Bank regulation under fire sale externalities*

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PRELIMINARY

Abstract

This paper examines the optimal design of capital and liquidity regulations when financial markets are incomplete and characterized by asset fire sale externalities. We show that when capital is regulated but liquidity is not, banks still hold liquid assets for micro-prudential reasons; they can use these resources to protect against liquidity shocks. Liquidity is advantageous from a macro-prudential standpoint as well: Higher liquidity holdings lead to less severe decreases in asset prices during times of distress. However, we assume that this externality is not internalized by individual banks. Therefore, banks' liquidity holdings are inefficiently low from a social point of view. Predicting this reaction from banks, the regulator raises the minimum capital ratio requirement to inefficiently high levels, which corresponds to a reduction in socially profitable long-term investments. Our results also indicate that the regulatory framework in the pre-Basel III period, which predominantly focused on capital adequacy requirements, was both inefficient and ineffective in addressing systemic instability caused by liquidity shocks.

Keywords: Bank capital regulation, liquidity regulation, fire sales, Basel III

*The analysis and the conclusions set forth are those of the authors and do not indicate concurrence by other members of the research staff or the Board of Governors of the Federal Reserve.

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1 Introduction

The recent financial crisis led to a redesign of bank regulations, with an emphasis on the macro-prudential aspects of regulation. Prior to the crisis, capital adequacy requirements were the dominant tool of bank regulators around the world. Capital requirements were traditionally used for two primary purposes: to enhance the stability of individual financial institutions and to create a level playing field for internationally active banks. The crisis, however, revealed that even financially sound institutions may face liquidity constraints which could undermine financial stability, especially when these constraints are faced by many institutions at the same time. Without the unprecedented liquidity and asset price supports of the leading central banks during the crisis, those liquidity problems could have resulted in a dramatic collapse of the financial system. The experience led to a renewed focus on the regulation of liquidity. A third generation of bank regulation principles, popularly known as Basel III, strengthens the previous Basel capital adequacy accord by adding liquidity requirements.

In this paper we investigate the optimal design of capital and liquidity regulations in a model characterized by systemic externalities generated by asset fire sales. We consider a three–period model in which a continuum of banks borrow from consumers in the initial period and invest in a long term asset. In the interim period, banks may face liquidity shocks, which could result in fire sale of their assets. Banks treat the asset price as given in this market. We investigate whether capital requirements would alone be sufficient to address the systemic externalities in our model, or if additional introduction of liquidity regulation could further improve financial stability and welfare. In order to do this, we compare and contrast three cases: competitive equilibrium without any regulation, regulation of only capital ratios (partial regulation), and regulation of both capital and liquidity ratios (complete regulation).

We show that the lack of complementary liquidity ratio requirements leads to inefficiently low levels of long-term investments and more severe financial crises, thus undermining the purpose of capital adequacy requirements. When capital is regulated but liquidity is not, banks still hold liquid assets for micro-prudential reasons; they can use these resources to protect against liquidity shocks. Liquidity is advantageous from a macro-prudential standpoint as well: Higher liquidity holdings lead to less severe decreases in asset prices during times of distress. However, this externality is not internalized by individual banks. Therefore, banks' liquidity holdings are inefficiently low from a social point of view. Predicting this reaction from banks, the regulator raises the minimum capital ratio requirement to inefficiently high levels, which leads to a reduction in socially profitable long-term investments.

We also show that banks react to the introduction of capital requirements by decreasing their liquidity ratios. If there is no regulation, banks choose a composition of risky and safe assets in their portfolio that reflects their privately optimal level of risk taking. When the level of risky investment limited by capital regulations, banks reduce the liquidity of their portfolio in order to get closer to their privately optimal level of risk.

Our results indicate that the regulatory framework in the pre-Basel III period, which predominantly focused on capital adequacy requirements, was both inefficient and ineffective in addressing systemic instability caused by liquidity shocks.

The paper proceeds as follows. Section 2 contains a brief summary of related literature. Section 3 provides the basics of the model and solves for the equilibrium of both unregulated and regulated economies. Section 4 presents the main results of the paper where we compare and contrast three cases: competitive equilibrium (no regulation), partially regulated equilibrium (only capital regulation), and the complete regulation equilibrium (both capital and liquidity regulations). Section 5 presents the conclusion. The appendix contains the closed–form solutions of the model and proofs omitted in the main text.

2 Literature review

Even though capital and liquidity regulations have been studied extensively on their own, we are aware of only two other papers that investigate the interaction between these two classical tools of regulators and their optimal determination. Kashyap, Tsomocos, and Vardoulakis (2014) consider an extended version of the Diamond and Dybvig (1983) model to investigate the effectiveness of several bank regulations in addressing two common financial system externalities:[1] excessive risk-taking due to limited liability and bank-runs. The central message of the paper is that a single regulation alone is never sufficient to correct for the inefficiencies created by these two externalities. The authors consider the effectiveness of a combination of capital and liquidity requirements in implementing the social planner's solution: Capital requirements can be optimally chosen to eliminate the possibility of a bank-run, while liquidity requirements would reduce the incentives to take excessive risk by essentially creating a "tax" on the risky investment. However, when the social planner equally cares about the all agents in the economy (depositors, bankers and entrepreneurs), such a combination results in lower social welfare compared to the social welfare attained by the use of capital requirements alone. Moreover, the optimal regulatory mix does not necessarily involve capital or liquidity regulations. Unlike this paper, their paper does not consider fire sale or pecuniary externalities. This causes a divergence in our results as well. We show that under pecuniary externalities, capital regulations are inefficient unless they are supplemented by liquidity requirements.

De Nicoló, Gamba, and Lucchetta (2012) consider a dynamic model of bank regulation and show that there exists an inverted-U-shaped relationship between bank lending, efficiency, welfare, and stringency of capital requirements. Unlike our paper, they find that when liquidity requirements are added to capital requirements, they eliminate the benefits of mild capital requirements

[1]They consider following regulations: deposit insurance, loan-to-value limits, dividend taxes, capital and liquidity ratio requirements.

because liquidity requirements reduce bank lending, efficiency, and social welfare by hampering bank maturity transformation. In their model, liquidity is only welfare reducing because, unlike our paper, they do not consider the role of liquidity in correcting negative externalities arising from fire sales.

Even though the literature on the interaction between capital and liquidity requirements is limited, there are studies that examine the interaction between different tools available to regulators. Acharya, Mehran, and Thakor (2010) show that simple capital requirements are not always sufficient to address both managerial shirking and asset-substitution (risk-shifting) externalities in banking simultaneously because there is an internal conflict between how the two problems can be addressed: Bank leverage should be high enough to create incentives for creditors to threaten liquidation and deter managerial shirking in monitoring and low enough to induce the bank's shareholders to avoid excessive risk taking. Therefore, the optimal capital regulation requires a two-tiered capital requirement with a part of bank capital invested in safe assets. The special capital should be unavailable to creditors upon failure so as to retain market discipline and be available to shareholders only contingent on good performance in order to contain risk-taking. Since the special capital is invested in safe assets, it resembles a liquidity requirement. However, it is different from reserve requirements due to the restriction on its distribution to creditors.

Acharya (2003) shows that convergence in international capital adequacy standards cannot be effective unless it is accompanied by convergence in other aspects of banking regulation, such as closure policies. Externalities in his model are in the form of the cost of investment in a risky asset. He assumes that a bank in one country increases costs of investment for itself and for a bank in the other country as it invests more in the risky asset and thereby creates externalities for the bank in the neighboring country.

Hellmann, Murdock, and Stiglitz (2000) show that while capital requirements can induce prudent behavior, they lead to Pareto-inefficient outcomes by reducing banks' franchise values, and hence providing incentives for gambling. Pareto-efficient outcomes can be achieved by adding deposit-rate controls as a regulatory instrument. The latter restores the prudent behavior by increasing franchise values.

A number of papers, especially after the global financial crisis, drew attention to the macro-prudential role of liquidity requirements, similar to the one considered in this paper. Calomiris, Heider, and Hoerova (2013) argue that the role of liquidity requirements should be conceived not only as an insurance policy that addresses the liquidity risks in distress times as proposed by Basel III, but also as a prudential regulatory tool to make crises less likely. However, their paper does not analyze how the liquidity requirements interact with prudential capital regulations.

This paper is also related to the literature that features financial amplification and asset fire sales which includes the seminal contributions of Fisher (1933), Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Krishnamurthy (2003, 2010), and Brunnermeier and Pedersen (2009). In our model, fire sales result from the combined effect of asset-specificity and correlated shocks that hit an entire industry or economy. This idea, which originated in Williamson (1988) and Shleifer and Vishny (1992), was later employed by fire sale models such as Lorenzoni (2008), Korinek (2011), Stein (2012), and Kara (2013). These later papers show that under pecuniary externalities arising from asset fire sales, there exists over-borrowing and hence over-investment in risky assets in a competitive setting compared to the socially optimal solution.

As opposed to the asset specificity idea discussed above, in Allen and Gale (1994, 1998) and Acharya and Yorulmazer (2008) the reason for fire sales is the limited amount of available cash in the market to buy long-term assets offered for sale by agents who need liquid resources immediately. The scarcity of liquid resources leads to necessary discounts in asset prices, a phenomenon known as "cash-in-the-market pricing."

The constrained inefficiency of competitive markets in this paper is due to the existence of pecuniary externalities under incomplete markets. The Pareto suboptimality of competitive markets when the markets are incomplete goes back at least to the work of Borch (1962). This idea was further developed in the seminal papers of Hart (1975), Stiglitz (1982), and Geanakoplos and Polemarchakis (1986) among others. Greenwald and Stiglitz (1986) extended the analysis by showing that pecuniary externalities by themselves, in general, are not a source of inefficiency, but can lead to significant welfare consequences when markets are incomplete or there is imperfect information.

In this paper, the incompleteness of markets arises from the financial constraints of bankers in the interim period. In particular, similar to Kiyotaki and Moore (1997) and Korinek (2011), we assume that a commitment problem prevents banks from borrowing the funds necessary for restructuring when liquidity shocks hit. If we completed the markets and allowed banks to borrow by pledging the future return stream from the assets, there would not be a reason for fire sales. In this first best world, there would not be a need for either capital or liquidity requirements because the systemic externality in the financial markets would be eliminated.

3 Model

This model contains three periods, $t = 0, 1, 2$; a continuum of banks and a continuum of consumers each with a unit mass and a financial regulator. There is also a unit mass of global investors. All agents are risk-neutral.

There are two goods in this economy: a consumption good and an investment good (i.e., the liquid and illiquid assets). Consumers are endowed with e units of consumption goods at $t = 0$ and $t = 2$, but none at $t = 1$.[2]

Banks have a technology that converts consumption goods into investment goods one-to-one at $t = 0$. Investment goods that are managed by a bank until the last period yield $R > 1$ consumption

[2] We assume that the initial endowment of consumers is sufficiently large, and it is not a binding constraint in equilibrium.

goods per unit. The investment good fully depreciates at $t = 2$ and it can never be converted into consumption goods.

Banks choose the level of investment, n_i, at $t = 0$ and borrow the necessary funds from consumers. We consider deposit contracts that are in the form of simple debt contracts, and assume that there is a deposit insurance operated by the regulator.[3] Therefore, banks can raise deposits at a constant and net zero interest rate. We also assume that banks are protected by limited liability.[4]

Banks also choose how much liquid assets to put aside for each unit of investment in the risky asset. The return on liquid asset is normalized to one. We denote the ratio of liquid assets to risky assets by b_i. Therefore, each bank raises a total of $(1 + b_i)n_i$ units of resources from consumers at $t = 0$. We assume that there is a cost of operating a bank, and this cost is increasing in the size of the balance sheet. The operational cost of a bank is captured by $\Phi((1 + b_i)n_i)$, where we assume that $\Phi'(\cdot) > 0$ and $\Phi''(\cdot) > 0$.

All uncertainty is resolved at the beginning of $t = 1$: The economy lands in good times with probability $1 - q$, and in bad times with probability q. In good times no banks are hit with shocks, therefore no further actions are taken. Banks keep managing their investment goods and realize the full returns from their investment, Rn_i, in the last period. They make the promised payment, $(1+b_i)n_i$, to consumers, and hence earn a net profit of $Rn_i+nb_i-(1+b_i)n_i-\Phi((1+b_i)n_i)$. However, in bad times, the investments of all banks are distressed. In case of distress, the investment has to be restructured in order to remain productive. Restructuring costs are equal to $c \leq 1$ units of consumption goods per unit of investment good. If c is not paid, the investment is scrapped (i.e., it fully depreciates).

There are no available domestic resources (i.e., consumption goods) with which to carry out the restructuring of distressed investment at $t = 1$. Only global investors are endowed with liquid resources at this point. Due to a commitment problem, banks cannot borrow the required resources from global investors. Our particular assumption is that individual banks cannot commit to pay their production to global investors in the last period.[5] The only way for banks to raise necessary funds for restructuring is to sell some fraction of the investment to global investors in an exchange of consumption goods.[6]

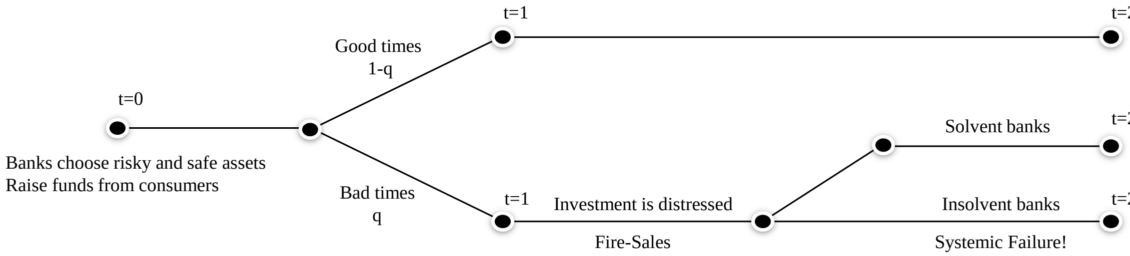
[3] There are two justifications for focusing on debt contracts. First, this assumption is realistic: The deposit contracts are in the form of simple debt contracts in practice. Second, debt contracts can be justified by assuming that depositors can observe banks' asset returns only at a cost. According to Townsend (1979), in the case of costly state verification, debt contracts will be optimal.

[4] Limited liability and deposit insurance assumptions are imposed to match reality and to simplify the analysis of the model. All qualitative results carry on when these assumptions are removed.

[5] For simplicity, we assume that the commitment problem is extreme (i.e., banks cannot commit to pay any fraction of their production to global investors). Assuming a milder but sufficiently strong commitment problem where banks can commit a small fraction of their production, as in Lorenzoni (2008) and Gai, Kapadia, Millard, and Perez (2008), does not change the qualitative results of this paper.

[6] An alternative story would be that households come in two generations as in Korinek (2011) and the assets produce a (potentially risky) return in the interim period in addition to the safe return in the final period. In this case, banks can borrow from the first generation households at the initial period because they have sufficient collateral to back their promises in the interim period, but banks cannot borrow from second generation households because

Figure 1: Timing of the model



The asset sales by banks will carry the features of a fire sale: The investment good will be traded below its fundamental value for banks, and the price will decrease as banks try to sell more assets. Banks will retain only a fraction of their assets after fire sales. If the asset price falls below a threshold, the expected return on the assets that can be retained by banks will be lower than the value promised to depositors; hence, banks will become insolvent.[7] We call this situation a "systemic failure". In this case, the deposit insurance fund covers the difference between the return on the assets retained by banks and the promised amount to depositors.[8] If fire sales are sufficiently mild, however, then banks will have enough assets to make the promised payments to the depositors. In this case, banks remain solvent, but compared to good times they make smaller profits. This sequence of events is illustrated in Figure 1.

Regulatory standards are set at the beginning of $t = 0$. The regulator determines the maximum investment allowed for banks in its jurisdiction, N, and the minimum liquidiy ratio B.[9] Investment levels and liquidity ratios of banks i have to satisfy $n_i \leq N$ and $b_i \leq B$ at $t = 0$. The regulatory standards are chosen to maximize the net expected returns on risky investments.[10]

the value of all assets are zero in the final period. In this alternative story, second generation households will be the buyers of assets from banks in the two countries and employ them in a less productive technology to produce returns in the final period similar to global investors here.

[7]Because all uncertainty is resolved at the beginning of $t = 1$, the expected return to a unit of investment good retained by banks after fire sales is equal to R.

[8]The deposit insurance is assumed to be funded by lump-sum taxes in the last period. If there were no deposit insurance, depositors would face real losses in this case; hence, the interest rate paid to deposits in equilibrium would be higher. The results of the paper hold regardless the existence of a deposit insurance.

[9]The first type of regulation becomes equivalent to a minimum capital ratio requirement when we introduce a costly bank equity capital to the model. We abstract from costly equity capital in the basic model in order to simplify the exposition.

[10]We consider the total welfare and we are silent on the distribution of this welfare between banks and consumers. This point is not relevant for our results because all agents are risk neutral and thus have the same utility function.

3.1 Global investors

Global investors are endowed with unlimited resources of consumption goods at $t = 1$.[11] They can purchase investment goods, y, from banks in each country at $t = 1$ and employ these assets to produce $F(y)$ units of consumption goods at $t = 2$. Let P denote the market price of the investment good at $t = 1$.[12] Because we have a continuum of global investors, each investor treats the market price as given, and chooses the amount of investment goods to purchase, y, to maximize net returns from investment at $t = 2$.

$$\max_{y \geq 0} F(y) - Py \tag{1}$$

The amount of assets they optimally buy satisfies the following first order conditions $F'(y) = P$. This first order condition determines global investors' (inverse) demand function for the investment good. Using this, we can define their demand function $Q^d(P)$ as follows:

$$y = F'(P)^{-1} \equiv Q^d(P) \tag{2}$$

We need to impose some structure on the return function of global investors and the model parameters in order to ensure that the equilibrium of this model is well-behaved.

Assumption 1 (Concavity)**.**

$$F'(y) > 0 \textit{ and } F''(y) < 0 \textit{ for all } y \geq 0, \textit{ with } F'(0) \leq R.$$

Assumption *Concavity* says that although global investors' return is strictly increasing the amount of assets employed ($F'(y) > 0$), they face decreasing returns to scale in the production of consumption goods ($F''(y) < 0$), as opposed to banks that are endowed with a constant returns to scale technology as described above. $F'(0) \leq R$ implies that global investors are less productive than banks at each level of investment goods employed. The concavity of the return function implies that the demand function of global investors for investment goods is downward sloping (see Figure 2). In other words, global investors will require higher discounts to absorb more assets from distressed banks at $t = 1$. The decreasing returns to scale technology assumption is a reduced way of modeling the existence of industry-specific heterogeneous assets, similar to Kiyotaki and Moore (1997), Lorenzoni (2008), Korinek (2011), and Stein (2012). In this more general setup, global investors would first purchase assets that are easy to manage, but as they purchased more assets, they would need to buy ones that required sophisticated management and operation skills.

[11] The assumption that there are some global investors with unlimited resources at the interim period when no one else has resources can be justified with reference to the empirical facts during the Asian and Latin American financial crises. Krugman (2000), Aguiar and Gopinath (2005), and Acharya, Shin, and Yorulmazer (2011) provide evidence that, when those countries were hit by shocks and their assets were distressed, some outside investors with large liquid resources bought their assets.

[12] Price of the investment good at $t = 0$ will be one as long as there is positive investment, and the price at $t = 2$ will be zero because the investment good fully depreciates at this point.

The idea that some assets are industry-specific, and hence less productive in the hands of outsiders, has its origins in Williamson (1988) and Shleifer and Vishny (1992).[13] These studies have claimed that when major players in such industries face correlated liquidity shocks and cannot raise external finance due to debt overhang, agency, or commitment problems, they may have to sell assets to outsiders. Outsiders are willing to pay less than the value in best use for the assets of distressed enterprises because they do not have the specific know-how to manage these assets well and therefore face agency costs of hiring specialists to run these assets.

Empirical and anecdotal evidence suggests the existence of fire sales of physical as well as financial assets.[14] Fire sales have been shown to exist in international settings as well. For example, Krugman (2000), Aguiar and Gopinath (2005), and Acharya, Shin, and Yorulmazer (2011) provide significant empirical and anecdotal evidence that during Asian and Latin American crises, foreign acquisitions of troubled countries' assets were very widely spread across industries and assets were sold at sharp discounts. This evidence suggests that foreign investors took control of domestic enterprises mainly because they had liquid resources whereas the locals did not, even though the locals had superior technology and the know-how to run the domestic enterprises. Further support for this argument comes from the evidence in Acharya, Shin, and Yorulmazer (2011): Many foreigners eventually flipped the assets they acquired during the Asian crisis to locals, and usually made enormous profits from such trades.

Assumption 2 (Elasticity)**.**

$$\epsilon^d = \frac{\partial Q^d(P)}{\partial P}\frac{P}{Q^d(P)} = \frac{F'(y)}{yF''(y)} < -1 \quad \textit{for all } y \geq 0$$

Assumption *Elasticity* says that global investors' demand for the investment good is elastic. This assumption implies that the amount spent by global investors on asset purchases, $Py = F'(y)y$, is strictly increasing in y. Therefore we can also write Assumption *Elasticity* as $F'(y) + yF''(y) > 0$.

If this assumption was violated, multiple levels of asset sales would raise a given amount of liquidity, and multiple equilibria in the asset market at $t = 1$ would be possible. This assumption is imposed by Lorenzoni (2008) and Korinek (2011) in order to rule out multiple equilibria under fire sales.[15]

Many regular return functions satisfy conditions given by Assumptions *Concavity* and *Elasticity*. Here are two examples that satisfy both assumptions: $F(y) = R\ln(1+y)$ and $F(y) = \sqrt{y + (1/2R)^2}$.

[13] Industry-specific assets can be physical, or they can be portfolios of financial intermediaries because many of these contain exotic tailor-made financial assets (Gai et al., 2008). Examples of industry-specific physical assets include oil rigs and refineries, aircraft, copper mines, pharmaceutical patents, and steel plants.

[14] Using a large sample of commercial aircraft transactions Pulvino (2002) shows that distressed airlines sell aircraft at a 14 percent discount from the average market price. This discount exists when the airline industry is depressed but not when it is booming. Coval and Stafford (2007) show that fire sales exist in equity markets when mutual funds engage in sales of similar stocks.

[15] Gai, Kapadia, Millard, and Perez (2008) provides the leading example where this assumption is not imposed and multiple equilibria in the asset market is therefore considered.

In our closed–form solutions below we will use the first of these examples for its analytical convenience. The following example satisfies Assumption *Concavity*, but not Assumption *Elasticity*: $F(y) = y(R - 2\alpha y)$ where $2\alpha y < R$ for all $y \geq 0$.

Assumption 3 (Range)**.**

$$R > 1 + qc$$

Assumption *Range* says that the net expected return to risky project is greater than zero. Here R is the $t = 2$ return on the risky project which requires one unit investment in terms of consumption goods at $t = 0$, and banks have to incur an extra cost c in the bad state, which arises with probability q.

3.2 Asset market equilibrium at date 1

First, we analyze the equilibrium at the interim period in each state of the world, for a given set of investment and liquid assets; then we consider the optimal choice of liquid and illiquid assets at $t = 0$. Note that, if good times are realized $t = 0$, no further actions need to be taken by any agent. Therefore, at $t = 1$ we need only to analyze the equilibrium of the model for bad times.

Consider the problem of a bank i if bad times are realized at $t = 1$. The bank reaches $t = 1$ with a level of investment equal to n_i and liquid assets of $b_i n_i$ which were chosen at the initial period. The investment is distressed and must be restructured using liquid resources. The investment will not produce any returns in the last period if it is not restructured.[16] The bank cannot raise external finance from global investors because it cannot commit to pay them in the last period. Therefore, the only way for the bank to raise the funds necessary for restructuring is to sell some fraction of the investment to global investors and use the proceeds to pay for restructuring costs, whereby it can retain another fraction of the investment.

At the beginning of $t = 1$ in bad times, a bank i decides what fraction of investment goods to restructure (χ_i) and what fraction of restructured assets to sell ($1 - \gamma_i$) to generate the resources for restructuring. Note that γ_i will then represent the fraction of assets that a bank keeps after fire sales.[17] Thus the bank takes the price of the investment good (P) as given, and chooses χ_i and γ_i to maximize total returns from that point on

$$\max_{0 \leq \chi_i, \gamma_i \leq 1} \pi_i = R\gamma_i \chi_i n_i + P(1 - \gamma_i)\chi_i n_i + b_i n_i - c\chi_i n_i \tag{3}$$

[16] For example, if the assets are physical, restructuring costs can be maintenance costs or working-capital needs.

[17] Following Lorenzoni (2008) and Gai, Kapadia, Millard, and Perez (2008), we assume that banks have to restructure an asset before selling it. Basically, this means that bank receive the asset price P from global investors, use a part, c, to restructure the asset, and then deliver the restructured assets to global investors. Therefore banks will sell assets only if P is greater than the restructuring cost, c. We could assume, without qualitatively changing our results, that it is the responsibility of global investors to restructure the assets that they purchase. However, the model is more easily solved using the current story.

subject to the budget constraint

$$P(1-\gamma_i)\chi_i n_i + n_i b_i - c\chi_i n_i \geq 0. \tag{4}$$

The first term in (3) is the (certain) total return that will be obtained from the unsold part of the restructured assets, which are $\chi_i n_i$, in the last period. The second term is the revenue raised by selling a fraction $(1-\gamma_i)$ of the restructured assets, which are $\chi_i n_i$, at the given market price P. The last term, $c\chi_i n_i$, gives the total cost of restructuring. Budget constraint (4) says that the sum of the liquid assets carried from the initial period and the revenues raised by selling assets must be greater than or equal to the restructuring costs.

By Assumption *Concavity*, the equilibrium price of assets must satisfy $P \leq F'(0) \leq R$, otherwise global investors will not purchase any assets. Later on, we will show that in equilibrium we must also have $c < P$. For the moment, we will assume that the equilibrium price of assets satisfies

$$c < P \leq R \tag{5}$$

Now, consider the first order conditions of the maximization problem (3) while ignoring the constraints

$$\frac{\partial \pi_i}{\partial \chi_i} = [R\gamma_i + P(1-\gamma_i) + b_i - c]n_i \tag{6}$$

$$\frac{\partial \pi_i}{\partial \gamma_i} = (R-P)\chi_i n_i \tag{7}$$

From (7) it is obvious that π_i is increasing in γ_i because $P \leq R$ by (5): When the price of investment goods is lower than the return that banks can generate by keeping them, banks want to retain a maximum amount. Choosing γ_i as high as possible implies that the budget constraint will bind. Hence, from (4) we obtain that the fraction of investment goods retained by banks after fire sales is

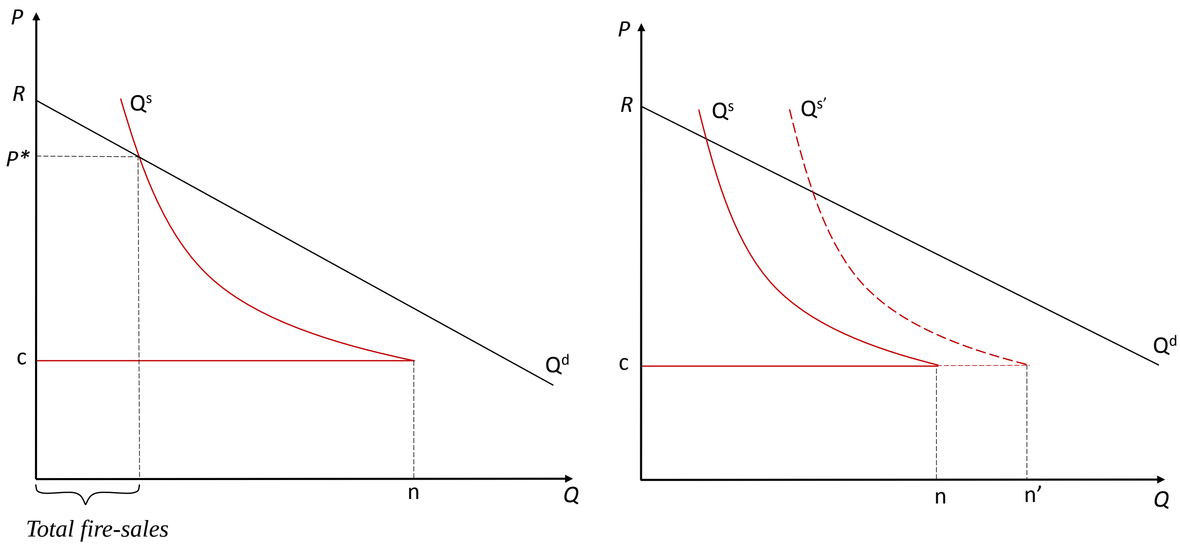
$$\gamma_i = 1 + \frac{b_i - c}{P} \tag{8}$$

The fraction banks retain after fire sales (γ_i) is increasing in the price of the investment good (P) and the liquidity ratio (b_i) and decreasing in the cost of restructuring (c). From (8) we can also obtain the total asset supply of a bank i as

$$Q_i^s(P, n_i, b_i) = (1-\gamma_i)n_i = \frac{c - b_i}{P} n_i \tag{9}$$

for $c < P \leq R$. This supply curve is downward-sloping and convex, which is standard in the fire sales literature. A negative slope implies that if there is a decrease in the price of assets banks have to sell more assets in order to generate the resources needed for restructuring. This is because banks are selling a valuable investment at a price below the fair value for them due to an exogenous

Figure 2: Equilibrium in the investment goods market and comparative statics



pressure (e.g., paying for restructuring costs).

On the other hand, using (8) we can write the first order condition (6) as

$$\frac{\partial \pi_i}{\partial \chi_i} = R\gamma_i n_i \geq 0 \tag{10}$$

Equation (10) shows that revenues are increasing in χ_i at $t = 1$. Therefore, banks will optimally choose to restructure the full fraction of the investment ($\chi_i = 1$). In other words, scrapping of investment goods will never arise in equilibrium.

Note that if the asset price is greater than R, banks want to sell all the investment goods they have because they can get at most R per unit by keeping and managing them. If the price is lower than $c - b$, however, they will optimally scrap all of their assets ($\chi_i = 0$). As discussed above, prices above R and below c will never arise in equilibrium. The total asset supply curve of banks is plotted in Figure 2 for an initial total investment level of n. The equilibrium price of investment goods, P^*, will be determined by the market clearing condition

$$E(P^*, n) = Q^d(P^*) - Q^s(P^*, n, b) = 0 \tag{11}$$

The condition above says that the excess demand in the asset market, denoted by $E(P, n_i, b_i)$, is equal to zero at the equilibrium price. $Q^d(P)$ in (11) is the demand function of global investors which was obtained from the first order conditions of global investors' problem as shown by (2). $Q^s(P, n, b)$ is the total supply of investment goods obtained by integrating (9) over i.

This equilibrium is illustrated in the left panel of Figure 2. Note that the equilibrium price of the investment good at $t = 1$ will be a function of the total initial investment in the risky asset and safe assets. Therefore, from the perspective of the initial period we denote the equilibrium price as $P(n, b)$.

How does a change in the initial risky investment level affect the price of investment good at $t = 1$? Lemma 1 shows that if investment into the risky asset increases at $t = 0$, a lower price for investment goods will be realized in the fire sales state at $t = 1$.

Lemma 1. *$P(n, b)$ is decreasing in n and increasing in b.*

Lemma 1 implies that higher investment in the risky asset or a lower liquidity ratio increases the severity of the financial crisis by lowering the asset prices. This effect is illustrated in the right panel of Figure 2. Suppose that initial risky investment level increases. In this case, banks will have to sell more assets at each price, as can be seen from individual supply function given by (9). Graphically, the total supply curve will shift to the right, as shown by the dotted-line supply curve in the right panel of Figure 2, which will cause a decrease in the equilibrium price of investment goods. A lower initial liquidity ratio will also have the same effect by increasing total supply. Lower asset prices, by contrast, will induce more fire sales by banks due to the downward-sloping supply curve. This additional result is formalized in Lemma 2.

Lemma 2. *Equilibrium fraction of assets sold, $1 - \gamma(n, b)$, is increasing in n and decreasing in b.*

Together lemmas 1 and 2 imply that a higher initial investment in the risky investment by one bank or a lower liquidity ratio creates negative externalities for other banks by making financial crises more severe (i.e., via lower asset prices according to Lemma 1) and more costly (i.e., more fire sales according to Lemma 2).

3.3 Competitive equilibrium

In this section, we solve for the risky and liquid asset holdings of banks at the initial period when there is no regulation. Each bank i at $t = 0$ chooses the level of investment in risky asset n_i and liquidity holdings, as a ratio of investment in risky asset b_i, to maximize expected profits given by

$$\max_{n_i, b_i} \Pi_i(n_i, b_i) = (1-q)\{R + b_i\}n_i + q\{I(b_i < c)R\gamma_i + I(b_i \geq c)[R + b_i - c]\}n_i - D(n_i(1 + b_i)) \quad (12)$$

subject to the budget constraint at $t = 1$

$$P(1 - \gamma_i)n_i + b_i n_i - cn_i \geq 0, \quad (13)$$

where $b_i n_i$ is the total liquidity holdings of bank i, and $D(n_i(1 + b_i)) = n_i(1 + b_i) + \Phi(n_i(1 + b_i))$ is the sum of the cost of funds and operational costs of a bank. Since we assumed that $\Phi(\cdot)$ is convex, it follows that $D(\cdot)$ is convex as well, that is, $D'(\cdot) > 0$ and $D''(\cdot) > 0$. We will interpret b_i as the liquidity ratio of bank i.

Whether the budget constraint given by (13) binds or not depends on banks' behavior as well as exogenous shocks. The budget constraint does not bind if there is no additional liquidity requirement, that is when there is no liquidity shock, at $t = 1$. In case of a liquidity shock, whether the constraint binds or not depends on how much liquidity holdings a bank has carried to $t = 1$. If a bank has chosen $b_i < c$, the constraint will be binding, and it will not be binding if $b_i \geq c$. Below we formally show that the optimal behavior of banks requires that the constraint binds.

Lemma 3. *Banks optimally take some fire sale risk, that is, $b_i < c$ for all banks in equilibrium.*

Proof. It is straightforward to show that we can never have excess liquidity in equilibrium, that is, $b_i > c$ because the return on liquidity is dominated by the expected return on the illiquid asset since $R > 1 + cq$ by Assumption *Range*. Therefore, for contradiction assume that $b_i = c$. Corresponding first order conditions of bank's problem with respect to n_i and b_i are respectively:

$$(1-q)(R+b_i) + qR \;=\; D^{'}(n_i(1+b_i))(1+b_i), \tag{14}$$

$$(1-q)n_i + qn_i \;=\; D^{'}(n_i(1+b_i))n_i \quad \Leftrightarrow \quad D^{'}(n_i(1+b_i)) = 1. \tag{15}$$

Combining the two equations and plugging $b_i = c$ implies that $R + (1-q)b_i = 1 + b_i \implies R + (1-q)c = 1 + c$, which contradicts with the assumption $R > 1 + cq$. Therefore, we must have $b_i < c$ for all $i \in [0, 1]$. □

This lemma allows us to focus on $b_i < c$. The first order conditions of the banks' problem (12) with respect to n_i and b_i in this case are respectively :

$$(1-q)(R+b_i) + qR\gamma_i \;=\; D^{'}(n_i(1+b_i))(1+b_i), \tag{16}$$

$$(1-q)n_i + qR\frac{1}{P}n_i \;=\; D^{'}(n_i(1+b_i))n_i, \tag{17}$$

where $\gamma_i = 1 + \frac{b_i - c}{P}$ as obtained in the previous section. Combining the two equations we get

$$(1-q)R + (1-q)b_i + qR + qR(\frac{b_i - c}{P}) = (1-q) + (1-q)b_i + \frac{qR}{P} + \frac{qR}{P}b_i.$$

Solving for P gives the competitive equilibrium price of assets at $t = 1$ as

$$P = \frac{qR(1+c)}{R - 1 + q}. \tag{18}$$

Note that, P is increasing in q and c, and decreasing in R. Furthermore, the analytical solution for P is independent of the functional form of the global investors' demand, and the operational cost of banks.

Lemma 4. *The equilibrium price of assets satisfies $R \geq P$.*

Proof.

$$R \geq P \implies R \geq \frac{qR(1+c)}{R-1+q} \implies 1 \geq \frac{q(1+c)}{R-1+q} \implies R-1+q \geq q(1+c)$$

which is guaranteed to hold by Assumption *Range*. □

Lemma 5. *The equilibrium price of assets satisfies $P > c$. Hence, there is no scrapping of investment goods in equilibrium.*

Proof.

$$P > c \implies qR + qRc > Rc - c + qc \implies c - cq > R(c - q - qc)$$

Replacing R with $1 + cq$ due to the assumption $R - cq > 1$,

$$c - cq > R(c-q-qc) > (cq+1)(c-q-qc) = c^2q - cq^2 - c^2q^2 + c - q - qc$$

implies $0 > c^2q - cq^2 - c^2q^2 - q$, which must hold given that $c < 1$ and $q < 1$. □

3.3.1 Functional form assumptions and a closed–form solution for the equilibrium

In order to obtain a closed–form solution of the full model, for the competitive equilibrium as well as the regulation cases that we will consider in the following sections, we need to choose functional forms for global investors' demand for long-term assets in the interim period and the operational cost of banks. On the demand side, suppose that the return function of global investors is given by $F(y) = R\ln(1+y)$. It is easy to check that this function satisfies Assumptions *Concavity* and *Elasticity*. For this return function we obtain the (inverse) demand function as

$$P = F'(y) = \frac{R}{1+y} \text{ and hence } y = F'^{-1}(P) = \frac{R}{P} - 1 \equiv Q^d(P) \tag{19}$$

Moreover, suppose that the operational costs of a bank are given by $\Phi(x) = dx^2$, and hence $\Phi^{'}(\cdot)$ is increasing, i.e. $\Phi^{'}(x) = 2dx$.

For convenience, insert the analytical solution for P from into demand side and define τ as follows

$$y = \frac{R}{P} - 1 = \frac{R-1+q}{q(1+c)} - 1 \equiv \tau \tag{20}$$

We define τ here in terms of exogenous variables. [It can also be written as $\frac{R}{P} = \tau + 1$, and we will use this below.] Equating this to supply side, $(1-\gamma)n = \tau$. Due to symmetric nature of the game and that there is a unit measure of banks, in equilibrium we have $b_i = b$ and $n_i = n$.

$$(c-b)n = P\tau \implies n = \frac{P\tau}{c-b}$$

Given b, this equation solves for n. Plugging $\frac{R}{P} = \tau + 1$ and $D^{'}(n(1+b)) = 1 + 2dn(1+b)$ into equation (17)

$$1 - q + q(\tau+1) = D^{'}(n(1+b)) = 1 + 2dn(1+b) \implies 1 - q + q\tau + q = 1 + q\tau = 1 + 2d\frac{P\tau}{c-b}(1+b)$$

where we also used $n = \frac{P\tau}{c-b}$. By substituting $P = \frac{R}{\tau+1}$ and simplifying we obtain the liquidity ratio in the competitive equilibrium

$$cq\tau - 2d\frac{\tau R}{\tau+1} = b(2d\frac{R}{\tau+1} + q)\tau \text{ solves for b}$$

$$b = \frac{cq\tau - 2d\frac{\tau R}{\tau+1}}{(2d\frac{R}{\tau+1} + q)\tau} = \frac{cq - 2d\frac{R}{\tau+1}}{2d\frac{R}{\tau+1} + q}$$

Having obtained the closed–form solutions for the competitive equilibrium, we can perform comparative statics of the equilibrium liquid and risky investment levels of banks with respect to model parameters. The results are summarized by the following two propositions.

Proposition 1. *The liquidity ratio in the competitive equilibrium* (b) *is increasing in the size of the liquidity shock* (c)*, the return to the risky asset* (R) *and the probability of the bad state* (q)*, and decreasing in the marginal cost of funds* (d)*.*

Proposition 2. *The risky holdings in the competitive equilibrium* (n) *are increasing in the return to the risky asset* (R)*, and decreasing in the size of the liquidity shock* (c)*, marginal cost of funds* (d)*, and the probability of the bad state* (q)*.*

In short, the two propositions above show that b and n move in the same direction as response to following parameters: R and d, while they move in opposite directions as response to c and q. This is intuitive since cq is the expected value of liquidity need at the interim period. As the expected liquidity need increases, the bank holds more liquidity and less risky asset. Of course that does not say whether the bank increases enough its liquidity holdings, from a socially optimal perspective.

3.4 Partial Regulation: Regulating only capital ratios

In this section, we assume that the regulator constrains the risky investment level of banks (n_i) but allows banks to freely choose their liquidity ratio (b_i). We consider this case to mimic the regulatory framework in the pre-Basel III period, which predominantly focused on capital adequacy requirements. Banks' level of risky investment has to satify $n_i \leq n$ where n is the maximum leverage level set by the regulator. We will start by assuming that banks leverage up to the allowed maximum level, i.e., banks' choice of n_i is *assumed* to be equal to n that is determined by the regulator. Later, we will prove this assertion.

Given $n_i = n$, banks choose the liquidity ratio (b_i) to maximize their expected profits given by (12). The first order condition of banks' problem (12) with respect to b_i is (implicitly) given by

$$(1-q) + qR\frac{1}{P} = D^{'}(n(1+b_i)) \implies b_i = \frac{D^{'-1}(1-q+q\frac{R}{P})}{n} - 1 \tag{21}$$

Taking this into account, the regulator's problem can be written as

$$\max_{n} \ W(n) = \ (1-q)\{R + b(n)\}n + qR\gamma n - D((1+b(n))n), \tag{22}$$

from which we can obtain the following first order conditions with respect to n as

$$(1-q)\{R + b(n) + nb^{'}(n)\} + qR\{\gamma + n\frac{d\gamma}{dn}\} = D^{'}(n(1+b))\{1 + b(n) + nb^{'}(n)\}$$

First, we study the reaction of banks to a tightening in capital requirements. For that we need the derivative of b_i with respect to n from banks' problem. We use the (implicit) reaction function given by (21), and take derivative of both sides of this equation with respect to n. Proposition 3 shows that banks reduce their liquidity ratios as the regulator tightens the limit on risky investments. The regulator attempts to correct excessive risk taking by banks using a limit on the long-term investment of banks, which is equivalent to the role of a risk-weighted capital ratio requirement. However, since this regulation prevents banks from reaching their privately optimal level of risk, they react by reducing their liquidity ratios. In other words, banks undermine the purpose of capital regulations by carrying less liquid portfolios.

Proposition 3. *Banks decrease their liquidity ratio as the regulator tightens capital requirements, that is,* $b^{'}(n) \geq 0$.

3.4.1 Closed–form solutions

In Section 6.1 in the Appendix we derive the solution to the partial regulation case as follows:

$$b^* = \frac{qc(\tau^*+1) - 2dR}{2dR + q(\tau^*+1)} \tag{23}$$

$$n^* = \frac{\tau^*}{\tau^*+1}\frac{2dR + q(\tau^*+1)}{2d(1+c)} \tag{24}$$

where

$$\tau^* \equiv \frac{R}{P^*} - 1, \tag{25}$$

and P^* is the only real (positive) root of the cubic equation below:[18]

$$2d\sigma P^{*3} + [qR\sigma - 2d\beta]P^* - q\beta = 0. \tag{26}$$

Here, we define σ and β in terms of the parameters of the model as follows:

$$\sigma \equiv \frac{R-1+q}{qR} \qquad \text{and} \qquad \beta \equiv R(1+c) \tag{27}$$

3.5 Complete Regulation: Regulating both capital and liquidity ratios

In this section, we study the case in which both risky investment levels and liquidity ratios of banks are regulated. The regulator imposes a minimum ratio of liquidity holding for each bank as a fraction of its risky asset ($b_i \geq b$) and a maximum level of risky investment ($n_i \leq n$). Both of these regulatory requirements have to bind in equilibrium due to the pecuniary externalities, that is, the fact that banks take asset prices in the interim period as given causes them to choose a higher risky investment level and a lower liquidity ratio compared to the socially optimal levels. The regulator incorporates this fact into its objective function as it determines these two regulatory limits. Therefore, the regulators problem is to choose the minimum liquidity ratio (b) and the maximum risky investment level (n) to maximize net expected returns to the banking system:[19]

$$\max_{n,b} \Pi_i(n_i, b_i) = (1-q)(R+b)n + qR\gamma n - D(n(1+b)) \tag{28}$$

subject to the budget constraint at $t = 1$

$$P(1-\gamma)n + bn - cn \geq 0$$

Corresponding first order conditions with respect to n and b are respectively;

$$(1-q)(R+b) + qR\Big\{\gamma + (b-c)P^{-2}(-1)\frac{\partial P}{\partial n}n\Big\} = D^{'}(n(1+b))(1+b) \tag{29}$$

$$(1-q)n + qR\Big\{\frac{1}{P} - (c-b)P^{-2}(-1)\frac{\partial P}{\partial b}\Big\}n = D^{'}(n(1+b))n, \tag{30}$$

where $\gamma = 1 + \frac{b-c}{P}$. Given $P = R + (b-c)n$, we have two equations and two unknowns (given

[18]The solution to this cubic equation can easily be obtained using Vieto's substitution. However, we will not show the solution here to save space, and also since we do not use the explicit solution in any of the proofs.

[19]Similar to Lemma 3 we can show that it is never optimal for the regulator to set the minimum liquidity ratio equal or greater than the size of the liquidity shock, c.

a functional form for $D(\cdot)$ as well). As shown in Section 6.2 in the Appendix we can obtain closed–form solutions for both n^{**}, b^{**} and P^{**} as follows:

$$n^{**} = \frac{2dr\tau^{**} + q\tau^{**}(\tau^{**}+1)(\tau^{**}+2)}{2d(1+c)(\tau^{**}+1)} \tag{31}$$

$$b^{**} = \frac{cq(\tau^{**}+1)(\tau^{**}+2) - 2dR}{2dR + q(\tau^{**}+1)(\tau^{**}+2)} \tag{32}$$

$$P^{**} = \sqrt{\frac{qR^2(1+c)}{R-1+q}}, \tag{33}$$

where

$$\tau^{**} = \frac{R}{P^{**}} - 1 = \sqrt{\frac{R-1+q}{q(1+c)}} - 1 \tag{34}$$

4 Results and Discussion

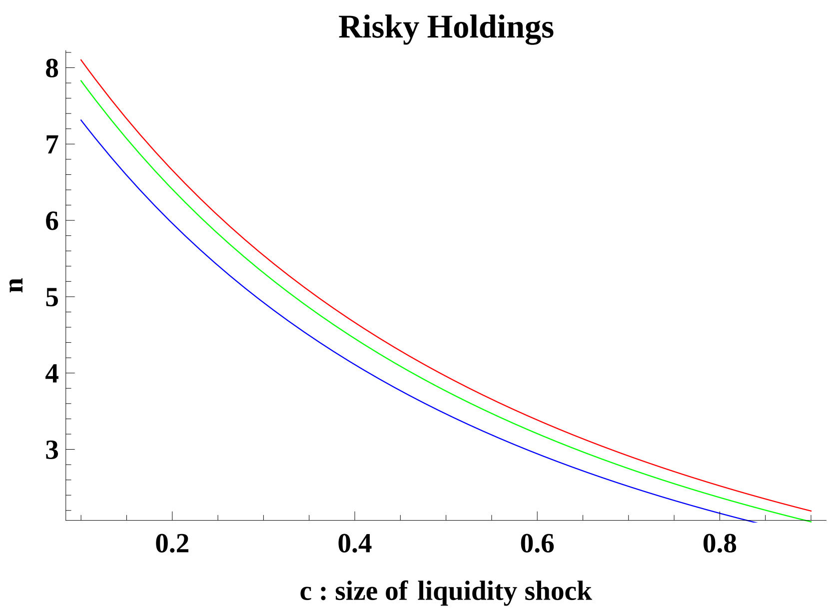


Figure 3: Risky Holdings: competitive, partial, and complete

In this section we discuss the main results of the paper and compare the properties of different regulatory schemes. Proposition 4 summarizes the results and we discuss these results with reference to Figures 3 to 7.

Proposition 4. *Risky investment levels, liquid asset holdings, and financial stability measures under competitive equilibrium (n, b, $1-\gamma$, P), partial regulation equilibrium (n^*, b^*, $1-\gamma^*$, P^*), and complete regulation equilibrium (n^{**}, b^{**}, $1-\gamma^{**}$, P^{**}) compare as follows:*

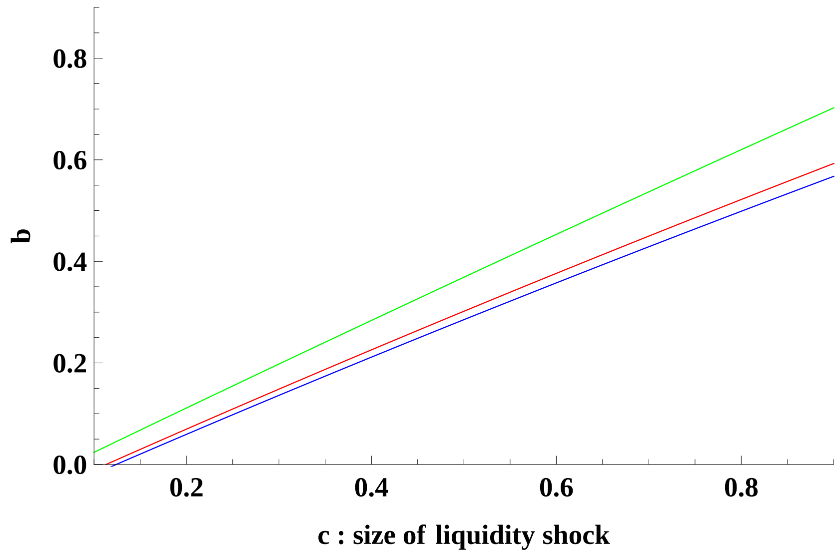


Figure 4: Liquidity Holdings: competitive, partial, and complete

a) $n > n^{**} > n^{*}$

b) $b^{**} > b > b^{*}$

c) Financial stability measures

i) $1-\gamma > 1-\gamma^{*} > 1-\gamma^{**}$

ii) $(1-\gamma)n > (1-\gamma^{*})n^{*} > (1-\gamma^{**})n^{**}$

iii) $P^{**} > P^{*} > P$

It is not surprising that $n > n^{**}$ and $b^{**} > b$, that is, in competitive equilibrium banks hold more risky asset and less liquid asset compared to the socially optimum levels, because banks do not internalize the fire sale externality. However, as also shown by Figure 3, socially optimum level of risky investment n^{**} is higher than the risky investment in the partial regulation case, that is, minimum capital ratio is inefficiently high under partial regulation. This result can be better explained when it is considered together with the comparison of liquidity holdings in these two cases. Figure 4 shows that the socially optimum level of liquidity is higher than the liquidity chosen by banks under the partial regulation. As a result we can conclude that holding more liquidity is costly. However having liquidity also makes it possible to hold more risky assets. Therefore, the socially optimal choice is to hold a higher level of risky investment that is supported with greater liquidity holdings. For example, consider a country that is transitioning from the partial regulation to the complete regulation by imposing new liquidity rules in addition to the existing capital rules. In

Figure 5: Price under Fire Sale: competitive, partial, and complete

part, this can be compared to moving from Basel II to Basel III regulatory approach.[20] Assuming that the capital regulation had been set optimally during the Basel II period, with the addition of liquidity requirements the capital requirements can be relaxed. Proposition 3 provides further clarification since it shows that banks' response to stricter capital regulation is to hold less liquidity. In the partial regulation case, where the regulator does not have the sufficient number of tools, the regulator predicts that the banks will hold less liquid assets, and hence imposes further tightening on the capital requirements.

How effective is the capital regulation when it is not accompanied by liquidity requirements? In order to answer this question, we can examine the prices of risky asset under fire sale in different cases. A lower price of the risky asset implies that the fire sales are more severe and the externality has a strong presence in the economy. Figure 5 shows that the fire sale price under partial regulation is not significantly different from the competitive equilibrium price, but it is far below the socially optimal price level. We reach a similar conclusion when we consider other measures of the severity of a crisis. For instance, Figure 6 shows the fraction of risky asset that must be sold to withstand the liquidity shock. These fractions are almost the same under the partial regulation and the no regulation environments, and they both are much higher than the fraction under the complete regulation. From this point of view, the partial regulation has almost no effect on limiting the fire sales. When we look at the total amount of risky asset sold under fire sales, as shown by Figure 7,

[20]To be more precise, liquidity requirements in Basel III are different than the simple liquidity requirements that we consider in this paper.

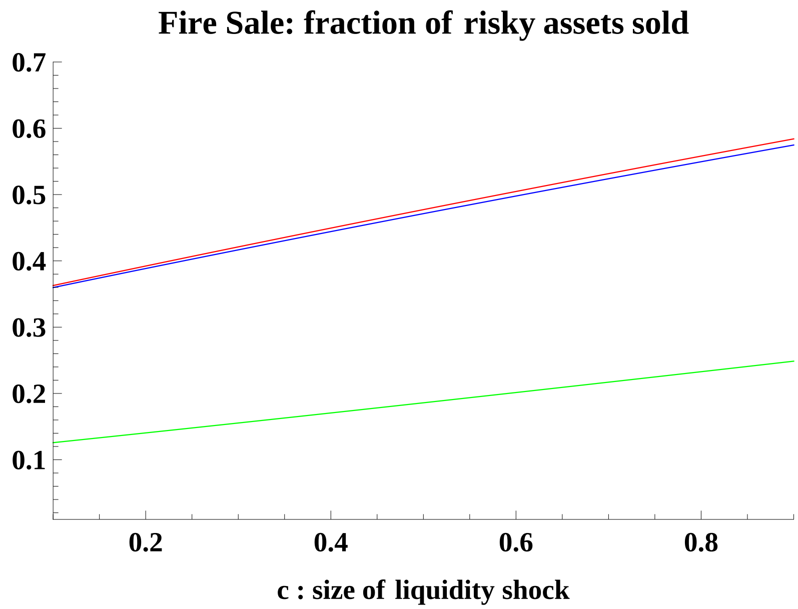


Figure 6: Fire Sale: competitive, partial, and complete

the message is the same:[21] Capital regulation has only a mild effect in terms of financial stability when it is not accompanied with liquidity requirements. The biggest improvement in terms of financial stability is achieved when both capital and liquidity are regulated. In other words, when capital is regulated by liquidity is not banks undermine the purpose of the regulation by decreasing their liquidity holdings.

Minimum capital rules may serve to several other purposes. In terms of limiting fire sales however, they are not very effective, especially if they are not supported with liquidity regulation rules. In relation to the regulation of liquidity, the important point is that banks hold a smaller fraction of their asset in terms of liquid items compared to no regulation case (competitive equilibrium). Since $b > b^*$, once the liquidity shock hits, banks will run out of liquidity earlier under partial regulation compared as opposed to the case where there is no regulation. Although banks have a greater exposure to fire sales under partial regulation in case of a liquidity shock, the regulated risky investment manifests itself in the amount of fire sales. Both in terms of fraction of the total risky asset and in terms of the total risky asset sold under fire sales, the amount of fire sales is smaller under partial regulation compared to competitive equilibrium, see Lemma 9 and 10.

The level of risky investment on its own is not very informative about the potential fire sales; a bank with a lower level of risky investment may end up conducting a larger fire sale compared to a bank with higher level of risky investment. This is due to low level of liquidity holding that accompanies the low level of risky investment, which is the case under the partial regulation.

[21] The difference between Figures 6 and 7 is due to different levels of risky investment.

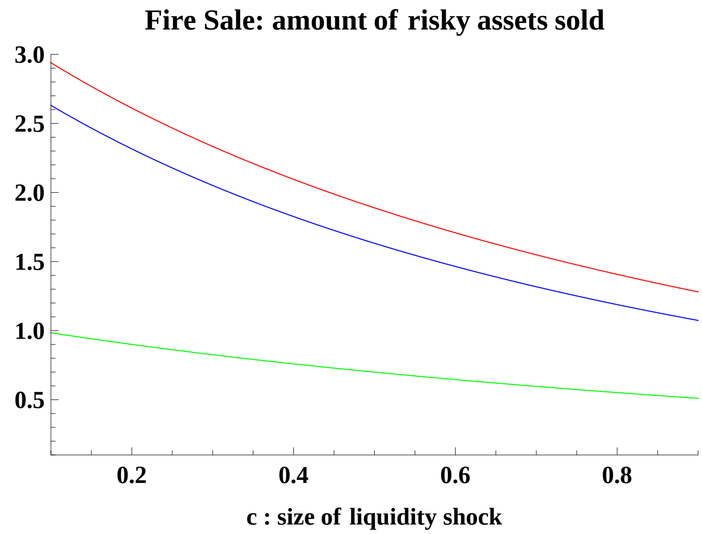


Figure 7: Fire Sale: competitive, partial, and complete

5 Conclusion

In this paper, we investigate the optimal design of bank regulation when financial markets are incomplete and characterized by fire sale externalities. In the model, banks need to fire sale their long term assets if aggregate liquidity shocks hit the economy. In case of liquidity shocks, each bank determines the optimal amount of assets to sell, without taking into account the effect of its asset sales on the price of assets. This creates a system wide externality under which there is, simultaneously, an over-investment in risky assets and under-provision of liquid assets in the competitive equilibrium. This fact creates the need for bank regulation in our setup.

First, we study the introduction of a capital requirement alone. The capital requirement limits the risky investment levels of banks. Ceteris paribus, these more stringent capital requirements would lead to less severe financial crises. However, we show that banks respond by decreasing their liquidity ratios - which, in turn, creates an opposing channel. The regulator, anticipating this response, sets capital ratios at even higher levels to offset the decrease in banks' liquidity ratios. Thus, under liquidity shocks, a well-capitalized banking system may experience greater losses than a less capitalized banking system.

Due to the inefficiently low liquidity ratios, seemingly high capital ratios will deteriorate rapidly in case of a liquidity shock. In other words, capital adequacy ratios are neither a robust measure of bank soundness nor a sufficient tool to regulate the excessive risk taking behavior of banks when the financial system is characterized by fire sale externalities. The optimal bank regulation should supplement capital adequacy ratios with liquidity requirements. In that regard, our results support the Basel III approach, which strengthens earlier capital adequacy accords by adding liquidity requirements.

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6 Appendix

6.1 Closed–form solutions for the partial regulation case

The regulator takes into acccount that for any given n, the banks optimally choose their liquidity ratio $b(n)$, and hence we can write the regulator's objective function as:

$$\max_{n} \; W(n) = \; (1-q)\{R+b(n)\}n + qR\gamma n - D((1+b(n))n),$$

from which we can obtain the following first order conditions with respect to n as

$$(1-q)\{R+b(n)+nb^{'}(n)\} + qR\{\gamma + n\frac{d\gamma}{dn}\} = D^{'}(n(1+b))\{1+b(n)+nb^{'}(n)\} \tag{35}$$

First, note that since

$$\gamma = 1 + \frac{b(n)-c}{P} = 1 + \frac{b(n)-c}{R+(b(n)-c)n},$$

we can obtain the total derivative of γ with respect to n as:

$$\begin{aligned}
\frac{d\gamma}{dn} &= \frac{\partial\gamma}{\partial b}b'(n) + \frac{\partial\gamma}{\partial n} \\
&= \frac{P-(b(n)-c)n}{P^2}b'(n) - \frac{(b(n)-c)^2}{P^2} \\
&= \frac{b'(n)}{P} - \frac{nb'(n)(b(n)-c)}{P^2} - \frac{(b(n)-c)^2}{P^2}
\end{aligned} \tag{36}$$

Substitute this into the first order conditions given by (35) and rearrange

$$\begin{aligned}
(1-q)\{R+b(n)\} + qR\left(1+\frac{b(n)-c}{P}\right) + nb^{'}(n)\left\{1-q+\frac{qR}{P} - D'(\cdot) - \frac{(b(n)-c)n}{P^2}qR\right\} \\
-qR\frac{n(b(n)-c)^2}{P^2} - D^{'}(\cdot)\{1+b(n)\} = 0
\end{aligned}$$

Note that from banks' first order condition we have $1-q+qR/P-D'(\cdot)=0$, hence the equation above further simplifies to

$$R-qR+(1-q)b(n)+qR+qR\frac{b(n)-c}{P}-qR\frac{n(b(n)-c)^2}{P^2}-b^{'}(n)\{\frac{(b(n)-c)n^2}{P^2}qR\}$$
$$-\left(1-q+\frac{qR}{P}\right)\{1+b(n)\}=0$$
$$R-1+q+\frac{qR}{P}[b(n)-c-1-b(n)]-qR\frac{n(b(n)-c)^2}{P^2}-b^{'}(n)\{\frac{(b(n)-c)n^2}{P^2}qR\}=0$$
$$R-1+q-\frac{qR(1+c)}{P}-qR\frac{n(b(n)-c)^2}{P^2}-b^{'}(n)\{\frac{(b(n)-c)n^2}{P^2}qR\}=0$$
$$R-1+q-\frac{qR(1+c)[R+(b(n)-c)n]-qR(b(n)-c)^2n}{P^2}-b^{'}(n)\{\frac{(b(n)-c)n^2}{P^2}qR\}=0$$
$$R-1+q-\frac{qR^2(1+c)}{P^2}-qR\frac{n(b(n)-c)(1+b(n))}{P^2}-b^{'}(n)\{\frac{(b(n)-c)n^2}{P^2}qR\}=0 \quad (37)$$

Divide the last equation by qR to obtain

$$\frac{R-1+q}{qR}-\frac{R(1+c)}{P^2}-\frac{n(b(n)-c)(1+b(n))}{P^2}-b^{'}(n)\{\frac{(b(n)-c)n^2}{P^2}\} \quad (38)$$

Let's define

$$\sigma=\frac{R-1+q}{qR} \quad (39)$$

Now, we can write the first order condition as

$$\frac{1}{P^2}\left\{\sigma P^2-R(1+c)-n(b(n)-c)(1+b(n))-b^{'}(n)(b(n)-c)n^2\right\}=0 \quad (40)$$

Since in equilibrium price must be positive, we focus on the term inside the curvy brackets. Use the expressions obtained for $1+b$, $b-c$ and $b'(n)$ in the proof of Proposition 3, to rewrite this term as:

$$\sigma P^2-R(1+c)+(R-P)\frac{q(R-P)}{2dPn}-\frac{q(R-P)^2}{n^2[2dP^2+qR]}\frac{R-P}{n}n^2=0$$
$$\sigma P^2-R(1+c)+\frac{q(R-P)^2}{n}\left[\frac{1}{2dP}+\frac{R-P}{2dP^2+qR}\right]=0$$

From the last equation we can obtain n in terms of P and the parameters of the model:

$$n=\frac{q(R-P)^2\left[\frac{1}{2dP}+\frac{R-P}{2dP^2+qR}\right]}{R(1+c)-\sigma P^2}\equiv\psi(P) \quad (41)$$

We can similarly obtain an expression for b in terms of P and the parameters of the model using the equilibrium price function $P = R + (b - c)n$, which implies that

$$b = \frac{P - R}{n} + c = \frac{P - R + cn}{n} = \frac{P - R + c\psi(P)}{\psi(P)} \tag{42}$$

Now, go back to the banks' first order condition, plug these expressions for n and b in order to obtain an equation that involves only P as an endogenous variable, from which we can solve for the equilibrium price P.

$$\begin{aligned}
-q + \frac{qR}{P} &= 2dn(1 + b) \\
\frac{qR}{P} &= 2d\psi(P)\left[\frac{P - R + c\psi(P)}{\psi(P)} + 1\right] + q \\
\frac{qR}{P} &= 2d\psi(P)\left[\frac{P - R + (1 + c)\psi(P)}{\psi(P)}\right] + q
\end{aligned}$$

Multiply the last equation with P and rearrange to obtain

$$\begin{aligned}
2d\left[P - R + (1 + c)\psi(P)\right] + qP - qR = 0 \\
-2dP(R - P) - q(R - P) + 2d(1 + c)P\psi(P) = 0
\end{aligned}$$

Rearrange the last equation and substitute for $\psi(P)$ from (41):

$$\begin{aligned}
2d(1 + c)P\psi(P) &= (R - P)(2dP + q) \\
2d(1 + c)P\frac{q(R - P)^2\left[\frac{1}{2dP} + \frac{R-P}{2dP^2+qR}\right]}{R(1 + c) - \sigma P^2} &= (R - P)(2dP + q) \\
2d(1 + c)qP(R - P)\left[\frac{1}{2dP} + \frac{R - P}{2dP^2 + qR}\right] &= \left[R(1 + c) - \sigma P^2\right](2dP + q) \\
(1 + c)q(R - P)\left[1 + \frac{2dP(R - P)}{2dP^2 + qR}\right] &= \left[R(1 + c) - \sigma P^2\right](2dP + q) \\
(1 + c)q(R - P)\left[\frac{2dP^2 + qR + 2dPR - 2dP^2}{2dP^2 + qR}\right] &= \left[R(1 + c) - \sigma P^2\right](2dP + q) \\
(1 + c)q(R - P)\left[\frac{R(2dP + q)}{2dP^2 + qR}\right] &= \left[R(1 + c) - \sigma P^2\right](2dP + q)
\end{aligned}$$

Lastly, simplifying $2dP + q$ from both sides and rearranging yields

$$(1 + c)q(R - P)R - (2dP^2 + qR)\left[R(1 + c) - \sigma P^2\right] = 0 \tag{43}$$

Rewrite this equation first by substituting $\beta \equiv R(1+c)$, and then expand it to obtain a polynomial equation in P:

$$
\begin{aligned}
q(R-P)\beta - (2dP^2 + qR)\left[\beta - \sigma P^2\right] &= 0 \\
qR\beta - q\beta P - 2d\beta P^2 + 2d\sigma P^4 - qR\beta + qR\sigma P^2 &= 0 \\
2d\sigma P^4 + (qR\sigma - 2d\beta)P^2 - q\beta P &= 0
\end{aligned}
$$

Since we are interested in non-zero and positive equilibrium price for the illiquid asset, divide this last equation by P to obtain a cubic equation in P:

$$2d\sigma P^3 + [qR\sigma - 2d\beta]P - q\beta = 0 \tag{44}$$

It is easy to show that this cubic equation has only one real root and two complex conjugate roots. The only real root can easily be obtained using Vieto's substitution for cubic equations. Since this real root gives the equilibrium price P in terms of the model parameters, we can also obtain a closed–form solution for the equilibrium level of risky investment n by substituting this value of P into (41), and for the equilibrium value of liquid asset ratio b by substituting the solution for P into (42).

6.2 Closed–form solutions for the complete regulation case

Solving for the complete regulation liquidity ratio (b^{**}):

The first order conditions of the regulator's problem with respect to n and b are respectively;

$$(1-q)(R+b) + qR\Big\{\gamma + \frac{\partial \gamma}{\partial n} n\Big\} = D^{'}(n(1+b))(1+b) \text{ where } \gamma = 1 + \frac{b-c}{P}$$

$$(1-q)n + qR\frac{\partial \gamma}{\partial b} n = D^{'}(n(1+b))n$$

Combining the two equations

$$(1-q)(R+b) + qR\Big\{\gamma + \frac{\partial \gamma}{\partial n} n\Big\} = \left[(1-q) + qR\frac{\partial \gamma}{\partial b}\right](1+b) = D^{'}(n(1+b))(1+b)$$

Plugging $\frac{\partial \gamma}{\partial n} = -\frac{(b-c)^2}{P^2}$ and $\frac{\partial \gamma}{\partial b} = \frac{R}{P^2}$, and later $P = R + (b-c)n$.

$$-\phi = -\frac{(1-q)[R-1]}{qR} = 1 + \frac{(b-c)P - (b-c)^2 n - R(1+b)}{P^2} \tag{45}$$

$$-\phi - 1 = \frac{(b-c)[R+(b-c)n] - (b-c)^2 n - R(1+b)}{P^2} = \frac{R(b-c-1-b)}{P^2} = \frac{-R(c+1)}{P^2} \tag{46}$$

$$\Longrightarrow P^2 = \frac{R(c+1)}{\phi+1} = \frac{q(c+1)R^2}{R-1+q} \Longrightarrow P^{**} = R\sqrt{\frac{q(c+1)}{R-1+q}}$$

Note that $P = \frac{P^{**2}}{R}$.

Going back to the first order condition w.r.t. b

$$\begin{aligned}
(1-q) + qR\frac{\partial\gamma}{\partial b} &= D^{'}(n(1+b)) \\
1 - q + qR\frac{R}{P^2} &= 1 + 2dn(1+b) \\
1 - q + q(\frac{R}{P})^2 &= 1 + 2dn(1+b) \\
1 - q + q(\tau^{**}+1)^2 &= 1 + 2dn(1+b) \\
q\{(\tau^{**}+1)^2 - 1\} &= 2dn(1+b) \\
q\{(\tau^{**}+1+1)(\tau^{**}+1-1\} &= 2d\frac{P\tau^{**}}{c-b}(1+b) \\
q\tau^{**}(\tau^{**}+2) &= 2d\frac{R}{\tau^{**}+1}\frac{\tau^{**}}{c-b}(1+b) \\
q(\tau^{**}+1)(\tau^{**}+2)(c-b) &= 2dR(1+b) \\
q(\tau^{**}+1)(\tau^{**}+2)c - 2dR &= b\{2dR + q(\tau^{**}+1)(\tau^{**}+2)\} \\
b^{**} &= \frac{cq(\tau^{**}+1)(\tau^{**}+2) - 2dR}{2dR + q(\tau^{**}+1)(\tau^{**}+2)} = \frac{cq - \frac{2dR}{(\tau^{**}+1)(\tau^{**}+2)}}{\frac{2dR}{(\tau^{**}+1)(\tau^{**}+2)} + q}
\end{aligned}$$

we used $\frac{R}{P^{**}} = \tau^{**} + 1$ and $n^{**} = \frac{P^{**}\tau^{**}}{c-b}$.

6.3 Proofs omitted in the main text

Lemma 1. *$P(n,b)$ is decreasing in n and increasing in b.*

Proof. The asset market clearing condition in the bad state at $t = 1$ is given as

$$Q^s(P) = \frac{c-b}{P}n = Q^d(P), \tag{47}$$

which can be written as

$$(c-b)n = PQ^d(P). \tag{48}$$

First, take the partial derivative of both sides of this last equation with respect to n:

$$\begin{aligned} c-b &= \frac{\partial P}{\partial n}Q^d(P) + P\frac{\partial Q^d(P)}{\partial P}\frac{\partial P}{\partial n} \\ &= \frac{\partial P}{\partial n}\left\{Q^d(P) + P\frac{\partial Q^d(P)}{\partial P}\right\} \\ &= \frac{\partial P}{\partial n}Q^d(P)\left\{1+\epsilon^d\right\} \end{aligned}$$

where

$$1+\epsilon^d = \frac{\partial Q^d(P)}{\partial P}\frac{P}{Q^d}, \tag{49}$$

is the price elasticity of demand function of the global investors. Rearranging the last equation gives

$$\frac{\partial P}{\partial n} = \frac{c-b}{Q^d(P)(1+\epsilon^d)} < 0 \tag{50}$$

since $1+\epsilon^d < 0$ by Assumption *Elasticity*, and $c-b > 0$ by Lemma 3. For the second part of the proof take the partial derivative of both sides of (48) with respect to b:

$$\begin{aligned} -n &= \frac{\partial P}{\partial b}Q^d(P) + P\frac{\partial Q^d(P)}{\partial P}\frac{\partial P}{\partial b} \\ &= \frac{\partial P}{\partial b}\left\{Q^d(P) + P\frac{\partial Q^d(P)}{\partial P}\right\} \\ &= \frac{\partial P}{\partial b}Q^d(P)\left\{1+\epsilon^d\right\} \end{aligned}$$

Rearranging the last equation gives

$$\frac{\partial P}{\partial b} = -\frac{n}{Q^d(P)(1+\epsilon^d)} > 0 \tag{51}$$

since $1+\epsilon^d < 0$ by Assumption *Elasticity*.

□

Lemma 2. *Equilibrium fraction of assets sold,* $1-\gamma(n,b)$, *is increasing in* n *and decreasing in* b.

Proof. Using (8) we can write banks' asset sales in equilibrium as $1-\gamma(n,b) = (c-b)/P(n,b)$. Note that

$$\frac{\partial(1-\gamma)}{\partial n} = \frac{\partial(1-\gamma)}{\partial P}\frac{\partial P}{\partial n} > 0, \tag{52}$$

because $\partial(1-\gamma)/\partial P = -c/P^2 < 0$ from (8) and by Lemma 1 we have that $\partial P/\partial n < 0$. Similarly, we can obtain

$$\frac{\partial(1-\gamma)}{\partial b} = -\frac{1}{P} + \frac{\partial(1-\gamma)}{\partial P}\frac{\partial P}{\partial b} < 0, \tag{53}$$

since $\partial(1-\gamma)/\partial P < 0$ as shown above, and by Lemma 1 we have that $\partial P/\partial b > 0$. □

Proposition 1. *The liquidity ratio in the competitive equilibrium* (b) *is increasing in the size of the liquidity shock* (c), *the return to the risky asset* (R) *and the probability of the bad state* (q), *and decreasing in the marginal cost of funds* (d).

Proof.

$$\begin{aligned}
\frac{\partial b}{\partial c} &= \frac{\{q - 2dR(\tau+1)^{-2}(-1)\frac{\partial\tau}{\partial c}\}[2d\frac{R}{\tau+1}+q] - [cq - 2d\frac{R}{\tau+1}]2dR(\tau+1)^{-2}(-1)\frac{\partial\tau}{\partial c}}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{2dR(\tau+1)^{-2}(-1)\frac{\partial\tau}{\partial c}[-2d\frac{R}{\tau+1} - q - cq + 2d\frac{R}{\tau+1}] + q[2d\frac{R}{\tau+1}+q]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{2dR(\tau+1)^{-2}(-1)\frac{\partial\tau}{\partial c}(-q)(1+c) + q[2d\frac{R}{\tau+1}+q]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{2dR(\tau+1)^{-2}\frac{\partial\tau}{\partial c}q(1+c) + q[2d\frac{R}{\tau+1}+q]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{2dR(\frac{q(1+c)}{R-1+q})^2\frac{R-1+q}{q(1+c)^2}(-1)q(1+c) + q[2dR\frac{q(1+c)}{R-1+q}+q]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{2dR\frac{q(1+c)}{R-1+q}(-1)q + q[2dR\frac{q(1+c)}{R-1+q}+q]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{q^2}{[2d\frac{R}{\tau+1}+q]^2} > 0.
\end{aligned}$$

$$\begin{aligned}
\frac{\partial b}{\partial d} &= \frac{[-2\frac{R}{\tau+1}][2d\frac{R}{\tau+1}+q]-[cq-2d\frac{R}{\tau+1}][2\frac{R}{\tau+1}]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{[-2\frac{R}{\tau+1}][2d\frac{R}{\tau+1}]-[2\frac{R}{\tau+1}]q-[2\frac{R}{\tau+1}]cq+[2\frac{R}{\tau+1}][2d\frac{R}{\tau+1}]}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{-\frac{2R}{\tau+1}q(1+c)}{[2d\frac{R}{\tau+1}+q]^2} < 0.
\end{aligned}$$

$$\frac{\partial b}{\partial q} = \frac{2(1+c)^2 dR}{(-1+q+R+2dR+2cdR)^2} > 0$$

$$\begin{aligned}
\frac{\partial b}{\partial R} &= \frac{\{-2d\frac{(\tau+1)-R\frac{\partial(\tau+1)}{\partial R}}{(\tau+1)^2}\}[2d\frac{R}{\tau+1}+q]-[cq-2d\frac{R}{\tau+1}]\{2d\frac{(\tau+1)-R\frac{\partial(\tau+1)}{\partial R}}{(\tau+1)^2}\}}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{\{-2d\frac{\frac{R-1+q}{q(1+c)}-\frac{R}{q(1+c)}}{(\tau+1)^2}\}[2d\frac{R}{\tau+1}+q]-[cq-2d\frac{R}{\tau+1}]\{2d\frac{\frac{R-1+q}{q(1+c)}-\frac{R}{q(1+c)}}{(\tau+1)^2}\}}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{\{\frac{2d(1-q)}{q(1+c)(\tau+1)^2}\}[2d\frac{R}{\tau+1}+q]-[cq-2d\frac{R}{\tau+1}]\{\frac{-2d(1-q)}{q(1+c)(\tau+1)^2}\}}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{\{\frac{2d(1-q)}{q(1+c)(\tau+1)^2}\}2d\frac{R}{\tau+1}+\{\frac{2d(1-q)}{q(1+c)(\tau+1)^2}\}q+\{\frac{2d(1-q)}{q(1+c)(\tau+1)^2}\}cq-\{\frac{2d(1-q)}{q(1+c)(\tau+1)^2}\}2d\frac{R}{\tau+1}}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{\{\frac{2d(1-q)}{q(1+c)(\tau+1)^2}\}q(1+c)}{[2d\frac{R}{\tau+1}+q]^2} \\
&= \frac{\frac{2d(1-q)}{(\tau+1)^2}}{[2d\frac{R}{\tau+1}+q]^2} > 0
\end{aligned}$$

□

Proposition 2. *The risky holdings in the competitive equilibrium (n) are increasing in the return to the risky asset (R), and decreasing in the size of the liquidity shock (c), marginal cost of funds (d), and the probability of the bad state (q).*

Proof.

$$\frac{\partial n}{\partial c} = \frac{(-1+c)q-2(-1+(1+d+cd)R)}{2(1+c)^3 d} < 0, \text{ not obvious but clear after little algebra}$$

$$\frac{\partial n}{\partial d} = \frac{1 + cq - R}{2(1+c)^2 d^2} < 0 \text{ due to assumption } R - cq > 1$$

$$\frac{\partial n}{\partial R} = \frac{(-1+q+R)^2 - 2(1+c)d(cq^2 - (-1+R)^2 - q(-1+c+2R))}{2(1+c)^2 d(-1+q+R)^2} > 0$$

$$\begin{aligned}
\underbrace{(-1+q+R)^2}_{\text{positive}} - 2(1+c)d(cq^2 - (-1+R)^2 - q(-1+c+2R)) &= . \\
&= . \\
(-1+q+R)^2 - 2(1+c)d(cq\underbrace{(q-1)}_{\text{negative}} - (-1+R)^2 - \underbrace{q(-1+2R)}_{\text{positive}}) > 0
\end{aligned}$$

$$\frac{\partial n}{\partial q} = -\frac{2d(-1+R)R + 2c^2 d(-1+R)R + c(q^2 + 2q(-1+R) + (-1+R)(-1+R+4dR)}{2(1+c)^2 d(-1+q+R)^2} < 0$$

□

Proposition 3. *Banks decrease their liquidity ratio as the regulator tightens capital requirements, that is,* $b'(n) \geq 0$.

Proof. Using the banks' FOC above and plugging in the functional form for the cost function we can obtain b as an implicit function of n (note that P is a function of both b and n).

$$\begin{aligned}
(1-q) + q\frac{R}{P} &= 1 + 2dn(1+b) \\
-q + \frac{R}{P} &= 2dn(1+b) \\
-qP + qR &= P2dn(1+b) \\
q(R-P) &= [R + (b-c)n]2dn(1+b) \\
-q(b-c)n &= 2dn(1+b)R + 2dn(1+b)(b-c)n \\
-(b-c)[q + 2dn(1+b)] &= 2d(1+b)R
\end{aligned}$$

Now, take derivative of both sides with respect to n, and collect terms that involve $b'(n)$:

$$\begin{aligned}
-b'(n)[q + 2dn(1+b)] - 2d(b-c)[1 + b + nb'(n)] &= 2dRb'(n) \\
-b'(n)[q + 2dn(1+b)] - 2d(b-c)(1+b) - 2d(b-c)nb'(n) &= 2dRb'(n) \\
-b'(n)[2dR + 2dn(b-c) + q + 2dn(1+b)] &= 2d(b-c)(1+b) \\
-b'(n)[2dR + q + 2dn(2b+1-c)] &= 2d(b-c)(1+b)
\end{aligned}$$

From the last equation we obtain:

$$b'(n) = \frac{-2d(b-c)(1+b)}{2dR + q + 2dn(2b+1-c)} > 0 \tag{54}$$

The sign of $b'(n)$ is positive because $b - c < 0$. We can further simplify $b'(n)$ in order to obtain a more convenient expression to use in the following proofs. In order to do this note that first, from the equilibrium price function $P = R + (b-c)n$ we can obtain that

$$b - c = -\frac{R-P}{n}, \tag{55}$$

and second, from the banks' first order condition we can obtain:

$$\begin{aligned} 1 - q + qR\frac{1}{P} &= 1 + 2dn(1+b) \\ q\left(\frac{R}{P} - 1\right) &= 2dn(1+b) \\ 1 + b &= \frac{q}{2dn}\left(\frac{R}{P} - 1\right). \end{aligned} \tag{56}$$

Eventually, plug these values for $1+b$ and $b-c$ into (54) to obtain:

$$\begin{aligned} b'(n) &= \frac{-2d(-1)\frac{R-P}{n}\frac{q}{2dn}\left(\frac{R}{P}-1\right)}{2dR + q - 2d(R-P) + 2d\frac{q}{2d}\left(\frac{R}{P}-1\right)} \\ &= \frac{\frac{q}{n^2 P}(R-P)^2}{\frac{1}{P}[2dRP + qP - 2dP(R-P) + q(R-P)]} \\ &= \frac{q(R-P)^2}{n^2[2dRP + qP - 2dRP + 2dP^2 + qR - qP]} \\ &= \frac{q(R-P)^2}{n^2[2dP^2 + qR]} \end{aligned} \tag{57}$$

□

Proposition 4. *Risky investment levels, liquid asset holdings, and financial stability measures under competitive equilibrium (n, b, $1-\gamma$, P), partial regulation equilibrium (n^*, b^*, $1-\gamma^*$, P^*), and complete regulation equilibrium (n^{**}, b^{**}, $1-\gamma^{**}$, P^{**}) compare as follows:*

a) $n > n^{**} > n^*$

b) $b^{**} > b > b^*$

c) Financial stability measures

i) $1-\gamma > 1-\gamma^* > 1-\gamma^{**}$

ii) $(1-\gamma)n > (1-\gamma^*)n^* > (1-\gamma^{**})n^{**}$

iii) $P^{**} > P^* > P$

Proof. Proof of this proposition is established through a series of lemmas below. □

Lemma 6. $P^{**} > P^* > P$

Proof. **Part 1:** $P^{**} > P^*$. Note that, in the solution to the complete regulation case above, we have obtained

$$P^{**} = \sqrt{\frac{qR^2(1+c)}{R-1+q}} = \sqrt{\frac{\beta}{\sigma}} \tag{58}$$

where σ is defined by (39) and $\beta \equiv R(1+c)$. Start from the cubic equation obtained in the solution for the partial case that gives P^*. We repeat this cubic equation below for convenience:

$$2d\sigma P^{*3} + [qR\sigma - 2d\beta]P^* - q\beta = 0 \tag{59}$$

Divide this equation by σ to obtain:

$$2dP^{*3} + \left[qR - 2d\frac{\beta}{\sigma}\right] P^* - q\frac{\beta}{\sigma} = 0 \tag{60}$$

Note that $\beta/\sigma = P^{**2}$, and substitute this into the equation above and manipulate:

$$2dP^{*3} + \left[qR - 2dP^{**2}\right] P^* - qP^{**2} = 0 \tag{61}$$

$$2dP^{*3} + qRP^* - 2dP^{**2}P^* - qP^{**2} = 0 \tag{62}$$

$$(2dP^{*2} + qR)P^* = (2dP^* + q)P^{**2} \tag{63}$$

Multiply both sides of this equation by P^* to obtain:

$$(2dP^{*2} + qR)P^{*2} = (2dP^{*2} + qP)P^{**2} \tag{64}$$

From this last equivalence we can obtain the square of the price ratios in the two cases as:

$$\left(\frac{P^{**}}{P^*}\right)^2 = \frac{2dP^{*2} + qR}{2dP^{*2} + qP} > 1, \tag{65}$$

since we must have $R > P^*$ in equilibrium. Therefore, $P^{**} > P^*$.

Part 2: $P^* > P$. First, note that we obtained the competitive equilibrium price of assets in the main text as:

$$P = \frac{qR(1+c)}{R-1+q} = \frac{\beta}{R\sigma}, \tag{66}$$

using the definitions of σ, β as defined above. Now, take the cubic equation given by (59) and divide it $R\sigma$ to obtain:

$$\frac{2d}{R}P^{*3} + \left[q - 2d\frac{\beta}{R\sigma}\right]P^{*} - q\frac{\beta}{R\sigma} = 0 \tag{67}$$

Note that $\beta/R\sigma = P$, and substitute this into the equation above and manipulate:

$$\frac{2d}{R}P^{*3} + \left[q - 2dP\right]P^{*} - qP = 0 \tag{68}$$

$$\left(\frac{2d}{R}P^{*2} + q\right)P^{*} = (2dP^{*} + q)P \tag{69}$$

From this last equivalence we can obtain the price ratios in these two cases as:

$$\frac{P}{P^{*}} = \frac{\frac{2d}{R}P^{*2} + q}{2dP^{*} + q} = \frac{2dP^{*2} + qR}{2dRP^{*} + qR} < 1, \tag{70}$$

The last inequality holds because $P^{*2} < RP^{*}$, which is in turn true since we must have $P^{*} < R$ in equilibrium. Therefore, $P^{*} > P$. □

Lemma 7. $b^{**} > b > b^{*}$

Proof. **Part 1:** $b^{**} > b$. Note that the closed–form solutions for the liquidity ratios in these two cases were obtained above as:

$$b = \frac{cq - 2d\frac{R}{\tau+1}}{2d\frac{R}{\tau+1} + q} \tag{71}$$

$$b^{**} = \frac{cq - 2d\frac{R}{(\tau^{**}+1)(\tau^{**}+2)}}{2d\frac{R}{(\tau^{**}+1)(\tau^{**}+2)} + q} \tag{72}$$

where

$$\tau = \frac{R - 1 + q}{q(1+c)} - 1 = \eta^2 - 1 \tag{73}$$

$$\tau^{**} = \frac{R}{P^{**}} - 1 = \sqrt{\frac{R - 1 + q}{q(1+c)}} - 1 = \eta - 1 \tag{74}$$

Therefore, $(\tau^{**} + 1)(\tau^{**} + 2) = \eta(\eta + 1) > \tau + 1 = \eta^2 \implies b^{**} > b$.

Part 2: $b > b^*$. Start from first order conditions of banks' problem in the partial case:

$$\begin{aligned}
1 - q + \frac{qR}{P} &= 1 + 2dn(1 + b(n)) \\
-q + \frac{qR}{P} &= 2dn(1 + b(n)) \\
q(\frac{R}{P} - 1) &= 2d\frac{P\tau}{c - b(n)}(1 + b(n)) \\
q\tau &= 2d\frac{P\tau}{c - b(n)}(1 + b(n)) \\
\implies q(c - b(n)) &= 2dP(1 + b(n)) = 2d(1 + b(n))\frac{R}{\tau + 1} \\
\implies qc(\tau + 1) - qb(n)(\tau + 1) &= 2dR + 2db(n)R \implies qc(\tau + 1) - 2dR = b(n)\{2dR + q(\tau + 1)\} \\
\implies b(n) &= \frac{qc(\tau^* + 1) - 2dR}{2dR + q(\tau^* + 1)}
\end{aligned}$$

where we used $n = \frac{P\tau}{c - b(n)}$ and $P = \frac{R}{\tau+1}$.

The competitive equilibrium liquidity ratio (b) and $b(n)$ in the partial regulation case have the same functional form, the only difference is τ vs τ^*.

$$\frac{db}{d\tau} = \frac{qc[2dR + q(\tau + 1)] - q[qc(\tau + 1) - 2dR]}{[2dR + q(\tau + 1)]^2} = \frac{2dRq(1_c)}{[2dR + q(\tau + 1)]^2} > 0 \tag{75}$$

Therefore, $b > b^*$ for $\tau > \tau^*$. □

Lemma 8. *$n > n^{**} > n^*$*

Proof. **Part 1:** $n > n^{**}$. We will use $n^{**} = \frac{P^{**}\tau^{**}}{c-b}$ and $c - b^{**} = \frac{2dR(1+b^{**})}{q(\tau^{**}+1)(\tau^{**}+2)}$ which we derived as we solve for b^{**}.

$$n^{**} = R\frac{\tau^{**}}{\tau^{**} + 1}\frac{q(\tau^{**} + 1)(\tau^{**} + 2)}{2dR(1 + b^{**})} = \frac{q}{2d}\frac{\tau^{**}(\tau^{**} + 2)}{1 + b^{**}}$$

With similar algebra for the competitive equilibrium we can derive the following $n = \frac{q}{2d}\frac{\tau}{1+b}$.

$$\frac{\tau^{**}(\tau^{**} + 2)}{1 + b^{**}} = \frac{(\eta - 1)(\eta + 1)}{1 + b^{**}} = \frac{\eta^2 - 1}{1 + b^{**}} < \frac{\eta^2 - 1}{1 + b} = \frac{\tau + 1}{1 + b} \text{ since } b^{**} > b$$

Therefore, $n > n^{**}$.

Part 2: $n^{**} > n^*$. For the second part of this lemma, we will use the fact that $P^{**} > P^*$ as proven by Lemma 6 above. Take the equation 65 that gives the square of the price ratios in these two cases and replace $P = \frac{R}{\tau+1}$ to obtain:

$$\left(\frac{P^{**}}{P^{*}}\right)^2 = (\frac{\tau^*+1}{\tau^{**}+1})^2 \equiv \kappa = \frac{2dR+q(\tau^*+1)^2}{2dR+q(\tau^*+1)} \tag{76}$$

$n^{**} = \frac{\tau^{**}}{\tau^{**}+1}\frac{2dR+q(\tau^{**}+1)(\tau^{**}+2)}{2d(1+c)}$ and $n^{*} = \frac{\tau^{*}}{\tau^{*}+1}\frac{2dR+q(\tau^{*}+1)}{2d(1+c)}$

$$n^{**} - n^* = \frac{1}{2d(1+c)}\{\frac{\tau^{**}(\tau^*+1)[2dR+q(\tau^{**}+1)(\tau^{**}+2)] - \tau^*(\tau^{**}+1)[2dR+q(\tau^*+1)]}{2d(1+c)(\tau^*+1)(\tau^{**}+1)}\} \tag{77}$$

The numerator can be simplied as follows

$$2dR\underbrace{[\tau^{**}(\tau^*+1)-\tau^*(\tau^{**}+1)]}_{\tau^{**}-\tau^*} + q(\tau^*+1)(\tau^{**}+1)\underbrace{[\tau 2(\tau^{**}+2)-\tau^*]}_{(\tau^{**}+1)^2-(\tau^*+1)} \tag{78}$$

From 76 we have $(\tau^{**}+1)^2 = (\tau^{**}+1)^2\frac{2dR+q(\tau^*+1)}{2dR+q(\tau^*+1)^2}$, plugging this into the last part of numerator $(\tau^{**}+1)^2 - (\tau^*+1) = (\tau^*+1)\{(\tau^*+1)\frac{2dR+q(\tau^*+1)}{2dR+q(\tau^*+1)^2} - 1\} = \frac{(\tau^*+1)2dR\tau^*}{2dR+q(\tau^*+1)^2}$

Plugging back to 78 we get $2dR[\tau^{**}-\tau^*] + \frac{q(\tau^*)(\tau^{**}+1)(\tau^*+1)\tau^* 2dR}{2dR+q(\tau^*+1)^2}$

We want this equation to be positive, which requires

$$\begin{aligned}
\frac{2dRq\tau^*(\tau^*+1)^2(\tau^{**}+1)}{2dR+q(\tau^*+1)^2} &> 2dR(\tau^*-\tau^{**}) \\
\implies q\tau^*(\tau^*+1)^2(\tau^{**}+1) &> 2dR(\tau^*-\tau^{**}) + q(\tau^*-\tau^{**})(\tau^*+1)^2 \\
\implies (\tau^*+1)^2 q\underbrace{[\tau^*(\tau^{**}+1)-\tau^*+\tau^{**}]}_{=\tau^{**}(\tau^*+1)} &> 2dR(\tau^*-\tau^{**})
\end{aligned}$$

We want to show that $(\tau^*+1)^2 q\tau^{**}(\tau^*+1) > 2dR(\tau^*-\tau^{**})$ holds.

$(\tau^{**}+1)^2 = \eta^2 = \frac{2dR(\tau^*+1)^2+q(\tau^*+1)^3}{2dR+q(\tau^*+1)^2}$ and $\tau^{**} = \eta - 1 \implies \tau^{**}(\tau^{**}+2) = \eta^2 - 1 \implies \eta^2 = \tau^{**}(\tau^{**}+2)+1$

$$\begin{aligned}
\eta^2 = \tau^{**}(\tau^{**}+2)+1 &= \frac{2dR(\tau^*+1)^2 + q(\tau^*+1)^3}{2dR + q(\tau^*+1)^2} \\
2dR\eta^2 + q(\tau^*+1)^2\eta^2 &= 2dR(\tau^*+1)^3 \\
2dR[\eta^2 - (\tau^*+1)^2] &= q(\tau^*+1)^3 - q(\tau^*+1)^2\eta^2 = q(\tau^*+1)^2[\tau^*+1-\eta^2] \\
2dR &= \frac{q(\tau^*+1)^2[\tau^*+1-\eta^2]}{\eta^2-(\tau^*+1)^2}
\end{aligned}$$

$$\begin{aligned}
(\tau^*+1)^3 q\tau^{**} - 2dR(\tau^*-\tau^{**}) &= (\tau^*+1)^3 q\tau^{**} - 2dR[(\tau^*+1)-(\tau^{**}+1)] \\
&\implies \frac{\tau^*+1}{\tau^{**}+1}(\tau^*+1)^2 q\tau^{**} + 2dR - 2dR\frac{\tau^*+1}{\tau^{**}+1} \\
&= \frac{\tau^*+1}{\tau^{**}+1}[(\tau^*+1)^2 q\tau^{**} - 2dR] + 2dR
\end{aligned}$$

Let's focus on $q\tau^{**}(\tau^*+1)^2 - 2dR$ because the remaining term is positive

$$q\tau^{**}(\tau^*+1)^2 > 2dR = \frac{q(\tau^*+1)^2[\tau^*+1-\eta^2]}{\eta^2-(\tau^*+1)^2}$$

$$q(\tau^*+1)^2[\tau^{**} - \frac{\tau^*+1-\eta^2}{\eta^2-(\tau^*+1)^2}] = q(\tau^*+1)^2[\tau^{**} + \frac{\eta^2-(\tau^*+1)}{\eta^2-(\tau^*+1)^2}]$$

$$\implies q\tau^{**}(\tau^*+1)^2 - 2dR = q(\tau^*+1)^2[\eta - 1 + \frac{\eta^2-(\tau^*+1)}{\eta^2-(\tau^*+1)^2}]$$

$\frac{\eta^2-(\tau^*+1)}{\eta^2-(\tau^*+1)^2} > 1 \implies \eta - 1 + \frac{\eta^2-(\tau^*+1)}{\eta^2-(\tau^*+1)^2} > 0.$

□

Lemma 9. $1-\gamma > 1-\gamma^* > 1-\gamma^{**}$

Proof.

$$1-\gamma = \frac{c-b}{P} \text{ together with } b^{**} > b^* \text{ and } P^{**} > P^* \implies 1-\gamma^* > 1-\gamma^{**}$$

For $(1-\gamma) > (1-\gamma^*)$, or $\frac{\frac{c-b}{P}}{\frac{c-b^*}{P^*}} > 1$

Given $b = \frac{qc(\tau+1)-2dR}{2dR+q(\tau+1)} \implies c-b = \frac{2dR(1+c)}{2dR+q(\tau+1)}$, and similarly $c-b^* = \frac{2dR(1+c)}{2dR+q(\tau^*+1)}$

$$\frac{\frac{c-b}{P}}{\frac{c-b^*}{P^*}} = \frac{c-b}{c-b_1}\frac{P}{P^*} = \frac{2dP^*+q}{2dP+q}\frac{P}{P^*}\frac{P^*}{P} > 1$$

The last inequality is due to $P^* > P$.

□

Lemma 10. $(1-\gamma)n > (1-\gamma^*)n^* > (1-\gamma^{**})n^{**}$

Proof. This is about the total amount of fire sales. Given that the demand function for risky assets in the interim period is downward sloping (continuous and differentable as well), the prices will be informative about the amount of fire sales.

$$(1-\gamma)n = \tau = \frac{R}{P} - 1 \text{ and } P^{**} > P^* > P \implies (1-\gamma^{**})n^{**} < (1-\gamma^*)n^* < (1-\gamma)n$$

□