option with changing loan to value ratios. The significant increase in the value of a default option can be associated with the ascent of indebtedness. The higher the loan-to-value ratio, the higher the default probability that a participating mortgage has. Higher rates of loan-to-value ratio increase the share of investors in the new project. Thus defeasance becomes more advantageous and value of defeasance increases significantly. The effect becomes stronger with an increased short rate. Similar to the default case, the option value of the prepayment penalty also increases as a result of higher indebtedness. Additionally, the change in option values with respect to different term structures preserves the impacts mentioned in Table 3.

The impacts of participation rates (θ_P and θ_H) on option values are illustrated in Table 5. As mentioned in Section 3.2, the conditions of default are LTV (Equation (23)) and DSCR (Equation (24)). An increase in the participation rates affects the LTV condition in two opposite channels. First, all else equal, the OLB goes up with increasing participation rates. The other channel comes from the negative effects of participation rates on the mortgage rate, so increasing participation rates lower the OLB. Although these two opposite channels are neutralized in LTV condition, participation rate θ_P has a positive effect on DSCR condition of the default. Therefore, the value of default increases as participation rate θ_P for profit flow raises, but no significant impact of the participation rate θ_H for appreciation of property on the option of default is found.

Table 5 also indicates that there is no significant effect of θ_P on the value of defeasance option in a given simulation setup. However, the defeasance option suffers from increasing value of θ_H. The advantage of defeasance provided by increased liquidity decreases with an increase in the participation rate for the property appreciation. Thus the value of defeasance option decreases. For the flat term structure of short rates, 20% increase in θ_H results to decrease of defeasance option value from 11.9931 to 9.0697. In the case of prepayment penalty, both participation rates have positive impact on the value of prepayment option. An explanation depends on the parameters. It can be stated that as shares given to the lender increase, willingness to prepay increases since the difference between the saving and penalty increases. The saving due to prepayment becomes more advantageous in the case of rising participation rates.

We allow that the short term rates are correlated with both profit cash flow and property value. Table 6 explains the behavior of each American option with different correlations in case of interest rate with both profit process and property value. The default option values are less when correlations are positive in comparison to negative correlations case. The default option is mainly valued according to the difference between OLB and prices. With increasing correlations, OLB and prices move in opposite directions. Under the positive correlation case, falling prices with decreasing short rates cause to increase the difference between OLB and prices. Therefore, values of the default option increases.

As similar of the default case, the defeasance option values increase with increasing correlations. Here, higher real estate value with higher short rates raises the benefit of defeasance while the cost of defeasance decreases. Thus defeasance option becomes more valuable. However in the prepayment penalty case, option values diminish as correlations increase. When the correlation between short rates and prices go up, they move to same direction. In other words, when short rates reduce, prices decrease as well. So the prepayment saving due to reduction in short rates, suffers from decreasing prices. Furthermore, co-movement of short rates and prices lower the saving value. Thus, option values of prepayment decreases with increasing correlations.

We present the implications of changes in credit spread in Table 7. Here, we examine the effect of using riskier agency securities, instead of treasury securities in the defeasance option. For this reason, we arrange the difference of long term mean rates $\theta - \tilde{\theta}$ by changing the long term value $\tilde{\theta}$ of lower risk free rate. Increasing credit spread decreases the defeasance option value in each term structure. For example, for $r_0 = 0.05$, the value of defeasance option equals to 11.9931 as credit spread is 0.03. The option value decreases to 8.4356 when the credit spread increases to 0.05. Furthermore, we use only defaultable risk free rate and assume that the credibility of the mortgage borrower remains the same throughout the time on the default and prepayment penalty options. Thus, changing credit spread does not have any impact on the value of the options of default and prepayment penalty.

Table 8 presents the results of different option values for different term structure cases with various state variable volatilities. The volatilities of profit flow σ_P and property value σ_H are called *price volatilities*, and σ_r refers to volatility of short term rates.

For all term structure cases, default option value becomes more valuable with increasing prices and short rate volatilities. These results are quite intuitive because increasing volatilities raise the difference between OLB and prices. For the defeasance option, increases in the volatilities of short rate and real estate property value enhance the benefit of defeasance more than the cost of it. Therefore the defeasance option value moves up for all term structures. However, volatilities have different effects on the option of prepayment. While an increase in the volatility of profit flow raises the value of the prepayment option, volatility of property value has negative effect on prepayment. This result can be interpreted by referring the initial values of profit flow and property value. Lastly, the difference between saving and penalty in the prepayment option goes up by increasing of variation of short rates. Thus, a positive change in the short rate volatility raises the option value of prepayment.

The impact of changing maturity time of the mortgage on different options is analyzed in Table 9. Since the values of each option heavily depends on the prices in the mortgage, higher levels of prices increase as maturity time increases. As a result, using early termination options become more advantageous for all term structures. Therefore, we find that all option values increase with increasing maturity time of participation mortgage under all term structure cases.

5 Conclusion

A participation mortgage is a type of loan which portions of the excess payoffs are shared across borrower and lender for any kind of real estate. In order to get a mortgage rate below market interest rate, the borrower pays a proportion of net operating income and sale proceeds of the property. This kind of risk sharing strategy brings beneficial outcomes for both parties in a mortgage contract especially in emerging economies. Thus, participating mortgages surpass conventional mortgage instruments for a high return investment. The Borrower owes a rate less than market coupon rate and lender obtains yield more than the conventional mortgage rate.

Mortgage rate, which is one of the most important variables in mortgage financing, is constructed and then examined under base case parameters. Mortgage rate reduces to 0.89% from 8.04%in the base case, giving 10% participation to the lender in a given mortgage. It is also shown that reduction in the mortgage rate varies according the type of participating mortgage. Additionally, early termination options such as default, defeasance and prepayment penalty are configured harmoniously with the modeling of participating mortgages.

We have also conducted sensitivity analysis for option values under changing parameters in order to reach practical results. The slope of short rate term structure is influential for all prepayment options. One of the most interesting results of this paper is that the parameters of loan to value ratio, maturity time of the mortgage and variance of short term rates positively affect the values of each early termination option. That is, increasing either indebtedness or life time of the mortgage or volatility of short rates are critical factors that raise the values of all options in any kind of participating mortgage. For the purposes of a more comprehensive analysis, changes in almost all parameters are used to investigate the sensitivities of option values.

References

Alvayay, J., Harter, C., Smith, W., 2005. A theoretic analysis of extent of lender participation in a participating mortgage. Review of Quantitative Finance and Accounting 25, 383–411.

Ambrose, B., Sanders, A., 2003. Commercial mortgage-backed securities: Prepayment and default. Journal of Real Estate Finance and Economics 26, 179–196.

Bakshi, G., Cao, C., Chen, Z., 1997. Empirical performance of alternative option pricing models. Journal of Finance 52, 2003–2049.

Brigo, D., Mercurio, F., 2006. Interest rate models: Theory and practice. Springer Verlag, New York N.Y. .

Ciochetti, B., Vandell, K., 1999. The performance of commercial mortgages. Real Estate Economics 27, 27–61.

Deng, Y., 1997. Mortgage termination: An empirical hazard model with stochastic term structure. The Journal of Real Estate Finance and Economics 14, 309–331.

Deng, Y., Quigley, J., Van Order, R., 2000. Mortgage terminations, heterogeneity and the exercise of mortgage options. Econometrica 68, 275–307.

Dierker, M., Quan, D., Torous, W., 2005. Valuing the defeasance option in securitized commercial mortgages. Real Estate Economics 33, 663–680.

Dougherty, A., Van Order, R., Villani, K., 1982. Pricing shared-appreciation mortgages. Housing Finance Review 1, 361–375.

Dougherty, A., Van Order, R., Villani, K., 1990. A practical analysis of shared-appreciation mortgages. Housing Policy Debate. Office of Housing Policy Research, Fannie Mae,Washington 2, 43–48.

Ebrahim, M., 1996. On the design and pareto-optimality of participating mortgages. Real Estate Economics 24, 407–419.

Ebrahim, M., Hussain, S., 2010. Financial development and asset valuation: The special case of real estate. Journal of Banking and Finance 34, 150–162.

Ebrahim, M., Hussain, S., 2011. Participating mortgages and the efficiency of financial intermediation. Journal of Banking and Finance 35, 3042–3054.

Hayre, L., 2001. Guide to mortgage-backed and asset-backed securities. Wiley Finance.

Hilliard, J., Kau, J., Slawson, C., 1998. Valuing prepayment and default in a fixed-rate mortgage: A bivariate binomial options pricing technique. Real Estate Economics 26, 431–468.

Kalotay, A., Yang, D., Fabozzi, F., 2004. Ancoption-theoretic prepayment model for mortgages and mortgage-backed securities. International Journal of Theoretical and Applied Finance 7, 949–978.

Kau, J., Keenan, D., Muller, W., Epperson, J., 1992. A generalized valuation model for fixed-rate residential mortgages. Journal of Money, Credit and Banking 24, 279–299.

Longstaff, F., Schwartz, E., 2001. Valuing american options by simulation: A simple least-squares approach. Review of Financial Studies 14, 113–147.

Merton, R., 1973. Theory of rational option pricing. Bell Journal of Economics and Management Science 4, 141–183.

Page, F., Sanders, A., 1986. On the pricing of shared-appreciation mortgages. Housing Finance Review 5, 49–57.

Pennacchi, G., 1988. Loan sales and the cost of bank capital. Journal of Finance 43, 375–396.

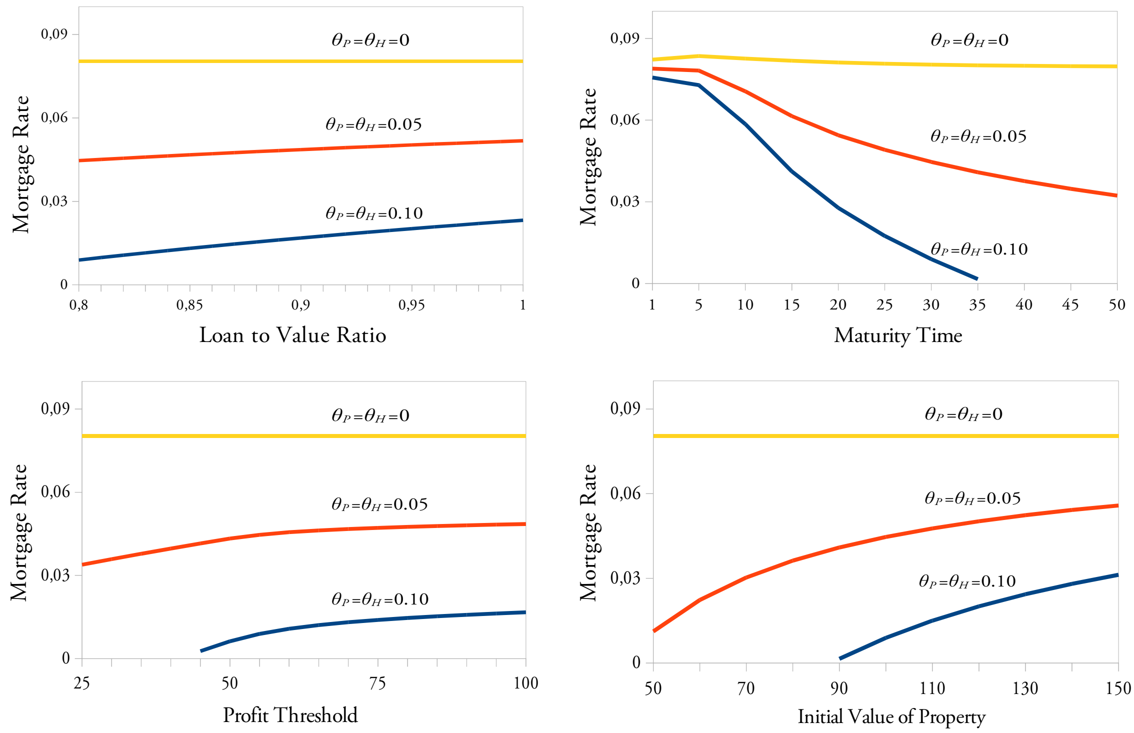
Riddiough, T., Thompson, H., 1993. Commercial mortgage pricing with unobservable borrower default costs. AREUEA Journal 21, 265–291.

Sanders, A., Slawson, C., 2005. Shared appreciation mortgages: Lessons from the uk. Journal of Housing Economics 14, 178–193.

Vandell, K., Barnes, W., Hartzell, D., Kraft, D., Wendt, W., 1993. Commercial mortgage defaults: Proportional hazards estimation using individual loan histories. Journal of the American Real Estate and Urban Economics Association 21, 451–480.

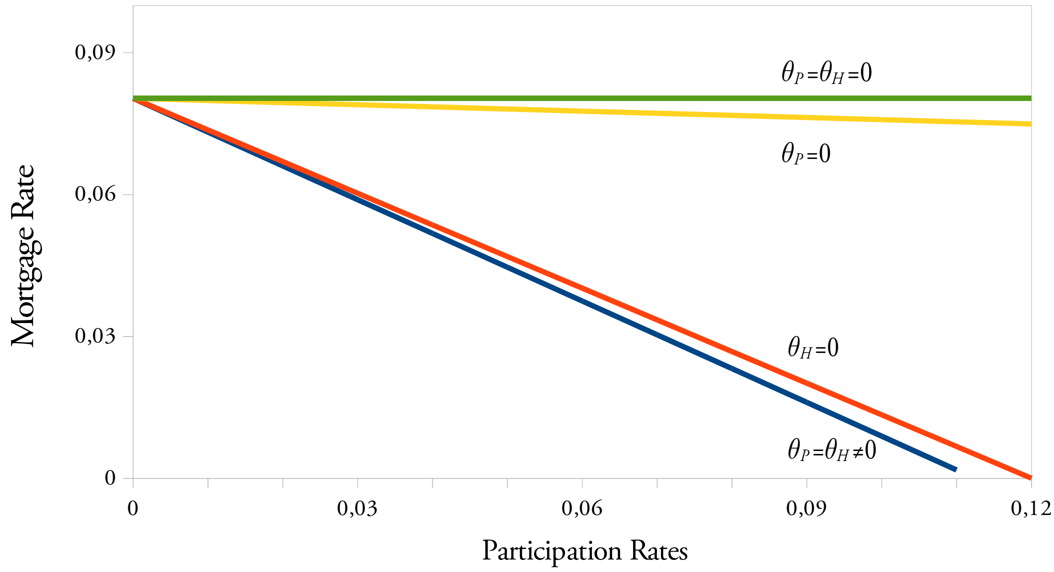
Vasicek, O., 1977. An equilibrium characterization of the term structure. Journal of Financial Economics 5, 177–188.

Figure 1: Effects of some state variables on the mortgage rate



Dependence of mortgage rate on various state variables in the model. The level of mortgage rate is shown in each sub-figures with changing participation rates ($\theta_P = \theta_H = 0, 0.05, 0.10$).

Figure 2: Effects of participating mortgage type on the mortgage rate



Each line refers to different type of participating mortgages. The line with $\theta_P = \theta_H = 0$ stands for Conventional Mortgages, $\theta_P = 0$ indicates Shared Appreciation Mortgages, $\theta_H = 0$ means Shared Income Mortgages and $\theta_P = \theta_H \neq 0$ shows Shared Equity Mortgages.

Table 1: Base case parameters for the calculation of mortgage rate

Parameter	Definition	Value
P_0	Initial value of profit flow	10
H_0	Initial value of property	100
K	Threshold for profit flow	11
r_0	Initial value of short term rate	7.5%
δ_P	Dividend rate	3%
σ_P	Volatility of profit flow	10%
δ_H	Service flow rate	3%
σ_H	Volatility of property value	10%
σ_r	Volatility of short term rate	1%
α	Speed of mean reversion	5%
θ	Long run mean of short term rates	7.5%
ρ^{Pr}	Correlation between profit flow and short term rate	-0.5
ρ^{Hr}	Correlation between property value and short term rate	-0.5
T	Life time of the loan	30
L	Loan to value ratio	80%
θ_P	Share rate for excess profit flow	10%
θ_H	Share rate for appreciation of property value	10%

Values of all necessary parameters to calculate mortgage rate in participating mortgages

Table 2: Base case parameters for option valuation

Parameter	Definition	Value
P_0	Initial value of profit flow	10
H_0	Initial value of property	100
K	Threshold for profit flow	11
r_0	Initial value of short term rate	7.5%
δ_P	Divident rate	3%
σ_P	Volatility of profit flow	10%
δ_H	Service flow rate	3%
σ_H	Volatility of property value	10%
σ_r	Volatility of short term rate	1%
$\sigma_{\tilde{r}}$	Volatility of risk free rate	0.6%
α	Speed of mean reversion	5%
θ	Long run mean of short term rates	7.5%
$\tilde{\theta}$	Long run mean of risk free rates	4.5%
ρ^{Pr}	Correlation between profit flow and short term rate	-0.5
ρ^{Hr}	Correlation between property value and short term rate	-0.5
T	Life time of the loan	30
L	Loan to value ratio	80%
θ_P	Share rate for excess profit flow	10%
θ_H	Share rate for appreciation of property value	10%
p	Penalty rate	2%
μ	Excess return of the new project	1%

Values of all necessary parameters to calculate the values for each option in participating mortgages

Table 3: Values of options under the base case scenario

	Default	Defeasance	Prepayment
$r_0 = 5\%$	0.8534	11.9931	6.6164
$r_0 = 7.5\%$	0.4437	17.0651	7.7488
$r_0 = 10\%$	0.1829	20.6101	8.6868

Comparison of option values for each termination case with different term structures.

Table 4: Values of options with changing loan to value ratio

	Default	Defeasance	Prepayment	
$L = 70\%$	0.3054	6.7757	6.2158	
$L = 80\%$	0.8534	11.9931	6.6164	$r_0 = 5\%$
$L = 90\%$	1.9907	30.7558	7.1196	
$L = 70\%$	0.1238	9.0337	7.2452	
$L = 80\%$	0.4437	17.0651	7.7488	$r_0 = 7.5\%$
$L = 90\%$	0.0964	42.1895	8.3525	
$L = 70\%$	0.0408	10.9990	8.0642	
$L = 80\%$	0.1829	20.6101	8.6868	$r_0 = 10\%$
$L = 90\%$	0.5483	50.6134	9.3543	

Effects of increasing level of loan to value ratio on early termination options under different term structures.

Table 5: Values of options with changing participation rates

		Default	Defeasance	Prepayment	
$\theta_P = 10\%$	θ_H=10%	0.8534	11.9931	6.6164	
$\theta_P = 30\%$	θ_H=10%	1.4216	13.0467	8.7632	$r_0 = 5\%$
$\theta_P = 10\%$	θ_H=30%	0.8690	9.0697	12.0961	
$\theta_P = 10\%$	θ_H=10%	0.4437	17.0651	7.7488	
$\theta_P = 30\%$	θ_H=10%	1.1012	15.9279	11.3137	$r_0 = 7.5\%$
$\theta_P = 10\%$	θ_H=30%	0.4195	13.2134	12.8392	
$\theta_P = 10\%$	θ_H=10%	0.1829	20.6101	8.6868	
$\theta_P = 30\%$	θ_H=10%	0.8644	19.5973	13.4426	$r_0 = 10\%$
$\theta_P = 10\%$	θ_H=30%	0.1782	15.6894	13.3058	

Effects of increasing level of participation rates on early termination options under different term structures.

Table 6: Values of options with changing correlation coefficients

		Default	Defeasance	Prepayment	
$\rho^{Pr} = -0.5$	$\rho^{Hr} = -0.5$	0.8534	11.9931	6.6164	$r_0 = 5\%$
$\rho^{Pr} = 0.5$	$\rho^{Hr} = 0.5$	3.1315	14.3988	6.0481	
$\rho^{Pr} = -0.5$	$\rho^{Hr} = -0.5$	0.4437	17.0651	7.7488	$r_0 = 7.5\%$
$\rho^{Pr} = 0.5$	$\rho^{Hr} = 0.5$	1.8715	18.4612	6.8133	
$\rho^{Pr} = -0.5$	$\rho^{Hr} = -0.5$	0.1829	20.6101	8.6868	$r_0 = 10\%$
$\rho^{Pr} = 0.5$	$\rho^{Hr} = 0.5$	1.1167	21.8334	7.1012	

Effects of increasing correlation coefficients on early termination options under different term structures.

Table 7: Values of options with changing difference between long term mean values of risk free rates

	Default	Defeasance	Prepayment	
$\theta - \tilde{\theta} = 0.03$	0.8534	11.9931	6.6164	
$\theta - \tilde{\theta} = 0.04$	0.8917	10.6622	6.4741	$r_0 = 5\%$
$\theta - \tilde{\theta} = 0.05$	0.8268	8.4356	6.4953	
$\theta - \tilde{\theta} = 0.03$	0.4437	17.0651	7.7488	
$\theta - \tilde{\theta} = 0.04$	0.4609	14.9528	7.7365	$r_0 = 7.5\%$
$\theta - \tilde{\theta} = 0.05$	0.4559	14.3699	7.5067	
$\theta - \tilde{\theta} = 0.03$	0.1829	20.6101	8.6868	
$\theta - \tilde{\theta} = 0.04$	0.1679	20.3464	8.4887	$r_0 = 10\%$
$\theta - \tilde{\theta} = 0.05$	0.1780	19.9594	8.6719	

Effects of increasing difference between long term mean values of risk free rates on early termination options under different term structures.

Table 8: Values of options with changing volatilities of state variables

			Default	Defeasance	Prepayment	
$\sigma_P = 10\%$	$\sigma_H = 10\%$	$\sigma_r = 1\%$	0.8534	11.9931	6.6164	$r_0 = 5\%$
$\sigma_P = 20\%$	$\sigma_H = 10\%$	$\sigma_r = 1\%$	1.2015	11.7590	7.3633	
$\sigma_P = 10\%$	$\sigma_H = 20\%$	$\sigma_r = 1\%$	5.6092	20.1896	6.2236	
$\sigma_P = 10\%$	$\sigma_H = 10\%$	$\sigma_r = 2\%$	4.4566	28.7231	12.8692	
$\sigma_P = 10\%$	$\sigma_H = 10\%$	$\sigma_r = 1\%$	0.4437	17.0651	7.7488	$r_0 = 7.5\%$
$\sigma_P = 20\%$	$\sigma_H = 10\%$	$\sigma_r = 1\%$	0.7134	16.1875	8.4013	
$\sigma_P = 10\%$	$\sigma_H = 20\%$	$\sigma_r = 1\%$	4.3864	24.0176	7.3692	
$\sigma_P = 10\%$	$\sigma_H = 10\%$	$\sigma_r = 2\%$	2.5754	30.5156	12.8927	
$\sigma_P = 10\%$	$\sigma_H = 10\%$	$\sigma_r = 1\%$	0.1829	20.6101	8.6868	$r_0 = 10\%$
$\sigma_P = 20\%$	$\sigma_H = 10\%$	$\sigma_r = 1\%$	0.3919	20.8501	9.8570	
$\sigma_P = 10\%$	$\sigma_H = 20\%$	$\sigma_r = 1\%$	3.3076	27.7592	8.4250	
$\sigma_P = 10\%$	$\sigma_H = 10\%$	$\sigma_r = 2\%$	1.5976	30.1940	12.5702	

Effects of increasing volatilities of profit flow, property value and short term rates on early termination options under different term structures.

Table 9: Values of options with changing maturity time

	Default	Defeasance	Prepayment	
$T = 10$	0.5218	0.8602	4.2827	
$T = 30$	0.8534	11.9931	6.6164	$r_0 = 5\%$
$T = 50$	1.2556	41.7609	8.2540	
$T = 10$	0.1263	1.1212	4.9797	
$T = 30$	0.4437	17.0651	7.7488	$r_0 = 7.5\%$
$T = 50$	0.7179	48.3432	9.0958	
$T = 10$	0.0225	1.8707	5.6473	
$T = 30$	0.1829	20.6101	8.6868	$r_0 = 10\%$
$T = 50$	0.4104	51.9537	10.0268	

Effects of increasing maturity time of property on early termination options under different term structures.